Consumption and Real Exchange Rates under Rational Inattention*

Wei Li†
Shanghai Advanced Institute of Finance

Yulei Luo‡
University of Hong Kong

Xiaowen Wang§
Liaoning University

February 26, 2026

Abstract

In this paper, we study households' joint consumption–portfolio choice under limited information-processing capacity (rational inattention or RI) in a two-country general equilibrium model with Epstein–Zin recursive utility. Optimally inattentive households adjust gradually to asset-return shocks and internalize the induced long-run consumption risk, which compresses the volatility of stochastic discount factors and lowers their cross-country comovement. Despite the high correlation of international equity returns, the model reproduces key international macro–finance facts: (i) substantially reduced correlation between relative consumption growth and real exchange rate depreciation (the Backus–Smith puzzle); (ii) smooth real exchange rate dynamics; and (iii) consumption growth that is smoother than real exchange rates.

JEL Classification Numbers: C61, D81, E21, F31.

Keywords: Rational Inattention, Long-run Consumption Risk, Consumption-Portfolio Rule, Real Exchange Rates.

---

*We are grateful to Charles Engel, Zhen Huo, Samuel Kortum, Kim Ruhl, Dan Cao, Xiaoji Lin, Fei Tan and Jieran Wu for their helpful comments and discussions. We also thank the seminar and conference participants at Fudan University, Shandong University, Liaoning University, the SHUFE Summer Workshop in International Economics, the Shanghai Macroeconomics Workshop, the International Symposium on Financial Openness and Liberalization in Emerging Economies in Shanghai, the China Meeting of the Econometric Society, the Econometric Society Australasian Meeting, and the IAER Macroeconomics Workshop in Dalian. Yunxiao Zhao provided excellent research assistance.

†Shanghai Advanced Institute of Finance. E-mail: veraliwei@gmail.com.

‡Faculty of Business and Economics, The University of Hong Kong, Hong Kong. E-mail: yulei.luo@gmail.com.

§China Economic Research Institute, Liaoning University, China. E-mail: wang.xiaowen47@gmail.com.

1 Introduction

Exchange-rate dynamics are central to international finance, yet canonical full-information rational expectations (FIRE) models with complete risk sharing deliver predictions that are sharply at odds with the data. Under the standard FIRE framework, the real exchange rate (RER) should move closely with cross-country relative consumption growth; empirically, however, correlations between relative consumption growth and the RER are near zero or negative. Backus and Smith (1993) highlight this discrepancy—now known as the Backus–Smith puzzle—and a broader "exchange-rate disconnect" literature (Meese and Rogoff 1993; Kollmann 1995; Obstfeld and Rogoff 2001) documents weak comovement between RERs and other macroeconomic aggregates. Standard two-country FIRE models also struggle to reconcile the low volatility of RERs and the limited covariation of consumption growth that would be required to account for international relative price movements (Brandt et al. 2006; Colacito and Croce 2011). These shortcomings are central to what is often termed the "international asset pricing puzzle."

Conventional consumption-based asset pricing with expected utility (EU) fails to explain the exchange rate disconnect puzzle, as well as the low volatilities of the exchange rate. Given the high observed risk premia in advanced economies, marginal utility growth must be very volatile. In standard EU models with power utility, this implies that marginal utility growth must also be highly correlated across countries to satisfy the cross-country Euler equations linking the RER to domestic and foreign marginal utilities. The Backus–Smith condition then predicts that relative consumption growth should be strongly and positively correlated with the RER (so that relative consumption is high when the relative price is low), a prediction that is starkly contradicted by the data. Moreover, focusing only on contemporaneous consumption growth when pricing risky assets can be misleading if consumption adjusts gradually in reality. One manifestation of this is the stock-market participation puzzle: observed equity holdings are much lower than implied by a standard Merton (1969) portfolio choice problem calibrated to the equity premium and a plausible degree of risk aversion. Mankiw and Zeldes (1991) show that only a small fraction of households participate in the stock market. Using capital income and labor share data, we estimate that the equity share of total wealth is 0.26 in the U.S. and 0.31 in the U.K. over our sample period, whereas a standard Merton model with complete participation would imply substantially higher equity shares.

A large literature has proposed mechanisms to account for international finance puzzles, including various forms of financial frictions and noise traders (e.g., Itskhoki and Mukhin 2011). However, these frictions are typically introduced in reduced form and are not tightly disciplined by micro-level evidence. At the same time, a substantial body of survey evidence points to pervasive information rigidities for households, firms, and professional forecasters. Coibion and

Gorodnichenko (2015), using arguably the most "model-free" approach to measuring information rigidity, find that sluggish updating of expectations is both quantitatively large and consistent across a wide range of survey and forecast data.

This paper shows that incorporating rational inattention (RI) into a standard two-country consumption–portfolio framework with recursive utility (RU) can reconcile these empirical regularities. RI introduces endogenous limits to information processing, leading agents to adjust their perceptions of wealth and consumption only gradually. Recursive utility allows for a separation between risk aversion and the intertemporal elasticity of substitution, making the timing of the resolution of uncertainty economically relevant—both features are crucial for our mechanism. For tractability, we assume complete home bias in asset holdings but imperfect international risk sharing. Because the representative agent with RU cares about the entire future path of consumption, this environment is well suited to studying the implications of RI: limited information-processing capacity induces slow, hump-shaped adjustments of consumption to shocks. When RI is embedded in a two-country model with complete home bias, the RI-induced long-run component of consumption risk lowers the optimal share of risky assets and thereby reduces the volatility of consumption growth and of the stochastic discount factor (SDF). At the same time, RI dampens the response of consumption to equity return shocks, lowering RER volatility and weakening the correlation between contemporaneous relative consumption growth and the RER. In this way, RI provides a unified explanation for both the exchange-rate disconnect and the Backus–Smith puzzle.

Our model resolves these two puzzles through related but distinct channels. For the "exchange rates too smooth" puzzle, the mechanism operates through portfolio choice. Under RI, households recognize that consumption adjusts only gradually to wealth shocks, which generates long-run consumption risk. With recursive utility and a preference for early resolution of uncertainty, this long-run risk is strongly priced, inducing households to reduce their exposure to risky assets. Lower holdings of risky assets translate into smoother consumption growth paths, which reduce the volatility of SDFs and hence of real exchange rates. For the Backus–Smith puzzle, the key is a timing mismatch: portfolio returns respond immediately to equity shocks, whereas consumption adjusts gradually as agents filter information about permanent and transitory components of wealth. Under recursive utility, the RER depends not only on relative consumption growth but also on relative portfolio returns. The decoupling of the timing of portfolio returns and consumption responses therefore breaks the tight contemporaneous link between Δe and $\Delta c - \Delta c^{*}$, allowing their correlation to fall well below one, in line with the data.

We further disentangle the different components of RI that shape consumption dynamics and RER behavior. The impact of RI can be decomposed into two forces: (i) a dampened contemporaneous impulse response of consumption to shocks and (ii) more gradual and persistent consumption adjustments. The first effect reduces the immediate sensitivity of consumption to equity return innovations; holding primitive risk aversion fixed, this weakens the consumption-smoothing benefit of risky assets and lowers the optimal risky share. The second effect—RI-induced long-run consumption risk—arises because agents internalize the persistent consequences of shocks for future consumption growth. This leads to an increased effective coefficient of risk aversion. Under RU, long-run risk is especially costly when agents prefer early resolution of uncertainty ($\gamma > 1/\psi$), which raises the effective price of risk. Both RU and RI-induced long-run risk are necessary to jointly match the facts. Without RU, only current consumption matters for RER determination, and the Backus–Smith puzzle remains unresolved. With RI but without RU, the primary effect is a change in the mapping from shocks to consumption, which by itself tends to increase the effective exposure to risky assets and hence the volatility of consumption growth and SDFs. In contrast, the combination of RI and RU allows us to generate low and weakly correlated consumption growth, realistic equity holdings, and smooth yet informative RER dynamics.

We complement our theoretical analysis with direct empirical evidence on information frictions. Using professional forecasters' inflation expectations for the U.S. and the U.K, we adopt the methodology of Coibion and Gorodnichenko (2015) to infer the degree of information rigidity. By relating ex post mean forecast errors to revisions in the cross-sectional mean forecast, we obtain a coefficient on forecast revisions that maps directly into the underlying degree of inattentiveness. The estimated rigidity is substantial in both countries and broadly consistent with the degree of rational inattention implied by our model calibration.

In the quantitative analysis, we jointly estimate the degree of RI, the coefficient of relative risk aversion (γ), and the intertemporal elasticity of substitution (ψ) to match key moments of the joint distribution of consumption, asset holdings, and real exchange rates. Our baseline calibration yields a moderate risk aversion coefficient of 1.061 and an intertemporal elasticity of substitution of 0.989, implying a preference for early resolution of uncertainty ($\gamma > 1/\psi$). Allowing information-processing constraints to differ across countries, we find that the degree of information rigidity is higher in the U.S. than in the U.K. These differences help account for the relatively low equity shares we infer for each country, as well as the observed exchange-rate disconnect and the low cross-country correlation of consumption growth.

Related Literature Our paper is related to two main strands of the literature. First, it contributes to research on uncertainty-induced, endogenous long-run risk and its asset-pricing implications. Luo and Young (2016) introduce state uncertainty arising from RI in a closed economy and study how RI-induced long-run consumption risk affects portfolio choice. Bidder and Dew-Becker (2016) show that when the investor is uncertain about the data-generating process for consumption and adopts a pessimistic model, the resulting asset-pricing model under model uncertainty necessarily features long-run risk. Our paper builds on this notion of RI-induced long-run risk and is, to our knowledge, the first to apply it to exchange rate anomalies.

A large literature argues that long-run risk is a more relevant measure of fundamental risk in asset markets. Hansen, Heaton, and Li (2008), Parker (2001, 2003), and Parker and Julliard (2005) emphasize the importance of low-frequency consumption risk for explaining equity premia. In particular, Parker (2003) finds that the correlation between 12-quarters-ahead long-run marginal utility and stock returns is maximized, suggesting that the long-run Euler equation is the appropriate benchmark for determining risk premia. In line with this argument, the variance of the short-run stochastic discount factor (SDF) in our model can be substantially smaller than what is implied by the mean–standard-deviation ratio of the risk premium, i.e., the Hansen–Jagannathan bound. By generating low short-run SDF volatility in equilibrium, our framework offers a novel perspective on the "exchange rates are too smooth" puzzle (Brandt et al., 2006). In this respect, it complements existing work that seeks to rationalize high cross-country correlations of SDFs (Colacito and Croce, 2011; Chien et al., 2020). Colacito and Croce (2011), for example, employ an exogenous long-run risk specification with highly persistent and highly correlated long-run consumption components across countries; both features are critical in their model for producing highly correlated SDFs. In contrast, our approach relies on relatively low volatilities of marginal utilities rather than high cross-country correlation. In our setting, long-run risk emerges endogenously from incomplete information and gives rise to smoother short-run SDFs.

Second, our paper contributes to the extensive literature on international macroeconomic puzzles, including Backus and Smith (1993), Obstfeld and Rogoff (2000), Gourinchas and Tornell (2004), Djeutem and Kasa (2013), Cao et al. (2020), Itskhoki and Mukhin (2021), and Sandulescu et al. (2021). We introduce RI and the associated endogenous long-run risk as an alternative mechanism for addressing exchange-rate-related puzzles. Our work is particularly close to Colacito and Croce (2011) and Sandulescu et al. (2021). Colacito and Croce (2011) propose exogenous long-run risk as a solution to the international equity premium puzzle and, as noted, rely on a high cross-country correlation of SDFs. Similarly, Brandt et al. (2006) also depend on highly correlated international SDFs to reconcile low variation in international relative prices with weak comovement in macroeconomic aggregates. By contrast, and more in line with Trojani and Vedolin (2020), our model delivers moderate levels of SDF volatility and international comovement, with the key mechanism operating through information frictions rather than extreme cross-country synchronization of risks. Our main contribution is to incorporate micro evidence on information frictions into an international asset-pricing framework and to show that RI-induced long-run risk can help resolve several exchange rate puzzles while remaining consistent with plausible levels of SDF volatility and international comovement.

The paper is organized as follows. Section 2 presents a canonical FIRE model and provides the model's key predictions on the joint dynamics of consumption growth and real exchange rates. Section 3 introduces RI into the otherwise standard model and examines the effects of RI on consumption, portfolio choices, and real exchange rates via the long-run consumption risk. Section 4 presents the calibration and estimation results and examines to what extent RI can help explain the empirical evidence. Section 5 provides some discussions on the effects of inattentiveness (another type of cognitive limitation) and stochastic volatility. Section 6 concludes.

2 A Full-Information Rational Expectations (FIRE) Model of Consumption and Real Exchange Rates

As noted in Backus et al. (2000), Brandt et al. (2006), and Colacito and Croce (2011), real exchange rates are linked to domestic and foreign marginal utility growth (or stochastic discount factors). Specifically, with no arbitrage opportunities, the growth of the logarithm real exchange rate would be equal to the difference of the logarithm pricing kernels for foreign currency- and domestic currency-denominated assets:

$$\Delta e_{t+1} = m_{t+1}^{*} - m_{t+1}. \tag{1}$$

We take the U.S. as the home country and define the nominal exchange rate, E, as U.S. dollars per unit of foreign currency. The real exchange rate is then given by: $e = E \cdot P^{*}/P$, where P and P^{*} denote the domestic and foreign price levels, respectively. Thus, the real exchange rate (RER) measures the units of domestic goods required to obtain one unit of foreign goods, and an increase in corresponds to a depreciation of the home country's RER. According to Equation (1), the real exchange rate is pinned down by the differential in pricing kernels across countries, and the volatility of its growth rate is governed by the volatilities of the stochastic discount factors (SDFs) in the home and foreign economies, as well as their correlation.

In this section, we solve a standard intertemporal consumption–portfolio choice problem separately for each country, using the log-linear approximation methods of Campbell (1993), Viceira (2001), and Campbell and Viceira (2002). We then use the resulting optimal consumption and portfolio policies to derive the SDFs and the implied international moments. Building on this benchmark model, the next section incorporates rational inattention (RI) into the standard framework and solves for optimal portfolio and consumption choices after taking endogenous long-run consumption risk into account. We then characterize the RI-implied SDFs and other key moments for real exchange rates and consumption growth.

The two countries are assumed to be symmetric and endowed with identical Epstein–Zin recursive preferences in our model economy. There are two country-specific goods, and agents exhibit complete home bias, consuming only domestically produced goods.[1] The model further assumes no trade in goods or assets across borders; in equilibrium, agents hold only domestic assets. Nonetheless, foreign and domestic equity returns are positively correlated. While complete home bias simplifies the environment, the correlation of cross-country equity returns captures deeper economic linkages, such as synchronized business cycles and trade-related shocks. This modeling strategy is consistent with Dumas et al. (2003), who attribute stock return correlations to common components in national output, and with Lewis and Liu (2015), who show that cross-country correlations in the persistent component of asset returns enhance international risk sharing. Hence, even under complete home bias, correlated equity returns provide an implicit channel for risk sharing across countries.

We assume that the utility function for the home representative consumer over a nondurable consumption good C_t takes the following recursive form:

$$U_t = \left\{ (1-\delta) C_t^{1-1/\psi} + \delta \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{(1-1/\psi)/(1-\gamma)} \right\}^{1/(1-1/\psi)}, \tag{2}$$

where γ is the coefficient of relative risk aversion (CRRA), ψ is the intertemporal elasticity of substitution (IES), and δ is the subjective discount rate. The agent's wealth evolves according to the random process:

$$A_{t+1} = (A_t - C_t) R_{p,t+1}, \tag{3}$$

where the market portfolio $R_{p,t+1}$ consists of a fraction of α share of wealth invested in a risky asset with gross return $R_{e,t}$ and a fraction of $1-\alpha$ invested in the riskless asset with gross return R_f. Denote $r_{e,t} = \ln(R_{e,t})$ and $r_f = \ln(R_f)$. As in Campbell (1993), the logarithm of the portfolio return can be written as:

$$r_{p,t+1} = \alpha(r_{e,t+1} - r_f) + r_f + \frac{1}{2}\alpha(1-\alpha)\omega^2. \tag{4}$$

In this model economy, for tractability, we do not model non-tradable labor income explicitly, and implicitly assume that the labor income can be spanned by marketable assets. As a result, the financial markets in our economy are complete. In addition, as argued in Merton (1971), the two-asset model (namely the risky market portfolio and the risk-free asset) can be used to achieve complete securities markets by dynamically and frictionlessly trading the two financial assets.

Given the above model specification, it is well known that this simple discrete-time model cannot be solved analytically. We then employ a log-linearization method as in Campbell (1993),

[1] As verified by our benchmark results in this section and by Chien et al. (2020), home bias by itself cannot resolve the Backus–Smith puzzle.

Viceira (2001), and Campbell and Viceira (2002) to obtain a closed-form solution to the problem. Specifically, we log-linearize the budget constraint around the unconditional mean of the log consumption-wealth ratio ($c - a = \mathbb{E}[c_t - a_t]$) to obtain

$$\Delta a_{t+1} = \left(1 - \frac{1}{\phi}\right)(c_t - a_t) + \Lambda + r^p_{t+1}, \tag{5}$$

where $\phi = 1 - \exp(c - a)$ and $\Lambda = \log(\phi) - (1 - 1/\phi)\log(1 - \phi)$. By applying the homogeneity of the planning problem (2) as in Epstein and Zin (1991), the optimal consumption and portfolio choice can be solved as:

$$c_t = b_0 + a_t, \tag{6}$$

$$\alpha = \frac{\mu - r_f + 0.5\omega^2}{\gamma\omega^2}, \tag{7}$$

where $b_0 = -\log\left(1 + \delta^{\psi}\left(\mathbb{E}_t\left[\phi_{t+1}R_{p,t+1}\right]^{1-\gamma}\right)^{1/\rho}\right)$, $\rho = (1-\gamma)/(1-1/\psi)$, and ϕ_{t+1} is constant when ψ is close to 1. The log risky return is assumed to be iid: $r_{e,t+1} = \mu + u_{t+1}$, $u_{t+1} \sim \mathbb{N}(0, \omega^2)$. Using (5), the consumption growth is also iid:

$$\Delta c_{t+1} = \alpha u_{t+1}. \tag{8}$$

The equation above implies that, under FIRE with only equity return shocks, consumption growth absorbs a constant fraction of stock market innovations and follows a random walk. Optimal investor behavior keeps the consumption–wealth ratio constant. Given the portfolio rule in Equation (7), we can compare the model's implications to U.S. data. Two points are worth emphasizing: (i) the relative risk aversion parameter must be as high as 13.2 to simultaneously match a 6.8% equity risk premium and a relatively low risky asset share of 26%, and (ii) the impulse response of consumption to an equity return shock is immediate and complete, failing to replicate the hump-shaped consumption responses observed empirically. Because the optimal consumption rule is a constant fraction of total wealth and returns are iid, the log-linear approximation is exact in this setting.

We now examine how the real exchange rate is determined by consumption growth and asset returns. As shown by Epstein and Zin (1991), under recursive preferences the logarithm of the stochastic discount factor (pricing kernel), m_{t+1}, can be expressed as a linear combination of log consumption growth, Δc_{t+1}, and the log return on the aggregate consumption claim (or market portfolio), $r_{p,t+1}$, in an FIRE environment:

$$m_{t+1} = \frac{1-\gamma}{1-1/\psi}\log\beta - \frac{1-\gamma}{\psi-1}\Delta c_{t+1} + \frac{1/\psi-\gamma}{1-1/\psi}r_{p,t+1}. \tag{9}$$

The corresponding expression for the foreign economy is analogous, with all variables indexed by an asterisk ($*$). Let ρ_u denote the correlation between foreign and domestic equity return innovations, $\rho_u = \mathrm{Corr}(u_{t+1}, u^*_{t+1})$. With recursive utility, the stochastic discount factor is thus a weighted average of shocks to consumption growth and to the market return.[2]

Assuming the two countries are symmetric and share identical preference parameters, the change in the real exchange rate, Δe_{t+1}, is driven by cross-country differences in consumption growth and market returns:

$$\Delta e_{t+1} = -\frac{1-\gamma}{\psi - 1}\left(\Delta c^*_{t+1} - \Delta c_{t+1}\right) + \frac{1/\psi - \gamma}{1 - 1/\psi}\left(r^*_{p,t+1} - r_{p,t+1}\right). \tag{10}$$

The decomposition on the right-hand side of the above equation allows us to characterize real depreciation dynamics using both goods-market and asset-market equilibrium conditions. In the special case where $\gamma = 1/\psi$ —that is, under time-additive expected utility—only the differential in current consumption growth matters. In this case, real exchange rate movements are perfectly negatively correlated with relative consumption growth. When $\gamma \neq 1/\psi$, a positive shock to the domestic portfolio return raises current consumption but also increases the home discount rate, leading to an appreciation of the home currency. The dynamics of the real exchange rate can therefore be derived from the endogenous consumption process implied by the underlying consumption–portfolio choice problem. Using these results together with Equation (1), we can compute: (i) the correlation between real exchange rate (RER) growth and the consumption growth differential, and (ii) the volatility of RER growth, as follows:

$$\mathrm{Corr}(\Delta e, \Delta c - \Delta c^*) = 1, \tag{11}$$

$$\mathrm{Var}(\Delta e) = \gamma^2(\alpha^2\omega^2 + \alpha^{*2}\omega^{*2} - 2\alpha\alpha^*\rho_u\omega\omega^*). \tag{12}$$

Equation (11) implies that real exchange rate growth and the cross-country differential in consumption growth are perfectly correlated—a stark contrast with the data. Table 1 reports international evidence for four major bilateral real exchange rates, taking the U.S. as the home country.[3] The sample spans January 19911 to December 2018. Empirically, the correlations between RER growth and consumption growth differentials are close to zero for all country pairs.

At the same time, the model-implied standard deviation of RER growth from Equation (12) is far too large. Given typical values of the equity premium, the implied annual variance of the stochastic discount factors (SDFs) in each country exceeds 33%. Matching an annualized RER

[2]This representation can be derived by log-linearizing around a constant consumption–wealth ratio and can also be interpreted in terms of revisions to expected aggregate consumption growth. See Campbell (2017) for a textbook treatment of Epstein–Zin preferences and stochastic discount factors.

[3]The parameter values used in generating the model's predictions are reported in Table 3.

growth volatility of roughly 10% therefore requires an implausibly high correlation between domestic and foreign SDFs. Yet, as shown in Table 1, the observed standard deviations of RER growth are only about 10% for the U.S.–U.K., U.S.–euro area, U.S.–Canada, and U.S.–Japan pairs. Most workhorse open-economy asset-pricing models address this by assuming SDFs that are nearly perfectly correlated across countries, but this is difficult to reconcile with the data: cross-country correlations in macroeconomic fundamentals are modest. For example, the correlation of per capita real consumption growth between the U.S. and the U.K. is 0.52, and effectively zero between the U.S. and Japan.

In summary, a standard FIRE model generates three central implications that conflict with the joint behavior of consumption growth, asset returns, and RERs:

1. Consumption growth is not as smooth as in the data.
2. The correlation between the consumption growth differential and the asset return differential is too high.
3. Households optimally hold too large a share of risky assets relative to the data, which further amplifies consumption volatility.

Taken together, these features produce excessively strong correlations between RER changes and consumption differentials, overly volatile SDFs, and hence too volatile RER dynamics.

Moreover, the standard Euler equation for an Epstein–Zin investor links the SDF to both contemporaneous consumption growth and anticipated future utility, the latter being expressible as revisions in expected future consumption growth. This perspective underlies the exogenous long-run risk literature, which accounts for the high Sharpe ratio of equity returns, the low risk-free rate, and key international macro-finance moments. In the next section, we analyze the consumption–portfolio choice problem of an RI agent taking long-run consumption risk into account. The RI agent updates beliefs about an imperfectly observed persistent component of consumption growth using a Kalman filter, which generates consumption dynamics with a persistent component akin to the exogenous long-run risk model of Bansal and Yaron (2004). Within this micro-founded framework, we can then study how rational inattention affects stock market participation, the equity premium, RER behavior, and broader international comovements.

3 Incorporating Rational Inattention due to Limited Information-Processing Capacity

Bansal and Yaron (2004) argue that long-run fluctuations in expected consumption growth provide a key channel for quantitatively accounting for a wide range of asset-pricing phenomena.

Subsequent work—Hansen et al. (2008), Parker (2001, 2003), and Parker and Julliard (2005)—shows that long-run risk better captures the true risk of the equity market, since the true consumption risk of a risky asset is its long-term impact on consumption. Luo (2010) and Luo and Young (2016) solve the optimal portfolio choice problem and demonstrate that the recursive preferences (RU) framework amplifies long-run risk and reduces the share invested in risky assets. In the RI model, long-run consumption risk naturally measures stock-market risk because consumption responds with lags. We show that this micro-founded framework can be used to examine both the international equity premium puzzle and the consumption–real-exchange-rate puzzle.

Specifically, following Parker (2001, 2003), we define long-run consumption risk as the covariance of asset returns and consumption growth over the period of the return and many subsequent periods. Furthermore, we apply the long-run Euler equation, shown below, to derive the optimal asset allocation and consumption rule:

$$\mathbb{E}_t \left[R_f^s \left(\frac{C_{t+1+s}}{C_t} \right)^{-\frac{1-\gamma}{\psi-1}} \left(R_{p,t+1} R_{p,t+2} \cdots R_{p,t+1+s} \right)^{\frac{1/\psi-\gamma}{1-1/\psi}} \left(R_{e,t+1} - R_f \right) \right] = 0, \tag{13}$$

where $R_{p,t+1+s}$ represents the cumulative return over future periods.

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above model. Under RI, agents have only finite Shannon channel capacity available to observe the true state of the world. Specifically, we assume that RI agents are unable to perfectly observe the current wealth level and instead receive a noisy signal of the true state a_{t+1}, as defined by the following equation:

$$\widetilde{a}_{t+1} = a_{t+1} + \xi_{t+1}, \tag{14}$$

where ξ_{t+1} is an observational noise. Following Sims (2003), we proceed in two steps. First, given the limited processing capacity (defined by Shannon's entropy), we calculate the minimum conditional variance of the state. Second, we use a Kalman filter to update the agent's perceived state and infer the minimum variance of the noise, $\text{Var}\left[\xi_{t+1}\right]$. The optimal posterior distribution for a_{t+1} by this approach is a Gaussian random variable:

$$a_{t+1} | \mathcal{I}_{t+1} \sim N \left(\widehat{a}_{t+1}, \Sigma_{t+1} \right), \tag{15}$$

where $\widehat{a}_{t+1} = \mathbb{E}\left[a_{t+1} | \mathcal{I}_{t+1}\right]$ and $\Sigma_{t+1} = \text{Var}\left(a_{t+1} | \mathcal{I}_{t+1}\right)$ are the conditional mean and variance of a_{t+1}, respectively. Denote $\Psi_t = \text{Var}_t(a_{t+1})$. The information constraint can be reduced to

$$\ln |\Psi_t| - \ln |\Sigma_{t+1}| \leq 2\kappa, \tag{16}$$

where κ is the investor's information channel capacity.[4]

[4]Since our model is univariate, we do not need to discuss optimal attention allocation problem.

The intertemporal budget constraint (5) then implies that

$$\mathbb{E}_t[a_{t+1}] = \mathbb{E}_t[r_{p,t+1}] + \Lambda + \widehat{a}_t, \tag{17}$$

and

$$\mathrm{Var}_t\,(a_{t+1}) = \mathrm{Var}_t(r_{p,t+1}) + \left(\frac{1}{\phi}\right)^2 \Sigma_t, \tag{18}$$

which establishes that the minimum conditional variance of the state $\Sigma = \mathrm{Var}_t(r_{p,t+1}) \,/\, \left[1 - \left(\frac{1}{\phi}\right)^2\right]$, where $\mathrm{Var}_t(r_{p,t+1}) = \alpha_{RI}^2 \omega^2$. Using the standard Kalman filtering technique, the perceived state can be written as:

$$\widehat{a}_{t+1} = (1-\theta)\left[\frac{1}{\phi}\widehat{a}_t + \left(1 - \frac{1}{\phi}\right)c_t + \Lambda + \mathbb{E}_t\left[r^p_{t+1}\right]\right] + \theta\widetilde{a}_{t+1}, \tag{19}$$

where $\theta = \Sigma\Delta^{-1}$ is the Kalman gain or observation weight on the noisy signal indicating how much uncertainty can be removed and is directly linked to channel capacity, Λ is the steady-sate value of the conditional variance of the noise, respectively. Combining with agent's information processing constraint, we have the Kalman gain $\theta = 1 - 1/\exp(2\kappa)$. As θ decreases from 1 to 0, the degree of rational inattention increases, indicating a lower capacity devoted to processing information. Then the minimum variance of the noise, $\mathrm{Var}\,(\xi_{t+1})$, is also pinned down.[5] According to the certainty equivalence principle, the optimal consumption decision under state uncertainty can be derived as:

$$c_t^{RI} = b_0 + \widehat{a}_t. \tag{20}$$

Household consumption remains a constant fraction of perceived wealth. Substituting the true state evolution Equation (5) into the definition of $\widetilde{a}_{t+1}$, the state transition equation for perceived state (19) now can be be modified accordingly:

$$\widehat{a}_{t+1} = \frac{1}{\phi}\widehat{a}_t + \left(1 - \frac{1}{\phi}\right)c_t + \Lambda + \mathbb{E}_t\left[r^p_{t+1}\right] + \eta_{t+1},$$

where $\eta_{t+1} = (\theta/\phi)\,(a_t - \widehat{a}_t) + \theta\,(\alpha_{RI} u_{t+1} + \xi_{t+1})$ is a weighted average of past observation errors, equity return risks, and RI-induced noises. For the optimal portfolio choice, by transforming Equation (13) and taking logs, we have

$$\mu - r_f + 0.5\omega^2 = -\,\mathrm{Cov}_{\,t}(m^s_{t+1}, r_{e,t+1}),$$

[5] For more details on the RI approach, see Luo and Young (2016).

and subsequently, as shown in Luo and Young (2016) and Online Appendix A, the ultimate consumption risk under RI can be expressed as:

$$
\begin{aligned}
-\operatorname{Cov}_t(m^s_{t+1}, r_{e,t+1}) &= -\operatorname{Cov}_t\left(-\frac{1-\gamma}{\psi-1}\sum_{j=0}^{s}\Delta c_{t+1+j} + \frac{1/\psi-\gamma}{1-1/\psi}\sum_{j=0}^{s} r_{p,t+1+j}, r_{e,t+1}\right) \\
&= \frac{\theta\,(1-\gamma)}{\psi-1}\left[1+\frac{1-\theta}{\phi}+\cdots+\left(\frac{1-\theta}{\phi}\right)^s\right]\cdot\alpha_{RI}\omega^2 - \frac{1/\psi-\gamma}{1-1/\psi}\cdot\alpha_{RI}\omega^2.
\end{aligned}
$$

When $s\to\infty$, this expression reduces to:

$$
\lim_{s\to\infty} -\operatorname{Cov}_t\left(-\frac{1-\gamma}{\psi-1}\sum_{j=0}^{s}\Delta c_{t+1+j} + \frac{1/\psi-\gamma}{1-1/\psi}\sum_{j=0}^{s} r_{p,t+1+j}, r_{e,t+1}\right) = \left(\frac{1-\gamma}{\psi-1}\zeta - \frac{1/\psi-\gamma}{1-1/\psi}\right)\alpha_{RI}\omega^2,
$$

where $\zeta = \frac{\theta}{1-(1-\theta)/\phi} > 1$, and $\phi = 1-\exp(c-a)$ is the steady state saving rate.

When confronted with state uncertainty, the agent updates her perceived state using a Kalman filter and adjusts her consumption gradually to equity return shocks. Following the same procedure in Luo and Young (2016), we can obtain the explicit expressions for the optimal share invested in the risky asset and the dynamics of consumption under RI when taking the long-run consumption risk into account. The following proposition summarizes the main results:

Proposition 1 *The optimal portfolio and consumption rule under RI can be written as:*

$$
\alpha_{RI} = \left(\frac{1-\gamma}{\psi-1}\zeta + \frac{\gamma-1/\psi}{1-1/\psi}\right)^{-1}\frac{\mu-r_f+0.5\omega^2}{\omega^2}, \tag{21}
$$

$$
\Delta c^{RI}_{t+1} = \theta\left[\frac{\alpha_{RI}u_{t+1}}{1-((1-\theta)/\phi)\cdot\mathbb{L}} + \left(\xi_{t+1} - \frac{(\theta/\phi)\,\xi_t}{1-((1-\theta)/\phi)\cdot\mathbb{L}}\right)\right]. \tag{22}
$$

And the portfolio choice (21) can be further decomposed into:

$$
\alpha_{RI} = \left(\frac{\gamma-1}{1-\psi}\cdot\underbrace{\theta}_{\text{Direct RI}}\cdot\underbrace{\frac{1}{1-(1-\theta)/\phi}}_{\text{Induced LRR}} + \underbrace{\frac{\gamma-1/\psi}{1-1/\psi}}_{\text{EarlyResolution}}\right)^{-1}\frac{\mu-r_f+0.5\omega^2}{\omega^2}. \tag{23}
$$

Proof. *See Online Appendix A.* ■

The equation above indicates that three forces shape the inattentive agent's portfolio choice: (i) the direct effect of inattention, (ii) the indirect effect of inattention through long-run risk, and (iii) preferences for early resolution of uncertainty. First, the degree of rational inattention, θ, reduces effective risk aversion. As the extent of rational inattention increases (i.e., as θ decreases), the optimal share invested in the risky asset rises. Second, inattention affects portfolio choice through long-run consumption risk. Because agents are averse to fluctuations in expected consumption growth, they demand a lower share of risky assets when long-run risk is more pronounced. Third, the separation of the intertemporal elasticity of substitution (ψ) and risk aversion (γ). Under a preference for early resolution of uncertainty ($\gamma > 1/\psi$), a higher elasticity of substitution raises the optimal allocation to the risky asset, as agents are more willing to tolerate intertemporal variation in consumption. The upper-left panel of Figure 1 depicts how the domestic portfolio share α varies with θ. Overall, when the indirect effect operating through induced long-run risk dominates the direct effect under recursive preferences, households' portfolio holdings of the risky asset are substantially reduced.

Endogenous long-run consumption growth. To better compare our inattention-induced consumption process with that of the exogenous long-run risk model, we can rewrite Equation (22) into a state-space representation:[6]

$$\Delta c_t = x_{t-1} + \varepsilon_{c,t}, \tag{24}$$

$$x_t = \rho_x x_{t-1} + \varepsilon_{x,t}, \tag{25}$$

where $\varepsilon_{c,t} = \theta\left(\alpha_{RI} u_t + \xi_t\right)$, $\varepsilon_{x,t} = \theta\left[\left((1-\theta)/\phi\right)\alpha_{RI} u_t - \left(\theta/\phi\right)\xi_t\right]$. In an RE model, when $\theta = 1$ and no endogenous noise is present, log consumption follows a random walk process. However, with rational inattention, there is a stochastic trend in the log consumption growth process, and the persistence $\rho_x = (1-\theta)/\phi$ increases with the degree of rational inattention. Note that the persistence of the long-run risk in consumption also rises with the inverse of steady-state saving rate, $1/\phi$, as consumption growth inherits the growth rate of perceived wealth level in Equation (19).

Assuming that each agent is subject to an idiosyncratic observational noise, by the law of large numbers, these idiosyncratic noises cancel out in aggregation. Thus, the aggregate consumption process can be written as

$$\Delta c_{t+1}^{RI} = \frac{\theta \alpha_{RI} u_{t+1}}{1 - \left((1-\theta)/\phi\right)\cdot \mathbb{L}}, \tag{26}$$

and we use Δc_{t+1}^{RI} to denote aggregate consumption henceforth. The aggregate consumption follows an AR(1) process, a special case of the long run risk model.[7] Compared with Equation (8), a representative rational inattentive consumer adjusts consumption with a lag in response to changes in wealth. This delay is a key implication of consumption choice under rational inattention, highlighting that when information is updated gradually, consumption also adjusts incompletely to exogenous shocks.

[6]Bansal and Yaron (2000) argue that this form of log consumption growth can be motivated empirically by the stochastic trend in the level of consumption that follows a exponential smoothing process and the cyclical component follows an AR(1) process.

[7]If we let $\varepsilon_{x,t} = \rho_x \varepsilon_{c,t}$, then the above state space representation of long run risk model reduces to an AR(1) process.

The above consumption process implies that the exchange rate in Equation (10) can be rewritten as:

$$\Delta e_{t+1}^{RI} = -\frac{1-\gamma}{\psi-1}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right) + \frac{1/\psi-\gamma}{1-1/\psi}\left(r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right), \tag{27}$$

which implies that because consumption responds with a lag to wealth shocks, the exchange rate also adjusts gradually, reflecting exposure to future long-run consumption risk. Conversely, the rate of the risky market portfolio on the right side of Equation (27) adjusts immediately to an equity return innovation, capturing a short-term shock to exchange rate growth.[8]

Using this equation, we can characterize the correlation between the real exchange rate (RER) and the consumption differential, $\Delta c - \Delta c^{*}$. Under RI, the contemporaneous consumption response to shocks is attenuated, leading to a muted change in current consumption growth. The first term on the right-hand side of Equation (27) captures the correlation between contemporaneous marginal utilities and consumption growth. RI weakens this correlation by dampening the immediate response of consumption, whereas the associated long-run risk strengthens it by inducing gradual, ongoing consumption adjustments. The second term on the right-hand side of Equation (27) reflects the direct correlation between the market return and consumption growth. This component is dampened by RI but is not affected by long-run risk. When the RI-induced attenuation dominates the amplification from long-run risk, the overall link between the RER and consumption is weakened. Moreover, under a preference for early resolution of uncertainty ($\gamma > 1/\psi$), the sign of this second correlation is negative, so that for a range of values of the degree of rational inattention, the model can generate a negative exchange rate–consumption correlation (an exchange rate disconnect).

To analyze more precisely how RI affects RER volatility, we follow the standard log-linear approximation approach of Campbell and Shiller (1988) and Colacito and Croce (2011) to derive the stochastic discount factors. This, in turn, allows us to express the innovations to RER growth as follows:

$$\begin{aligned}\Delta e_{t+1}^{RI} - \mathbb{E}_t[\Delta e_{t+1}^{RI}] &= \frac{\delta\left(1/\psi-\gamma\right)}{1-\rho_x\delta}\left(\varepsilon_{x,t+1}^{*} - \varepsilon_{x,t+1}\right) - \gamma\left(\varepsilon_{c,t+1}^{*} - \varepsilon_{c,t+1}\right) \\ &= \underbrace{\theta}_{\text{RI}}\left(\underbrace{\frac{\delta\left(1/\psi-\gamma\right)\rho_x}{1-\rho_x\delta}}_{\text{Long-run effect}} - \underbrace{\gamma}_{\text{Short-run effect}}\right)\left(\alpha_{RI}^{*}u_{t+1}^{*} - \alpha_{RI}u_{t+1}\right).\end{aligned} \tag{28}$$

Equation (28) characterizes the dynamics of the RER as being driven by the joint effects of short-run shocks to realized consumption growth and long-run shocks to the normalized contin-

[8] Gourinchas and Tornell (2004) analyze the role of expectation errors in generating predictable excess returns. In their paper, the forward premium is due to a signal extraction problem. Signals are updated partially, which is key to explain the observed "delayed-overshooting" in the data.

uation value. In the second line, the first term in the bracket captures the long-run effect on the normalized continuation value in the stochastic discount factor, arising from the RI-induced persistent long-run component. When preferences are time-separable ($\gamma = 1/\psi$), this term vanishes and exchange rate fluctuations become proportional to the difference in the degrees of RI across countries. When $\gamma \neq 1/\psi$, this long-run effect is nonlinear in θ: as the degree of RI increases (i.e., θ decreases), the persistence of the induced long-run risk, ρ_x, rises. Hence, for $\theta \in (0, 1)$, RER volatility is a nonlinear function of the degree of RI. The second term reflects the impact of immediate, short-run consumption news on the RER. Overall, RI dampens RER dynamics through weaker short-run responses while amplifying them through RI-induced long-run consumption risk. When the direct RI effect dominates the long-run risk effect, RER volatility is reduced. This feature differentiates our framework from standard long-run risk models: even when domestic and foreign consumption news are not highly correlated, RI lowers risky asset holdings, compresses wealth fluctuations, and thus reduces consumption growth volatility. Accordingly, RI alone—beyond long-run consumption risk—can generate low RER volatility.

3.1 Implications for Volatilities and Correlations

Equation (27) provides the basis for computing the moments of international comovements. The following proposition summarizes the main results regarding the joint dynamics of the RER and consumption growth across countries:

Proposition 2 *Under RI, when $\theta = \theta^*$ and the consumption-wealth ratios are $\phi = \phi^*$, the correlation of the RER growth and the consumption growth differential is:*

$$\operatorname{Corr}(\Delta e, \Delta c - \Delta c^*) = \sqrt{\frac{\Omega}{\varsigma}} \left(\frac{1-\gamma}{\psi - 1} \varsigma - \theta \frac{1/\psi - \gamma}{1 - 1/\psi} \right) \frac{1}{\sqrt{\operatorname{Var}(\Delta e)}}, \tag{29}$$

and the volatility of the RER is:

$$\operatorname{Var}(\Delta e) = \Omega \left[\left(\frac{1-\gamma}{\psi - 1} \right)^2 \varsigma + \left(\frac{1/\psi - \gamma}{1 - 1/\psi} \right)^2 - 2\theta \frac{1-\gamma}{\psi - 1} \frac{1/\psi - \gamma}{1 - 1/\psi} \right], \tag{30}$$

where $\varsigma = \frac{\theta^2}{1 - [(1-\theta)/\phi]^2}$ decreases as the degree of RI increases (i.e., θ decreases), and Ω is the volatility of the portfolio return differential:

$$\Omega = \alpha_{RI}^2 \omega^2 + \alpha_{RI}^{*2} \omega^{*2} - 2\rho_u \alpha_{RI} \alpha_{RI}^* \omega \omega^*. \tag{31}$$

Moreover, the volatilities and correlation of domestic and foreign consumption growth can be written as:

$$\operatorname{Var}(\Delta c) = \varsigma \alpha_{RI}^2 \omega^2, \operatorname{Var}(\Delta c^*) = \varsigma \alpha_{RI}^{*2} \omega^{*2}, \tag{32}$$

$$\operatorname{Corr}(\Delta c, \Delta c^*) = \rho_u, \tag{33}$$

respectively.

Proof. *See Online Appendix B for the derivation.* ■

From the proposition, when $\theta = \theta^* = 1$, the model nests the FIRE benchmark. Evaluating the expressions at $\theta = \theta^*$ allows us to study the behavior of the model-implied moments. We focus on values of θ above the U.K. consumption–wealth ratio, 0.23, so that the persistence of the long-run consumption component, $(1-\theta)/\phi$, remains below 1. As θ decreases, both α_{RI} and α^*_{RI}, as well as ς, decline monotonically, which in turn reduces the volatility of consumption growth. The upper-right panel of Figure 1 shows that the consumption–RER correlation, (29), first falls below zero and then rises above zero as θ decreases, implying non-monotonic behavior. This arises from the interaction between RI and long-run consumption risk. The volatility of RER growth is likewise nonlinear in θ. While the volatility of the portfolio return differential, Ω, falls as θ declines, the bracketed term $\left[\left(\frac{1-\gamma}{\psi-1}\right)^2 \varsigma + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2 - 2\theta \frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\right]$ increases with a lower θ. When $\theta \neq \theta^*$, allowing for heterogeneity in θ across home and foreign economies further weakens cross-country correlations in macro fundamentals. However, such heterogeneity also amplifies cross-country dispersion in consumption growth, raising the variance of the consumption differential and hence generating a more volatile RER.

Correlation of RER and the relative consumption growth. The upper-right panel of Figure 1 illustrates that combining RI with RU ($\gamma \neq 1/\psi$) breaks the perfect comovement between RER depreciation and relative consumption growth implied by the benchmark model. With a preference for early resolution of uncertainty ($\gamma > 1/\psi$), the RER depends not only on relative consumption growth ($\Delta c^* - \Delta c$) but also on relative portfolio returns ($r^* - r$). Consider a positive equity-return shock in the foreign stock market. This shock raises current foreign consumption, lowers foreign marginal utility, and puts downward pressure on the foreign currency. At the same time, as implied by the market return in Equation (27), the foreign currency appreciates on impact because the higher return increases the discount rate. In the absence of early resolution and RI, only the initial depreciation channel would be operative and $\text{Corr}(\Delta e, \Delta c - \Delta c^*) = 1$.[9]

When $\gamma > 1/\psi$, the second channel becomes operative. RI further attenuates the initial depreciation effect by rendering the contemporaneous consumption response incomplete. In particular, if the intertemporal elasticity of substitution is below one and the adjustment in consumption is smaller than the change in the interest rate, the second channel dominates the consumption channel, generating a negative correlation between the RER and the consumption differential. Consequently, the overall RER–consumption correlation is reduced and can even turn negative.

[9] Under power utilities, complete markets in state contingent assets ensure that growth rates in the marginal utility of currency are equalized across countries. See Obstfeld and Rogoff (2001).

Over a wide range of θ values, the model-implied Corr($\Delta e, \Delta c - \Delta c^*$) drops sharply from unity and becomes close to zero or negative for some intervals, consistent with empirical findings of near-zero or negative correlations. By contrast, the standard FIRE model, even when calibrated with a higher risk aversion γ to match a plausible risky-asset share, fails to reproduce these international moments: a higher γ does not lower Var(Δe), and Corr($\Delta e, \Delta c - \Delta c^*$) remains 1. These outcomes are depicted in the first row of the right panel of Figure 4. Hence, both RU ($\gamma \neq 1/\psi$) and RI ($\theta < 1$) are necessary to generate a lower Corr($\Delta e, \Delta c - \Delta c^*$).

Volatility of RER. The lower-right panel of Figure 1 and the right panel in the second row of Figure 4 display the standard deviation of the RER when $\theta = \theta^*$ and agents are endowed with Epstein-Zin utility or power utility, respectively. These figures clearly show that a lower degree of RI and the induced long-run consumption risk consistently reduces the RER volatility monotonically under power utility.[10] Introducing RU ($\gamma \neq 1/\psi$) increases the standard deviation of the RER compared to the power utility case and brings in non-monotonic features. Furthermore, when $\gamma > 1/\psi$, that is, when agents prefer early resolution of uncertainty under RU, the reduction in the RER volatility is mitigated within certain ranges of θ. This occurs because the induced long-run risk influences the RER volatility in the opposite direction. A more persistent long-run risk (due to a smaller θ) results in larger RER movements. However, when the effect of RI dominates, particularly at lower θ values, the RER volatility tends to decrease.

In the upper panels of Figure 2, we further disentangle the roles of RI and RI-induced long-run risk in shaping RER volatility. Note that, unlike in the proposition, we now allow for $\phi \neq \phi^*$ in all panels. A lower θ consistently reduces the volatility of the stochastic discount factors (SDFs), reflecting both the impact of long-run risk and the decline in optimal risky-asset holdings. At the same time, the cross-country correlation of SDFs—mirroring the behavior of the international correlation of aggregate consumption—also falls as θ decreases, with a marked drop as θ approaches 0.23. When θ initially falls from 1, the reduction in SDF volatilities dominates the decline in their international correlation, so that RER volatility decreases. As θ continues to fall toward 0.23, the cross-country correlation of SDFs declines more sharply, which then pushes up RER volatility. Consequently, RER volatility is non-monotonic in θ. The lower panels of Figure 2 document the corresponding behavior of consumption volatilities and their international correlations, providing additional insight into these mechanisms.

The impulse responses in Figure 3 further illustrate these dynamics. Following a positive innovation to the foreign equity return, the log portfolio return effect dominates on impact, and the foreign currency appreciates. In the second period, only the lagged consumption response remains operative, reducing marginal utility and exerting downward pressure on the foreign

[10] θ and θ^* are assumed to move simultaneously. Without loss of generality, other parameters are set at the calibrated values under RU.

currency. Thereafter, the RER exhibits a hump-shaped response before converging back to its long-run mean of zero. [11]

We next turn to the remaining moments in the special case $\gamma = 1/\psi$ and study how they vary with θ. Detailed derivations are provided in Online Appendix C. As shown in the other panels of Figure 4, all effects become monotone in θ under expected utility (EU). The EU case not only fails to generate an empirically plausible RER disconnect, but also implies an autocorrelation of RER growth that remains excessively high as θ decreases. When $\theta = \theta^*$, this autocorrelation is approximately $(1-\theta)/\phi$, reflecting the persistence of consumption growth, and it converges to the persistence of the long-run component in the consumption process. By contrast, under recursive utility, RER growth remains close to a random walk over a wide range of θ. Allowing $\gamma \neq 1/\psi$ is thus crucial for breaking the perfect link between the RER and the consumption differential.

It is also important to distinguish our results from those of Colacito and Croce (2011) and Brandt et al. (2006) by emphasizing that our framework substantially reduces the variance of the one-period SDF, m_t. The long-run Euler condition implies $\mathbb{E}[M_t^s R_{e,t}] = 0$, where the "long-run stochastic discount factor" M_t^s is

$$M_t^s = R_f^s \left(\frac{C_{t+1+s}}{C_t}\right)^{-\frac{1-\gamma}{\psi-1}} \left(R_{p,t+1}R_{p,t+2}\cdots R_{p,t+1+s}\right)^{\frac{1/\psi-\gamma}{1-1/\psi}}. \tag{34}$$

Therefore, the variance of M_t^s should satisfy $\mathrm{Std}(M_t^s)/\mathbb{E}\left[M_t^s\right] \geq \mathbb{E}\left[R_{e,t}\right]/\mathrm{Std}(R_{e,t})$. that $\mathbb{E}[R_{e,t}] = 6.8\%$ and $\mathrm{Std}\left[R_{e,t}\right] = 15\%$ with a gross risk-free rate near one, we have $\mathrm{Std}[M_t^s] \geqslant 0.45$, while $\mathrm{Std}[M_t]$ can be way smaller if we use $S = 11$ as in Parker (2003) and Parker and Julliard (2002). In a long-run risk environment, such "long-run SDFs" must be taken into account when pricing risks, in addition to the standard one-period SDFs; without long-run risk, the two coincide.

In our model, RER variance is damped through two main channels: (i) under RI, agents optimally hold fewer risky assets, and (ii) the response of consumption growth to equity-return shocks is hump-shaped rather than purely contemporaneous. As shown by Equations (27) and (65), these features reduce the variance of the relevant SDFs, thereby lowering RER volatility. This behavior of international SDFs is appealing because cross-country correlations of key macroeconomic aggregates tend to be very low across a range of frequencies.[12]

[11] Our model does not generate "delayed overshooting," as the transmission to the interest rate is immediate.

[12] For brevity, we do not emphasize cross-country consumption correlations in this paper, although RI can also account for their low empirical values, as discussed in Li et al. (2017). In Equation (33), allowing θ to differ across countries generates a lower consumption correlation than the corresponding stock-market correlation.

3.2 Implications for International Risk Sharing

Our model also provides a framework to discuss the implications of RI for international risk sharing. We adopt the risk-sharing index (RSI) constructed by Brandt et al. (2006) as well as used in Chien et al. (2020):

$$\text{RSI} = 1 - \frac{\text{Var}\left(\Delta e_{t+1}\right)}{\text{Var}\left(m_{t+1}^{*}\right) + \text{Var}\left(m_{t+1}\right)}, \tag{35}$$

which captures not only the correlation of the stochastic discount factors but also their difference in scale. We now calculate the index for our RI economy. As shown in Online Appendix C, the volatility of the stochastic discount factors can be written as:

$$\text{Var}(m_{t+1}) = \left(\frac{1-\gamma}{\psi-1}\right)^2 \text{Var}(\Delta c_{t+1}) + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2 \text{Var}(r_{p,t+1}) - \frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\text{Cov}(\Delta c_{t+1}, r_{p,t+1}).$$

Along with the volatility of the RER growth we derived in Equation (30) and $\text{Var}\left(r_p\right) = \alpha_{RI}^2\omega^2$, the Risk Sharing Index can be written as follows:

$$\text{RSI} = 1 - \frac{\left(\alpha_{RI}^2\omega^2 + \alpha_{RI}^{*2}\omega^{*2} - 2\rho_u\alpha_{RI}\alpha_{RI}^{*}\omega\omega^{*}\right)\left[\left(\frac{1-\gamma}{\psi-1}\right)^2\varsigma + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2 - 2\theta\frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\right]}{\left(\alpha_{RI}^2\omega^2 + \alpha_{RI}^{*2}\omega^{*2}\right)\left[\left(\frac{1-\gamma}{\psi-1}\right)^2\varsigma + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2 - \theta\frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\right]}.$$

When investors become less attentive, as indicated by a smaller value of θ, the degree of risk sharing between the two countries decreases. The risk sharing index is 0.99 under full information but drops to 0.83 under RI, which is more consistent with the reality. As shown in the formula, a smaller θ reduces the volatility of the stochastic discount factors (the denominator) more than the volatility of the RER (the numerator). This occurs because RI and the long-run risk jointly reduce the volatility of the pricing kernels and their correlation with the calibrated variable, θ.

Equivalently, using Equation (1), we can link the standard deviation of the change in the RER to the moments of the SDFs as follows:

$$\text{Std}\left(\Delta e_{t+1}\right) = \text{Std}\left(m_{t+1}\right)\sqrt{2\left(1 - \text{Corr}\left(m_{t+1}, m_{t+1}^{*}\right)\right)},$$

which implies that our model is capable of reducing the volatility of the RER. The model's success of producing this result mainly depends on the reduced volatility of the pricing kernels given the weakened contemporaneous consumption response. Nevertheless, reduced risk sharing—which results from decreased investment in internationally correlated assets— elevates RER volatility. These finding contrasts with that of Brandt et al. (2006), as they require a high correlation between international SDFs to suppress exchange rate volatility. To reiterate, the interaction between RI and the long-run risk leads to fewer risky-asset holdings, denoted by smaller α

and α^*. This adjustment weakens risk sharing by reducing the correlation between home and foreign portfolio returns. This effect, captured by the decline of the term $2\rho_u \alpha_{RI} \alpha_{RI}^* \omega \omega^*$ in the numerator, acts to offset the reduction in the RER volatility. The results are summarized in Figure 5 and are robust for various degrees of risk aversion.

4 Empirical Evidence and Calibration

4.1 Data and Baseline Calibration

In this section, we describe the data and the calibration strategy. Parameter values are chosen primarily to match key empirical moments for the U.S. and U.K. economies. Both countries are assumed to share symmetric preference parameters, following Bansal and Yaron (2004) and Colacito and Croce (2011). The calibration is conducted at a monthly frequency, as in Colacito and Croce (2011). Table 1 reports international evidence on consumption and the RER for the U.S.–U.K., U.S.–Euro Area, U.S.–Canada, and U.S.–Japan pairs. The baseline sample spans from 1991m1 to 2018m12, except for the Euro Area. All moments are annualized and computed from monthly data, with the exception of consumption moments. Real per capita consumption is measured as quarterly real private consumption expenditure divided by annual population (source: Datastream). Nominal exchange rates and consumer price indices are taken from the IMF's International Financial Statistics, and we use end-of-period bilateral exchange rates. Table 1 shows that the standard deviations of RER growth are relatively stable, at around 10% across all four pairs. RER growth is close to a random walk, with autocorrelation coefficients near zero. The correlations between consumption growth differentials and corresponding RER growth are negative or near zero in all cases, indicating the presence of the Backus–Smith puzzle over the sample. The data also reveal low consumption growth volatility and weak cross-country consumption correlations, implying that aggregate consumption is very smooth and that economies are only weakly correlated.[13] By contrast, the benchmark FIRE model implies excessively strong comovement between consumption and the RER, as well as overly volatile RERs and consumption growth.

Table 3 summarizes the baseline calibration of the RI model. The RI and FIRE models share the same parameters, except for the degrees of rational inattention, θ and θ^*. The subjective discount factor δ is set to be 0.998.. Average saving–wealth ratios are taken from Table 2. The first two moments of domestic and foreign risk premia are reported in Rows 4–7. Excess returns are computed from total return indices and one-month Eurocurrency deposit middle rates from Datastream; standard deviations are based on monthly returns from the same source.

[13] The correlation between Japan's and U.S. consumption growth is particularly low over 1991–2018. Over the longer period 1980–2018, the correlation is 0.11.

Cross-country correlations are calculated using total returns. Steady-state wealth–consumption ratios are constructed from annual data (1995–2018) on household financial assets divided by consumption expenditure, sourced from Datastream, Eurostat, and national accounts; Euro Area data begin in 1999. These data imply a saving–wealth ratio of roughly 80% across the countries/regions considered.

Following Gabaix and Laibson (2001), the equity share of total wealth, α^e, is computed using the labor share s, the risk-free rate r, and the return to capital R:

$$H = \int_0^{30} \exp(-rt)\, sY\, dt, \tag{36}$$

$$K = \frac{1-s}{R} Y, \tag{37}$$

$$\alpha^e = \frac{K}{K+H}. \tag{38}$$

The average annual risk-free rates in the U.S. and U.K. over the sample are 2.92% and 4.08%, respectively. Labor shares are measured as average annual labor compensation in GDP at current national prices from Datastream, over 1991–2017.[14] Following Caselli and Feyrer (2007), we set the return to capital to 0.09 in both countries. The implied equity shares are approximately 0.26 in the U.S. and 0.31 in the U.K.

We then jointly estimate the remaining parameters—the domestic and foreign θ and θ^*, the (symmetric) coefficient of relative risk aversion (γ), and the intertemporal elasticity of substitution (ψ)—to match key international moments for asset holdings, exchange rates, consumption dynamics, and their cross-country correlations. The estimated parameters are reported in Table 3. We use GMM to minimize the distance between model-implied and targeted data moments. The resulting calibration yields a coefficient of relative risk aversion of about 1.061 and an intertemporal elasticity of substitution of 0.989. These estimates are then fed into the RI model, which we evaluate by comparing model-implied moments with the data. Columns 3–6 of Table 4 show the degree of alignment between model and data. In the RE model, the representative agent holds an excessively large share of risky assets for a moderate level of risk aversion γ and faces large consumption fluctuations when consumption follows a random walk. This reduces the model-generated exchange-rate–to–consumption variance ratio. In the RE* column, we recalibrate parameters to best match the empirical moments under full information. A high γ ($\gamma = 13$) lowers the equilibrium risky asset share, but RER volatility remains too high because there is little consumption smoothing. Moreover, as discussed above, the RER–consumption correlation remains pinned at 1 irrespective of γ. Column 6 reports the RI results. Overall,

[14] See Galí, Gertler, and López-Salido (2001) and Piketty and Zucman (2014) for discussions of labor shares in the Euro Area.

the RI framework delivers a substantially better fit to the data than both the standard RE benchmark and its re-calibrated extension (RE*).

Finally, Tables 5–8 present calibration results for the U.S.–Japan and U.S.–Canada pairs. Across these additional country pairs, the RI model continues to outperform the RE benchmarks in matching the volatility of the RER, the RER–consumption correlation, and the volatility of consumption growth.

4.2 Direct Estimation of the Degrees of RI (θ)

To discipline the incorporation and magnitude of RI in our model economy, we calibrate the degree of information rigidity using data from professional forecasters' surveys in the U.S. and the U.K. Following Coibion and Gorodnichenko (2015), in models with imperfect information, the speed of learning—or Kalman gain, —can be inferred from the relationship between forecast revisions and forecast errors. Specifically, when averaging across agents, the ex-post mean forecast error for a macroeconomic variable and the ex-ante mean forecast revision satisfy:

$$x_{t+h,t} - \mathbb{E}_t\left[x_{t+h,t}\right] = \frac{1-G}{G}\left(\mathbb{E}_t\left[x_{t+h,t}\right] - \mathbb{E}_{t-1}\left[x_{t+h,t}\right]\right) + \delta z_{t-1} + error_t, \tag{39}$$

where G is the Kalman gain governing agents' updating, and $1-G$ measures the degree of information rigidity (or RI). Each agent i generates h periods ahead forecasts $\mathbb{E}_{i,t}\left[x_{t+h}\right]$ using standard Kalman filtering, and $\mathbb{E}_t\left[\cdot\right]$ denotes the average forecast across agents at time t. z_{t-1} is a set of relevant control variables.

We focus on year-on-year inflation forecasts π_t from the U.S. Survey of Professional Forecasters (SPF) and from HM Treasury's Forecasts for the UK Economy.[15] For inflation, we include as additional controls the average quarterly 3-month T-bill rate, the quarterly change in the log oil price, and the average unemployment rate. We set $h=3$, so that $\pi_{t+3,t}$ denotes the average inflation rate between quarter t and quarter $t+3$. The variables used in the empirical implementation are defined as follows:

$$\begin{aligned}
\pi_{t+3,t} &= \text{Q4-of-Year-}t \text{ inflation rate,} \\
\mathbb{E}_t\left[\pi_{t+3,t}\right] &= \text{Average forecasts of Q4 inflation rate as of February of the year,} \\
\mathbb{E}_t\left[\pi_{t+3,t}\right] - \mathbb{E}_{t-1}\left[\pi_{t+3,t}\right] &= \text{Average revisions in forecasts of Q4 inflation rate from November} \\
&\quad\ \text{of last year to February of this year,} \\
z_{t-1} &= \text{Controls in November of last year.}
\end{aligned}$$

[15] Candian and De Leo (2025) show that exchange rate forecast errors can be predicted using past depreciations, and that forecasters underweight past exchange rate changes relative to the rational-expectations benchmark. Our mechanism is consistent with their evidence, as well as with Angeletos et al. (2021) and related contributions documenting that aggregate forecast errors are positively related to lagged aggregate forecast revisions.

Relationship (39) can then be estimated using year-on-year inflation π, mean forecasts $\mathbb{E}[\pi]$, and horizon h through the empirical specification:

$$\begin{aligned}\pi_{t+3,t} - \mathbb{E}_t[\pi_{t+3,t}] = c + \beta(\mathbb{E}_t[\pi_{t+3,t}] - \mathbb{E}_{t-1}[\pi_{t+3,t}]) + \alpha_1 \cdot 3\text{-}MonthTbill_{t-1} \\ +\alpha_2 \cdot \Delta \log(Oil_{t-1}) + \alpha_3 \cdot Unemp_{t-1} + error_t,\end{aligned} \tag{40}$$

where $\beta > 0$ is indicative of RI. For the U.S., all macroeconomic and financial data are drawn from the Federal Reserve Economic Data (FRED) database. The short rate is the 3-month Treasury bill secondary market rate, and the oil price is the WTI spot crude oil price. For the U.K., inflation is measured by the retail price index (all items) from the Office for National Statistics (ONS). The short rate is the monthly average 3-month Treasury bill rate from the Bank of England; following the discontinuation of this series after 2017, we extrapolate using the U.K. nominal spot curve. The U.K. unemployment rate is from the ONS, and oil prices are measured by Brent crude oil futures. Using these data, we estimate Equation (40) by OLS. Over the 1980-2022 sample for the U.S., the regression in (39) yields $\widehat{\beta} = 1.07$ (s.e.=0.39), implying a Kalman gain $\widehat{G} = 1/\left(1+\widehat{\beta}\right) \approx 0.48$. Thus, agents put less than half the weight on new noisy signals and more than half on their prior forecasts. For the U.K. sample (1987-2022), we obtain $\widehat{\beta} = 1.83$ (s.e.=0.47), corresponding to a Kalman gain of approximately 0.35. The model-implied values of the Kalman gains, θ and θ^*, lie comfortably within the 95% confidence intervals associated with these empirical estimates.

5 Extensions

In this section, we consider two extensions to our benchmark framework: (i) inattentiveness, as an alternative form of cognitive limitation, and (ii) stochastic volatility in equity returns. We show that our core results are robust to both modifications. Moreover, in each case the resulting model delivers a substantially better quantitative fit than the corresponding FIRE benchmark.

5.1 Sticky Information due to Inattentiveness

The previous sections have shown that introducing rational inattention (RI) into a consumption–real exchange rate (RER) model with recursive preferences improves the model's ability to account for the joint dynamics of consumption and the RER in major industrial economies. In this section, we consider a competing hypothesis about cognitive limitations—"inattentiveness"—as developed by Mankiw and Reis (2002) and Reis (2006). The inattentiveness framework emphasizes behavioral or bounded-rationality considerations, whereby agents update or reoptimize only infrequently, often due to the costs of acquiring and processing information or of planning. In Reis's (2006) model, for instance, consumption between adjustment dates is determined by a deterministic optimization problem, while at adjustment dates it is chosen according to the standard stochastic consumption-savings problem. As a result, agents in an inattentiveness economy adjust infrequently but fully when they do adjust, and aggregate sluggishness arises purely from aggregating across heterogeneous adjustment times. By contrast, under RI, individuals update their plans frequently but only partially, and the sluggishness in aggregate consumption reflects incomplete adjustment at the individual level. In practice, both types of information frictions are likely to be present. Coibion and Gorodnichenko (2015), using SPF forecast data, empirically assess the degree of information rigidity implied by these two classes of models and show that both imply the same mapping between average ex-post forecast errors and average ex-ante forecast revisions: the coefficient on forecast revisions depends only on the degree of information rigidity. Using data from $1969 - 2010$, they strongly reject the null of full-information rational expectations and find a significantly positive coefficient on forecast revisions, consistent with both noisy-information and sticky-information specifications.

The objective of this section is to integrate these two related dimensions of inattention into a unified framework: (i) the extensive margin of inattention (whether and when agents adjust, i.e., adjustment occurs only at discrete dates) and (ii) the intensive margin of inattention (how much information is processed when agents choose to adjust). We then study how the interaction of these margins shapes the joint dynamics of consumption, portfolio choice, and the RER.

To capture the role of inattentiveness in affecting the joint dynamics of consumption, asset allocation and RERs, this section extends our benchmark model by incorporating inattentiveness. Specifically, we assume that planning dates are $D(i)$ where $i \in \mathbb{N}_0$, with $D(0) \equiv 0$, and inattentiveness intervals $d(i) = D(i) - D(i-1)$. At date $t+1$, there are d types of consumers, and the size of each type is assumed to be $1/d$ to preserve a stationary distribution. A fraction of $1/d$ agents adjust consumption at C_{t+1}, and lastly adjusts at $t+1-d$. The remaining population do not adjust at $t+1$, and lastly adjust at C_{t+2-d}, C_{t+3-d}, ..., and C_t, evenly. The latter group constitutes the $d-1$ types. The degree of inattentiveness, d, is symmetric in both countries. In the presence of inattentiveness, the filtration of the model economy is defined as $\mathfrak{J} = \{\mathfrak{J}_t, t \geq 0\}$, where:

$$\mathfrak{J}_t = \mathcal{I}_{D(i)} \text{ for } t \in [D(i), D(i+1)).$$

At planning date $D(i)$, the consumer with limited attention observes noisy signal: $\widetilde{a}_{D(i)} = a_{D(i)} + \xi_{D(i)}$ and faces the information-processed constraint:

$$\mathbb{H}(a_{D(i)}|\mathcal{I}_{D(i-1)}) - \mathbb{H}(a_{D(i)}|\mathcal{I}_{D(i)}) = \kappa,$$

where κ is finite channel capacity.

In the presence of both RI and Inattentiveness, consumption growth is deterministic between two adjusting periods and is governed by the following Euler equation:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\rho/\psi}\overline{R}_p^{\rho-1}R_f = 1.$$

where $\overline{R}_p$ denotes the expected portfolio return. The wealth evolution between planning dates is governed by:

$$A_{t+1} = R_{p,t+1}(A_t - C_t), \tag{41}$$

where $R_{p,t+1} = \alpha_t(R_{e,t+1} - R_f) + R_f$ is the return to the market portfolio. At planning dates, the wealth evolution is governed by:

$$A^+_{D(i)} = A^-_{D(i)}\left(1 - K\right),$$

where K is a proportional planning cost paid by the consumer.

At date t, the consumer who is adjusting chooses the next planning date $t+d$ and plans for the interval. The value function satisfies:

$$V(A_t) = \max_{\{C,\alpha,d\}} \mathbb{E}_t\left\{\sum_{i=t}^{t+d-1}\beta^{i-t}U(C_i) + \beta^{-d}\mathbb{E}_t[V(A_{t+d})]\right\},$$

where d is the chosen inattentiveness interval, the consumption plan (C_t) and portfolio share (α_t) are subject to (41) for t, $t+1$, ..., and $t+d-1$. In this problem, $A^+_t = A^-_t(1-K)$ is defined as the post-adjustment wealth (after paying proportional cost). Similarly, at the next planning date, we have $A^+_{t+d} = A^-_{t+d}(1-K)$. For the consumers who do not adjust, consumption follows the deterministic path:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\rho/\psi}\overline{R}_p^{\rho-1}R_f = 1.$$

Note that the no-Ponzi game condition, $\lim_{t\to\infty}\beta^{-t}A_t \geq 0$, holds.

Consider first the inattentive consumer's consumption path. If the consumer is inattentive between time t and $t+s$, the long-run Euler condition with recursive utility implies:

$$R_f^{s-1}\left(\frac{C_{t+s}}{C_t}\right)^{-\rho/\psi}\overline{R}_p^{s(\rho-1)}R_f = 1.$$

Taking logs and approximating around the steady state consumption-wealth ratio, the above

Euler equation reduces to:

$$c_{t+s} = c_t + As, \tag{42}$$

where $A = (\rho/\psi)\left[sr_f + s\left(\rho - 1\right)r_p\right]$, and $r_p = \log\left(\overline{R}_p\right)$. We have the following proposition:

Proposition 3 *In the optimal consumption-portfolio choice problem in which consumers with recursive utility face both inattentiveness and RI, the transition of the perceived state at the next planning date $t+d$ is given by:*

$$\widehat{a}_{t+d} = \frac{1}{\phi^d}\widehat{a}_t + \frac{\phi^d - 1}{\phi^d}c_t^o + \left[\frac{\phi\left(\phi^{1-d}-1\right)}{\left(1-\phi\right)^2} - \frac{\phi\left(d-1\right)}{1-\phi}\right]A + \frac{1-\phi^d}{\phi^{d-1}\left(1-\phi\right)}\Lambda - \log\left(1-K\right) + \eta_{t+d}, \tag{43}$$

where c_t^o denotes the optimal log consumption at the planning date t, Λ is defined in Section 3, the pure RI problem, and

$$\eta_{t+d} = \frac{\theta}{\phi^d}\left(a_t - \widehat{a}_t\right) + \theta\left(\sum_{i=0}^{d-1}\phi^i r_{p,t+1+i} + \xi_{t+d}\right).$$

Combining Equations (67) and (43), we have the following expression for the filtering error:

$$a_{t+d} - \widehat{a}_{t+d} = \frac{\left(1-\theta\right)\sum_{i=0}^{d-1}\phi^{i-d+1}r_{p,t+1+i}}{1-\left(1-\theta\right)/\phi^d\cdot\mathbb{L}^d} - \frac{\theta\xi_{t+d}}{1-\left(1-\theta\right)/\phi^d\cdot\mathbb{L}^d},$$

where $\mathbb{L}^d$ denotes a d-period lag operator.

Proof. See Online Appendix D for the derivation. ■

In the next step, we will discuss how to determine the optimal inattentiveness interval in the model economy. Compared to the pure RI problem, the current model economy evolves as if time is stretched and periods are effectively prolonged; consequently, wealth shocks accumulate over these extended intervals. Guess that the value function takes the form:

$$J\left(\widehat{A}_t\right) = \chi_t\widehat{A}_t, \tag{44}$$

where $\widehat{A}_t \equiv \exp\left(\widehat{a}_t\right)$ is the perceived state. We rewrite Equation (43) as

$$\Delta\widehat{a}_{t+d} = \left[\left(1-\frac{1}{\phi}\right)\left(c_t^o - \widehat{a}_t\right) + \Lambda\right]\frac{\phi^d - 1}{\phi^{d-1}\left(1-\phi\right)} + B + \eta_{t+d}, \tag{45}$$

where $B = \left[\frac{\phi\left(\phi^{1-d}-1\right)}{\left(1-\phi\right)^2} - \frac{\phi\left(d-1\right)}{1-\phi}\right]A - \log\left(1-K\right)$. By taking exponential on both sides of the above equation, we obtain

$$\widehat{A}_{t+d} = \left(\widehat{A}_t - C_t^o\right)^{\frac{\phi^d-1}{\phi^{d-1}\left(1-\phi\right)}}\widehat{A}_t^{1-\frac{\phi^d-1}{\phi^{d-1}\left(1-\phi\right)}}\exp\left(B + \eta_{t+d}\right). \tag{46}$$

Substituting the above equation into the FOC with respect to consumption, we get

$$\left(C_t^o/\widehat{A}_t\right)^{-1/\sigma}\left(1 - C_t^o/\widehat{A}_t\right)^{1-\left[\left(\phi^d-1\right)\left(1-1/\sigma\right)\right]/\left[\phi^{d-1}\left(1-\phi\right)\right]} = \left(\mathbb{E}_t\left[\phi_{t+d}\widehat{\eta}_{t+d}\right]^{1-\gamma}\right)^{\left(1-1/\sigma\right)//\left(1-\gamma\right)}, \tag{47}$$

where $\widehat{\eta}_{t+d} = \exp(B + \eta_{t+d})$, and $\frac{\phi^d - 1}{\phi^{d-1}(1-\phi)} > 1$ when $d > 1$. Substituting the FOC into the value function yields:

$$\chi_t = \left(C_t^o / \widehat{A}_t\right)^{1/(1/\sigma - 1)}, \tag{48}$$

which implies that the value function is increasing in the consumption-wealth ratio when $1/\sigma > 1$. The following proposition summarizes the results on the optimal choice of consumption and inattention period d^o:

Proposition 4 *When $1/\sigma > 1$, in the model with both inattentiveness and RI, the optimal consumption-wealth ratio is given by the FOC, (47). The optimal inattentiveness intervals d^o are then determined by maximizing the consumption-wealth ratio.*

Proof. *Maximizing Equation (48), given that $1/\sigma > 1$, yields the optimum of the homogeneous value function. The optimal inattentiveness intervals d^o therefore follow from maximizing the consumption-wealth ratio in the FOC.* ■

On the right-hand side of Equation (47), a longer inattentiveness interval d raises the cumulative return $\widehat{\eta}_{t+1}$, while simultaneously reducing the wealth that remains available at time t. The planning cost K and the degree of RI θ jointly shape the mean and variance of $\exp\{B + \eta_{t+d}\}$, implying that the optimal adjustment interval d^o is an implicit function of K and other structural parameters. Because the first-order condition does not yield a closed-form solution for d^o, the literature typically treats K as a free parameter disciplined by data—for example, Reis (2006) calibrates K such that d corresponds to eight quarters. In our case, we instead calibrate d and θ and jointly to match key data moments.

Table 9 compares model specifications that combine rational inattention with inattentiveness. The first column reports empirical moments from U.S.–U.K. data. The second column presents the FIRE benchmark with $\theta = \theta^* = 1$ (no information frictions) and $d = 1$ (no inattentiveness). The remaining columns report two sets of estimation results. "Baseline" uses the same weighting scheme and parameter estimates as in Table 4, whereas "FX-Weight GMM" assigns greater weight to exchange rate moments. Within each set, the "RI + Inattentiveness" specification allows the adjustment interval to be freely estimated, while the "RI Only" specification imposes .

Under our baseline estimates, the optimal inattentiveness interval converges to $d \approx 1.01$, effectively indistinguishable from the pure RI case. This indicates that the intensive margin of rational inattention—gradual and incomplete updating of perceived wealth—is the primary force behind the model's performance, while the extensive margin of infrequent adjustment contributes very little additional explanatory power.

Figure 6 illustrates the underlying trade-off. As the planning interval d increases, portfolio shares decline because less frequent rebalancing implies greater cumulative exposure to wealth shocks at planning dates, thereby raising perceived intertemporal risk through the term in the portfolio condition. At the same time, the Backus–Smith correlation rises. Under pure RI, gradual consumption adjustment generates a timing mismatch between exchange rate movements and consumption differentials; inattentiveness instead produces discrete catch-up adjustments that are contemporaneous with accumulated portfolio returns, pushing the co-movement back toward its FIRE benchmark. Exchange rate volatility is non-monotonic in d and eventually increases sharply: while shrinking portfolio shares dampen the portfolio-return channel, the variance of the consumption differential increases rapidly due to the persistence embedded in $\zeta(d)$, and for sufficiently large d this persistence-driven amplification dominates.

Under the alternative FX-Weight GMM specification, greater inattentiveness (larger d) can substitute for some of the friction otherwise captured by RI—estimated θ rises from 0.19 to 0.46 and from 0.22 to 0.52—yet the overall model fit is essentially unchanged relative to the pure RI case. In this sense, RI offers a parsimonious structural representation of a broader class of persistent informational frictions.

5.2 Stochastic Volatility

In this section, we consider the investor's problem when the risky return has stochastic volatility (SV). SV has been widely used in the finance literature to model the uncertainty of risky returns. Following Taylor (1986), we use a latent stochastic process to model the volatility of the equity return. Specifically, we assume that the equity return and its standard deviation are governed by the following two first-order autoregressive processes:

$$r_{e,t+1} = \mu + \exp\left(h_{t+1}/2\right)\varepsilon_{t+1},\ \varepsilon_{t+1} \sim \mathbb{N}(0,1), \tag{49}$$

$$h_{t+1} = \lambda + \rho_h h_t + \sigma_\eta \eta_{t+1},\ \eta_{t+1} \sim \mathbb{N}(0,1), \tag{50}$$

which is also called the log-normal SV model. We assume that both ε_{t+1} and η_{t+1} are iid normally distributed innovations, h_{t+1} and h^*_{t+1} are mutually independent, and $|\rho_h| < 1$ so the log-variance is stationary. With the stationarity condition, the log-variance follows an AR(1) process with unconditional mean $\mathbb{E}[h_t] = \lambda/(1-\rho_h)$ and unconditional variance $\operatorname{Var}(h_t) = \sigma_\eta^2/\left(1-\rho_h^2\right)$. Furthermore, the returns are conditionally normal:

$$r_{e,t+1}|h_{t+1} \sim \mathbb{N}\left(\mu, \exp\left(h_{t+1}\right)\right). \tag{51}$$

We derive and solve for the conditional moments as follows.[16] Applying the same argument as in the constant volatility case, the optimal portfolio choice and aggregate consumption growth

[16] The unconditional international moments under stochastic volatility move with θ in a similar pattern as the constant voaltility case, with the constant volatility being replaced by the unconditional mean and variance of the log-normal SV process.

are now:

$$\alpha_{RI}\left(h_{t+1}\right)=\left(\frac{1-\gamma}{\psi-1}\zeta+\frac{\gamma-1/\psi}{1-1/\psi}\right)^{-1}\frac{\mu-r_f+0.5\exp\left(h_{t+1}\right)}{\exp\left(h_{t+1}\right)}, \tag{52}$$

$$\Delta c_{t+1}^{RI}\left(h_{t+1}\right)=\theta\left[\frac{\pi\left(h_{t+1}\right)\exp\left(h_{t+1}\right)+\alpha_{RI}\left(h_{t+1}\right)\left(\exp\left(h_t/2\right)\varepsilon_{t+1}-r_f\right)}{1-\left((1-\theta)/\phi\right)\cdot L}\right], \tag{53}$$

respectively, where $\pi\left(h_{t+1}\right)=\alpha_{RI}\left(h_{t+1}\right)\left(1-\alpha_{RI}\left(h_{t+1}\right)\right)/2$. The first equation above shows that the investors hold fewer risky assets when the variance is higher. Furthermore, the interaction of RI and the long-run risk reduces the optimal share invested in the risky asset, consistent with the benchmark model. To better examine how the two forces influence the key moments we examine, we let $\theta=\theta^*$ and $\phi=\phi^*$. The following proposition summarizes the main results:

Proposition 5 *In the RI-SV model, when $\theta=\theta^*$ and the consumption-wealth ratios $\phi=\phi^*$, the correlation of the RER growth and consumption growth differential is:*

$$\operatorname{Corr}\left(\Delta e,\Delta c-\Delta c^*|h,h^*\right)=\sqrt{\frac{\Omega\left(h,h^*\right)}{\varsigma}}\left[\frac{1-\gamma}{\psi-1}\varsigma-\theta\frac{1/\psi-\gamma}{1-1/\psi}\right]\frac{1}{\sqrt{\operatorname{Var}\left(\Delta e|h,h^*\right)}}, \tag{54}$$

and the volatility of the RER growth is

$$\operatorname{Var}\left(\Delta e|h,h^*\right)=\Omega\left(h,h^*\right)\left[\left(\frac{1-\gamma}{\psi-1}\right)^2\varsigma+\left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2-2\theta\frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\right], \tag{55}$$

where $\varsigma=\frac{\theta^2}{1-[(1-\theta)/\phi]^2}$ decreases when the degree of RI increases (i.e., θ decreases), and Ω is the volatility of the portfolio return differential:

$$\Omega\left(h,h^*\right)=\alpha_{RI}^2\left(h\right)\exp\left(h\right)+\alpha_{RI}^{*2}\left(h^*\right)\exp\left(h^*\right)-2\rho_u\alpha_{RI}\left(h\right)\alpha_{RI}^*\left(h^*\right)\exp\left[\left(h+h^*\right)/2\right].$$

Finally, the volatilities and correlation of consumption growth in domestic and foreign countries and their correlation are:

$$\operatorname{Var}\left(\Delta c|h\right)=\varsigma\alpha_{RI}^2\left(h\right)\exp\left(h\right), \tag{56}$$

$$\operatorname{Var}\left(\Delta c^*|h^*\right)=\varsigma\alpha_{RI}^{*2}\left(h^*\right)\exp\left(h^*\right), \tag{57}$$

$$\operatorname{Corr}(\Delta c,\Delta c^*|h,h^*)=\rho_u, \tag{58}$$

respectively.

Proof. *The proof follows from simple algebra.* ■

As in the constant-volatility case, the proposition above shows that RI and LLR jointly influence the extent of RER disconnect. Conditional on current return volatilities, the volatilities of domestic and foreign consumption are reduced because more risk-averse investors optimally hold fewer risky assets in the stochastic-volatility (SV) environment as well. For the RER, it is also informative to examine how the volatility of the portfolio differential responds to changes in return volatility. To this end, we fix $\theta = \theta^*$ and very h and h^* in order to analyze how these two moments respond as these parameters change. The simulation procedure is as follows. First, because the likelihood function of the SV model is analytically intractable, we estimate a lognormal SV model for monthly equity returns in the U.S. and the U.K. over 1991m1-2018m12 using Bayesian methods. We assume flat priors (log prior equal to 0 on the support of the parameters and $-\inf$ elsewhere), cast the SV model in state-space form, and compute the posterior. For the U.S., the latent log-volatility process is estimated as

$$h_t = -0.3603 + 0.9137 h_{t-1} + 0.3577 \eta_t. \tag{59}$$

while for the U.K. the log stochastic volatility h_t^* evolves according to:

$$h_t^* = -0.7581 + 0.8242 h_{t-1}^* + 0.5090 \eta_t^*. \tag{60}$$

Second, starting from the unconditional means of h_t and h_t^*, we simulate their paths for $N = 50$ periods for each country, holding all other parameters at their benchmark RI values. We then compute and plot the standard deviation of RER growth and the variance of the interest rate differential as functions of θ, given the implied return volatilities $\exp(h)$ and $\exp(h^*)$.

Figure 7 summarizes these results. In the top panels, each solid line traces how a given moment varies with θ for a specific pair of simulated return volatilities. The upper-left panel shows that, in line with the constant-volatility benchmark, the variance of the interest rate differential falls as θ decreases (i.e., as RI strengthens). Because the term inside the brackets in Equation (55) is non-monotonic in θ, reflecting the interaction between RI and LLR, the standard deviation of RER growth is likewise non-monotonic, as illustrated in the upper-right panel. Overall, the core intuition from the benchmark model carries over: for any pair of return volatilities along the simulated paths, a lower (stronger RI) can reduce RER volatility.

To further assess the role of return volatility, the bottom panels of Figure 7 impose $h = h^*$ and jointly vary h and θ. A key additional insight is that, for any given θ, the standard deviation of RER growth first declines and then rises as return volatility increases. This pattern arises because the volatility of the interest rate differential is itself non-monotonic in the underlying risky return $\alpha \exp(h/2)\varepsilon$..

6 Conclusion

In this paper, we incorporate limited information-processing capacity (rational inattention, RI) into a two-country, consumption-based asset-pricing framework with recursive preferences and complete home bias, and study household consumption–portfolio decisions under imperfect risk sharing. The closed-form solution allows us to isolate two channels through which RI shapes consumption dynamics and the real exchange rate (RER): (i) dampened and incomplete consumption adjustment, and (ii) RI-induced long-run consumption risk. Under RI, households update their beliefs about asset returns gradually and internalize the resulting long-run consumption risk. This behavior reduces optimal risky-asset holdings, lowers consumption volatility, and—because risk sharing is imperfect—also reduces RER volatility. These two RI channels have opposing implications for the covariance between contemporaneous marginal utility and consumption growth: the adjustment-damping channel weakens this covariance, while the long-run-risk channel strengthens it. When the damping effect dominates, the model generates a weaker RER–consumption link and a standard deviation of RER growth closer to the data, thus offering a potential resolution to the Backus–Smith puzzle. We show that these findings are robust along two dimensions. First, augmenting RI with infrequent plan adjustment (inattentiveness) to capture both intensive and extensive information frictions, we find that the intensive margin—gradual within-period updating—is quantitatively dominant; joint estimation implies an optimal inattentiveness interval of approximately one period. Second, allowing for stochastic volatility in equity returns does not overturn the core mechanisms. Finally, the calibrated information frictions are consistent with survey evidence from professional forecasters, providing empirical support for the quantitative relevance of the proposed channels.

References

[1] Angeletos, G. M., Huo, Z., and Sastry, K. A. (2021), "Imperfect Macroeconomic Expectations: Evidence and Theory," *NBER Macroeconomics Annual*, 35(1), 1-86.

[2] Bacchetta, P. and E. van Wincoop (2006), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review*, 96(3), 552-576.

[3] Backus, D. K., Gregory, A. W., and Telmer, C. I. (1993), "Accounting for Forward Rates in Markets for Foreign Currency," *Journal of Finance*, 48(5), 1887-1908.

[4] Backus, D. K., and Smith, G. W. (1993), "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods," *Journal of International Economics*, 35(3-4), 297-316.

[5] Bansal, R., and Yaron, A. (2004), "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59(4), 1481-1509.

[6] Bidder, R., and Dew-Becker, I. (2016), "Long-run Risk Is the Worst-Case Scenario," *American Economic Review*, 106(9), 2494-2527.

[7] Brandt, M. W., Cochrane, J. H., and Santa-Clara, P. (2006), "International Risk Sharing Is Better Than You Think, or Exchange Rates Are Too Smooth," *Journal of Monetary Economics*, 53(4), 671-698.

[8] Campbell, J. Y. (1993), "Intertemporal Asset Pricing without Consumption Data," *American Economic Review*, 487-512.

[9] Campbell, J. Y. and Viceira, L. M. (2002), *Strategic Asset Allocation: Portfolio Choice for Long-term Investors.* Oxford University Press, USA.

[10] Campbell, J. Y. (2017), *Financial Decisions and Markets: A Course in Asset Pricing.* Princeton University Press.

[11] Candian, G. (2019), "Information Frictions and Real Exchange Rate Dynamics," *Journal of International Economics*, 116, 189-205.

[12] Candian, G., De Leo, P., and Gemmi, L. (2025), "Managing Expectations with Exchange Rate Policy," Working Paper.

[13] Canova, F., and M.O. Ravn (1996), "International Consumption Risk Sharing," *International Economic Review* 37(3), 573-601.

[14] Cao, D., Evans, M. D., and Luo, W. (2020), "Real Exchange Rate Dynamics Beyond Business Cycles," *Available at SSRN 3552189.*

[15] Caselli, F., and Feyrer, J. (2007), "The Marginal Product of Capital," *Quarterly Journal of Economic*s 122(2), 535-568.

[16] Chien, Y., Lustig, H., and Naknoi, K. (2020), "Why Are Exchange Rates So Smooth? A Household Finance Explanation," *Journal of Monetary Economics* 112, 129-144.

[17] Coibion, O. and Gorodnichenko Y. (2015), "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review* 105(8), 2644–2678.

[18] Colacito, R., and Croce, M. M. (2011), "Risks for the Long Run and the Real Exchange Rate," *Journal of Political Economy* 119(1), 153-181.

[19] Dasgupta, K. and Mondria, J. (2018), "Inattentive Importers," *Journal of International Economics* 112(C), 150-165.

[20] Djeutem, E., and Kasa, K. (2013), "Robustness and Exchange Rate Volatility," *Journal of International Economics* 91(1), 27-39.

[21] Dumas, B., Harvey, C. R., and Ruiz, P. (2003), "Are Correlations of Stock Returns Justified by Subsequent Changes in National Outputs?" *Journal of International Money and Finance* 22(6), 777-811.

[22] Engel C., and K. West (2005), "Exchange Rates and Fundamentals," *Journal of Political Economy*, 113, 485-517.

[23] Epstein, L. G., and Zin, S. E. (1991), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy* 99(2), 263-286.

[24] Gabaix, X., and Laibson, D. (2001), "The 6D Bias and the Equity-premium Puzzle," *NBER Macroeconomics Annual* 16, 257-312.

[25] Gourinchas, P. O., and Tornell, A. (2004), "Exchange Rate Puzzles and Distorted Beliefs," *Journal of International Economics* 64(2), 303-333.

[26] Hansen, L. P., Heaton, J. C., and Li, N. (2008), "Consumption Strikes Back? Measuring Long-run Risk," *Journal of Political Economy* 116(2), 260-302.

[27] Itskhoki, O., and Mukhin, D. (2021), "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy* 129(8), 2183-2232.

[28] Kollmann, R. (1995), "Consumption, Real Exchange Rates and the Structure of International Asset Markets," *Journal of International Money and Finance* 14(2), 191-211.

[29] Kollmann, R. (2015), "Exchange Rates Dynamics with Long-run Risk and Recursive Preferences," *Open Economies Review* 26, 175-196.

[30] Lewis, K. (1996), "What Can Explain the Apparent Lack of International Consumption Risk Sharing?" *Journal of Political Economy* 104(2), 267-297.

[31] Lewis, K. K., and Liu, E. X. (2015), "Evaluating International Consumption Risk Sharing Gains: An Asset Return View," *Journal of Monetary Economics* 71, 84-98.

[32] Li, W., Luo, Y., and Nie, J. (2017), "Elastic Attention, Risk Sharing, and International Comovements," *Journal of Economic Dynamics and Control* 79, 1-20.

[33] Luo, Y. (2010), "Rational Inattention, Long-run Consumption Risk, and Portfolio Choice," *Review of Economic Dynamics* 13(4), 843-860.

[34] Luo, Y., and Young, E. R. (2016). “Long-Run Consumption Risk and Asset Allocation under Recursive Utility and Rational Inattention,” *Journal of Money, Credit and Banking* 48(2-3), 325-362.

[35] Mankiw, N. G., and Zeldes, S. P. (1991), “The Consumption of Stockholders and Nonstockholders,” *Journal of Financial Economics*, 29(1), 97-112.

[36] Meese, R. A., and Rogoff, K. (1983), “Empirical Exchange Rate Models of the Seventies: Do They Fit out of Sample?” *Journal of International Economics* 14(1-2), 3-24.

[37] Merton, R. C. (1971), “Optimum Consumption and Portfolio Rules in a Continuous-Time Model,” *Journal of Economic Theory* 3(4), 373-413.

[38] Molavi, P., Tahbaz-Salehi, A., and Vedolin, A. (2024), ”Model Complexity, Expectations, and Asset Prices,” *Review of Economic Studies*, 91(4), 2462-2507.

[39] Obstfeld, M., and Rogoff, K. (2001), “The Six Major Puzzles in International Macroeconomics: Is There A Common Cause?” *NBER Macroeconomics Annual* 15, 339-390.

[40] Parker, J. A. (2001), “The Consumption Risk of the Stock Market,” *Brookings Papers on Economic Activity* 2001(2), 279-348.

[41] Parker, J. A. (2003), “Consumption Risk and Expected Stock Returns,” *American Economic Review* 93(2), 376-382.

[42] Parker, J. A., and Julliard, C. (2005), “Consumption Risk and the Cross Section of Expected Returns,” *Journal of Political Economy* 113(1), 185-222.

[43] Reis, R. (2006), “Inattentive Consumers,” *Journal of Monetary Economics*, 53(8), 1761-1800.

[44] Sims, C. A. (2003), “Implications of Rational Inattention,” *Journal of Monetary Economics* 50 (3), 665-690.

[45] Sims, C. A. (2010), “Rational Inattention and Monetary Economics,” *Handbook of Monetary Economics*.

[46] Sandulescu, M., Trojani, F., and Vedolin, A. (2021), “Model-free International Stochastic Discount Factors,” *Journal of Finance* 76(2), 935-976. (18-18).

[47] Taylor, S. J. (1986), *Modelling Financial Time Series*. Wiley, Chichester.

[48] Viceira, L. M. (2001), “Optimal Portfolio Choice for Long-horizon Investors with Nontradable Labor Income,” *Journal of Finance* 56(2), 433-470.

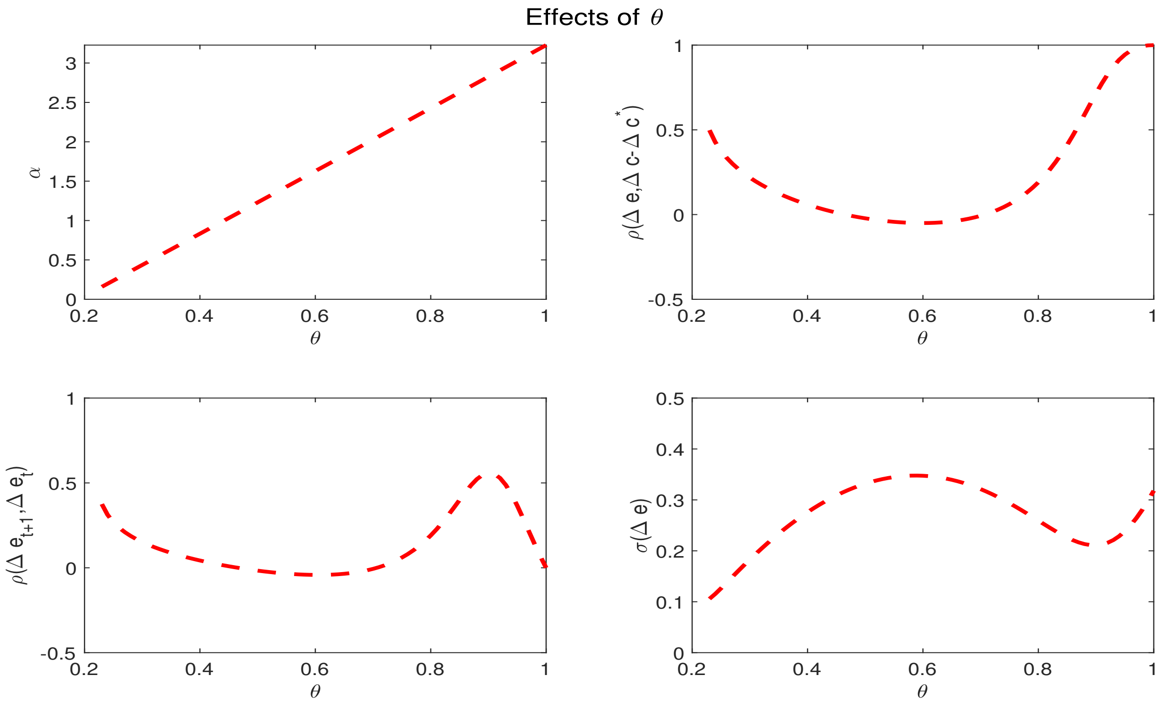

Figure 1: Effects of θ on α and Dynamics of the RER When $\gamma > 1/\psi$

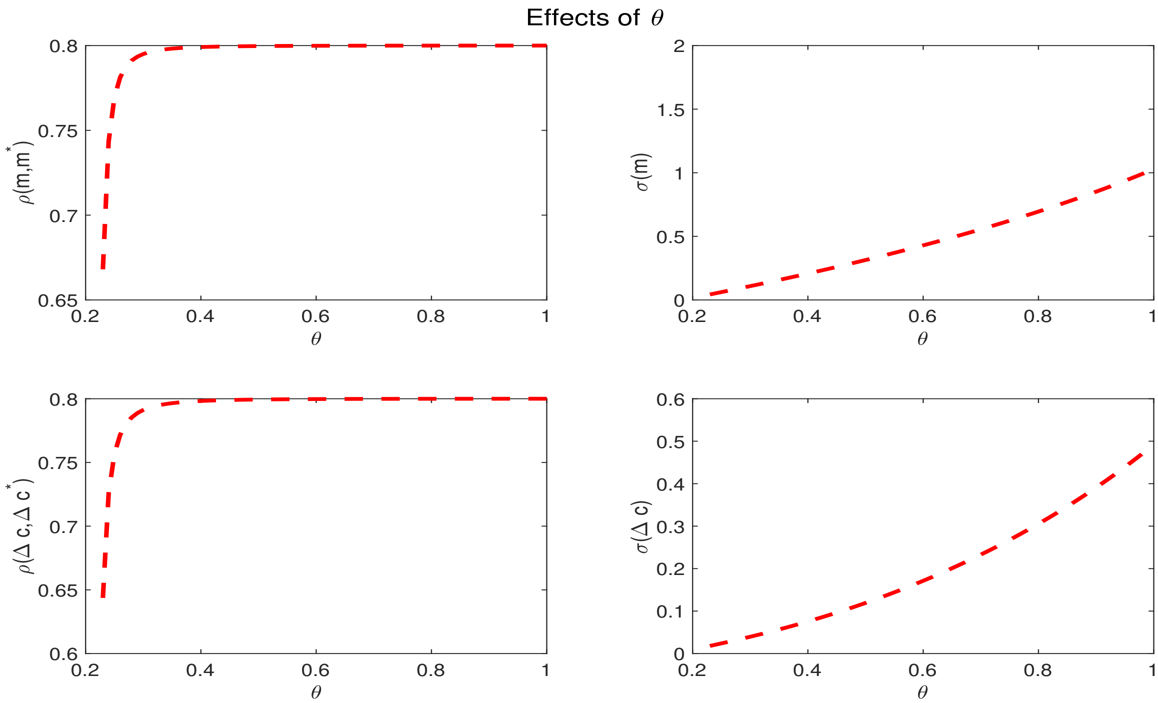

Figure 2: Effects of θ on SDFs and Consumption Growth When $\gamma > 1/\psi$

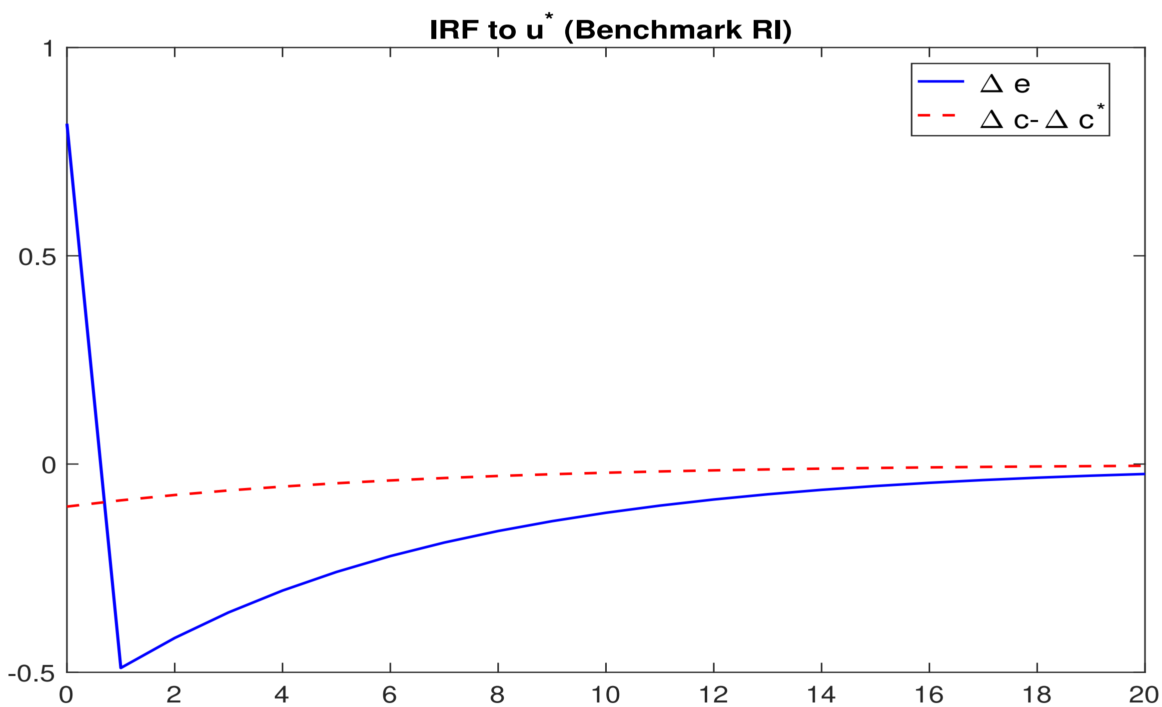

Figure 3: Impulse Responses of RER growth and Consumption Growth Differential to an Equity Return Shock

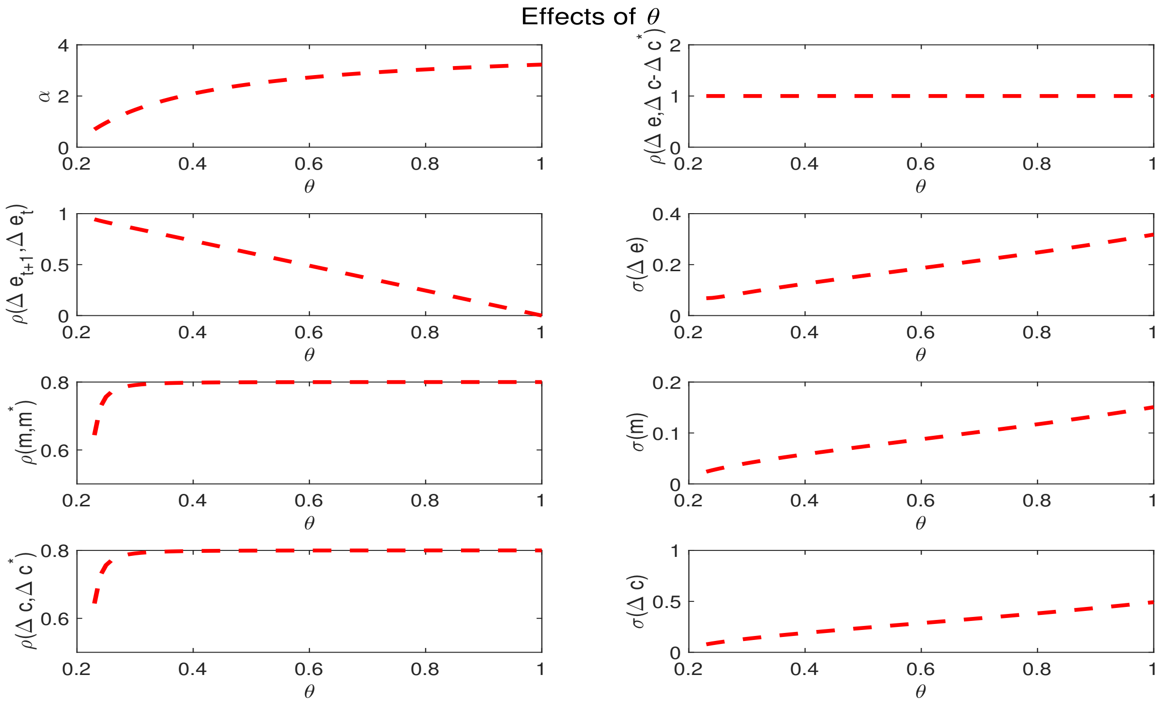

Figure 4: Key Moments as θ Varies, $\gamma = 1/\psi$ (EU)

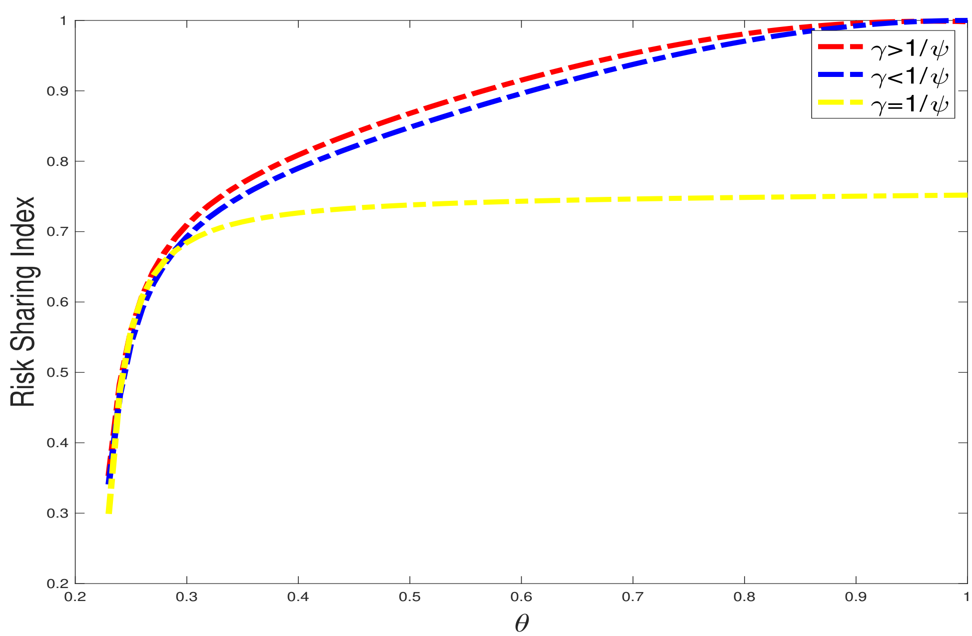

Figure 5: Risk Sharing Index as θ Varies

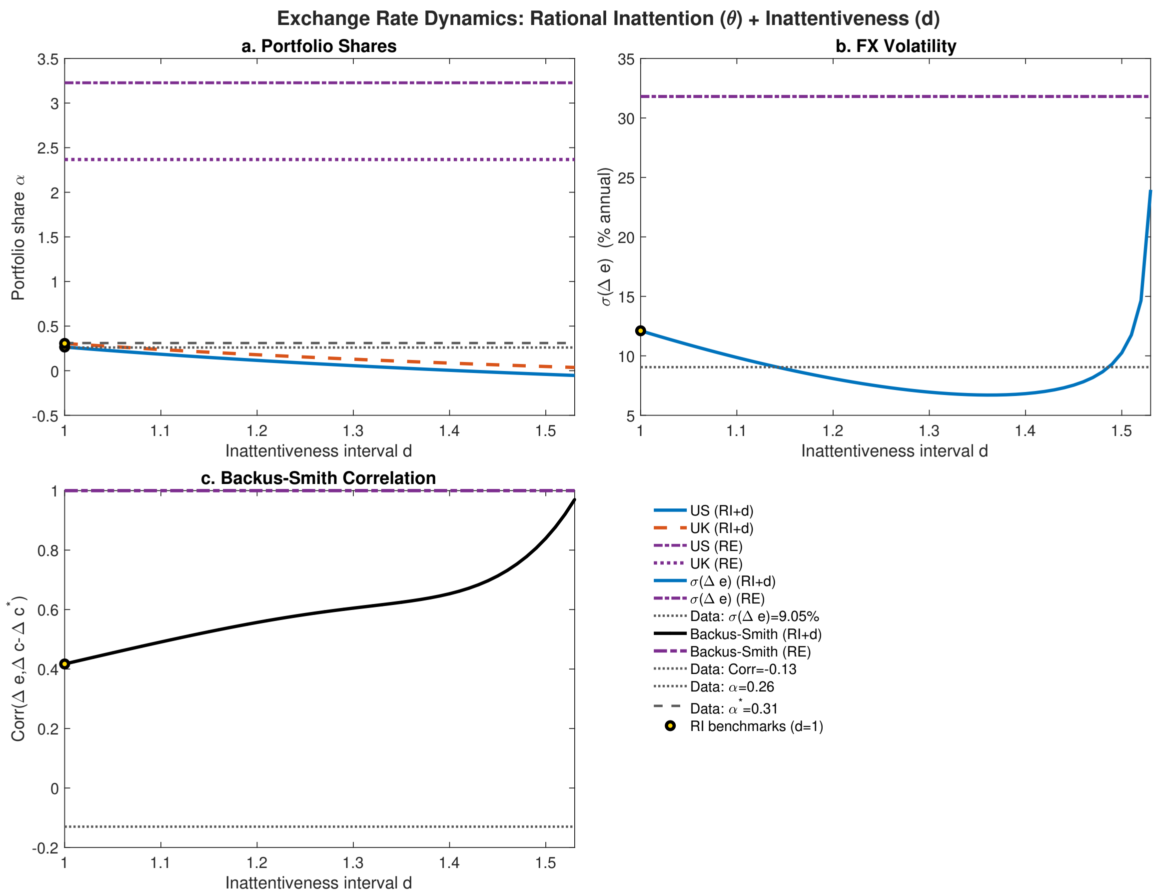

Figure 6: Targeted Moments as a function of inattentiveness interval d. Yellow circles mark the RI only benchmark ($d = 1$) from Table 4.

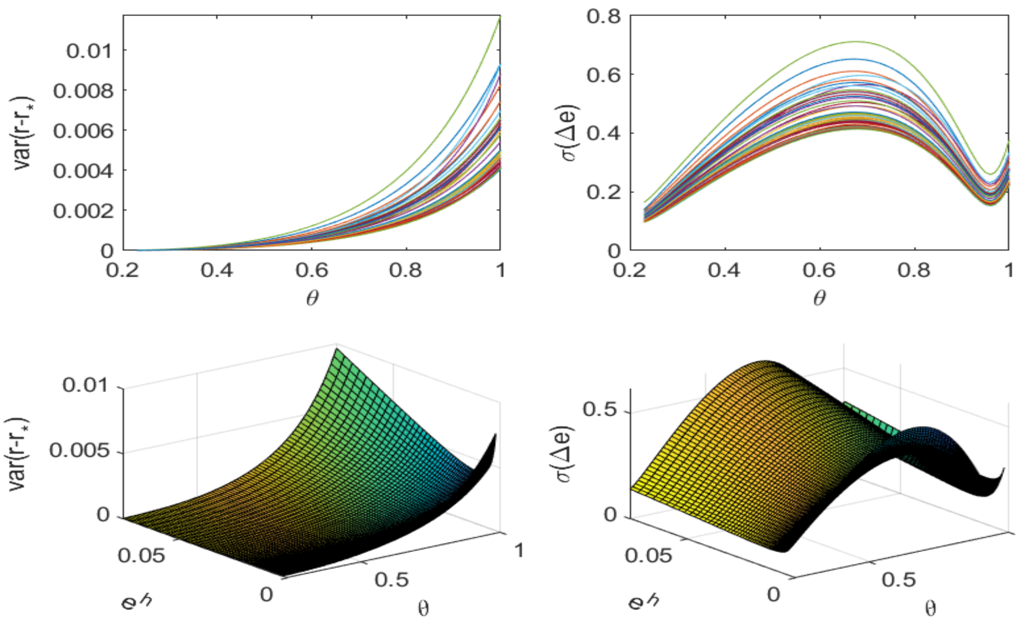

Figure 7: Standard Deviation of FX Growth and Variance of Interest Rate Differential as θ and Stochastic Volatilities Vary, $\gamma > 1/\psi$

Table 1: Some Puzzles in International Macro

Variable	Definition	United Kingdom	Euro Area	Canada	Japan
$\sigma(\Delta e)$	Standard deviation of RER growth				
	Data	9.05%	10.09%	8.05%	10.36%
$\rho(\Delta e_{t+1}, \Delta e_t)$	Autocorrelation of RER growth				
	Data	.06	.00	$-.09$	.10
$\rho(\Delta e, \Delta c - \Delta c^*)$	Correlation of RER growth and consumption growth differentials				
	Data	$-.13$	.04	$-.10$	.06
$\sigma(\Delta c)$	Standard deviation of consumption growth				
	Data	2.91%	2.02%	2.21%	3.52%
$\rho(\Delta c, \Delta c^*)$	Correlation of consumption growth				
	Data	.52	.64	.23	.00

Note: All data are from 1991m1 - 2018m12 except the Euro zone. The data source are International Financial Database(IFS) and Datastream. All moments are annualized. Norminal exchange rates data are monthly end-of-period exchange rates, CPI is consumer price index for all items and consumption data is quarterly real household expenditure per capita.

Table 2: Total Financial Assets Held by the Household Sector as Ratios of Consumption

	Total Financial Assets as ratios of Consumption				
Country/Area	U.S.	UK	Euro Zone	Canada	Japan
Ratios	5.39	4.49	3.46	5.05	5.53

Data source: Eurostat, IFS and national source.

Table 3: Baseline Calibration: U.S.-U.K.

Parameter	Definition	Value
Conventional parameter		
β	Subjective discount factor	.998
From data		
ϕ^*	Saving-wealth ratio for foreign economy	0.78
ϕ	Saving-wealth ratio for home economy	0.81
$\mu^* - r^*$	Average excess return for foreign economy	4.2%
$\mu - r$	Average excess return for U.S.	6.8%
ω^*	Standard deviation of foreign stock indices	14.45%
ω	Standard deviation of US stock indices	15.25%
ρ	Correlation of US stock and foreign stock	0.80
Calibrated parameters		
ψ	Intertemporal elasticity of substitution	0.989
γ	Risk aversion	1.061
θ^*	Kalman gain for foreign investors	0.34
θ	Kalman gain for US investors	0.26

Table 4: Model Comparison: U.S.-U.K.

Variable	Definition	U.S.	U.K.	RE	RI	RE*
$\rho(m^*, m)$	Correlation of pricing kernels	$\cdots$	$\cdots$	.80	.78	0.80
$\sigma(\Delta e)$	Standard deviation of RER growth	9.05%		31.80%	12.08%	31.68%
$\rho(\Delta e_{t+1}, \Delta e_t)$	Autocorrelation of RER growth	.06		0	.32	0
$\rho(\Delta e, \Delta c - \Delta c^*)$	Correlation of RER growth and consumption growth differentials	$-.13$		1	.42	1
$\sigma(\Delta c)$	std. of consumption growth	2.62%		170.50%	2.61%	14.00%
$\sigma(\Delta c^*)$	Foreign std. of consumption growth		2.92%	118.50%	2.83%	9.80%
α	Share of the risky asset	.26		3.23	.26	.26
α^*	Foreign share of the risky asset		.31	2.37	.31	.20

Notes: The RE column reports the full-information model evaluated at the same calibrated structural parameters (γ, ψ) as in the RI specification, with $\theta = \theta^* = 1$. The RE* column reports the best-fitting full-information model obtained by re-optimizing (γ, ψ) to minimize the target distance under rational expectations.

Table 5: Baseline Calibration: U.S.-Japan

Parameter	Definition	Value
Conventional parameter		
β	Subjective discount factor	.998
From data		
ϕ^*	Saving-wealth ratio for foreign economy	0.82
ϕ	Saving-wealth ratio for U.S.	0.81
$\mu^* - r^*$	Average excess return for foreign economy	0.51%
$\mu - r$	Average excess return for U.S.	6.8%
ω^*	Standard deviation of foreign stock indices	18.58%
ω	Standard deviation of U.S. stock indices	15.25%
ρ	Correlation of U.S. stock and foreign stock	0.53
Calibrated parameters		
ψ	Intertemporal elasticity of substitution	0.999996
γ	Risk aversion	1.00038
θ^*	Kalman gain for foreign investors	0.57
θ	Kalman gain for U.S. investors	0.23

Table 6: Model Comparison: U.S.-Japan

Variable	Definition	U.S.	Japan	RE	RI
$\rho(m^*, m)$	Correlation of pricing kernels	$\cdots$	$\cdots$	.53	.45
$\sigma(\Delta e)$	Standard deviation of RER growth	10.36%		46.84%	18.81%
$\rho(\Delta e_{t+1}, \Delta e_t)$	Autocorrelation of RER growth	.10		0	.15
$\rho(\Delta e, \Delta c - \Delta c^*)$	Correlation of RER growth and consumption growth differentials	.06		1	0.11
$\sigma(\Delta c)$	Std. of consumption growth	2.62%		180.43%	4.30%
$\sigma(\Delta c^*)$	Foreign std. of consumption growth		3.52%	41.65%	9.58%
α	Share of the risky asset	.26		3.41	.12
α^*	Foreign share of the risky asset		.21	2.52	.22

Table 7: Baseline Calibration: U.S.-Canada

Parameter	Definition	Value
Conventional parameter		
β	Subjective discount factor	.998
From data		
ϕ^*	Saving-wealth ratio for foreign economy	0.80
ϕ	Saving-wealth ratio for home economy	0.81
$\mu^* - r^*$	Average excess return for foreign economy	3.51%
$\mu - r$	Average excess return for U.S.	6.8%
ω^*	Standard deviation of foreign stock indices	14.31%
ω	Standard deviation of US stock indices	15.25%
ρ	Correlation of US stock and foreign stock	0.79
Calibrated parameters		
ψ	Intertemporal elasticity of substitution	0.99777
γ	Risk aversion	1.01363
θ^*	Kalman gain for foreign investors	0.31
θ	Kalman gain for US investors	0.26

Table 8: Model Comparison: U.S.-Canada

Variable	Definition	U.S.	Canada	RE	RI
$\rho(m^*, m)$	Correlation of pricing kernels	$\cdots$	$\cdots$	.793	.786
$\sigma(\Delta e)$	Standard deviation of RER growth	8.05%		33.19%	12.70%
$\rho(\Delta e_{t+1}, \Delta e_t)$	Autocorrelation of RER growth	$-.09$		0	0.27
$\rho(\Delta e, \Delta c - \Delta c^*)$	Correlation of RER growth and consumption growth differentials	$-.10$		1	0.37
$\sigma(\Delta c)$	std. of consumption growth	2.62%		51.39%	2.57%
$\sigma(\Delta c^*)$	Foreign std. of consumption growth		2.21%	31.23%	2.20%
α	Share of the risky asset	.26		3.37	.27
α^*	Foreign share of the risky asset		.27	2.18	.25

Table 9: Intensive vs. Extensive Margins of Information Friction: U.S.–U.K

	Data	RE	Baseline		FX-Weight GMM	
			RI+Inattentiveness $d \neq 1$	RI Only $d = 1$	RI+Inattentiveness $d \neq 1$	RI Only $d = 1$
$\sigma(\Delta e)$ (%)	9.05	31.80	12.08	12.08	8.92	8.87
$\text{Corr}(\Delta e, \Delta c - \Delta c^*)$	-0.13	1.00	0.42	0.42	0.20	0.20
α (U.S.)	0.26	3.23	0.26	0.26	0.10	0.10
α^* (U.K.)	0.31	2.37	0.31	0.31	0.72	0.71
$\sigma(c)$ (%)	2.62	170.50	2.61	2.61	12.80	13.03
$\sigma(c^*)$ (%)	2.92	118.50	2.83	2.83	26.10	26.43
d	–	1.00	1.01	1.00	2.87	1.00
θ (U.S.)	–	1.00	0.26	0.26	0.46	0.19
θ^* (U.K.)	–	1.00	0.34	0.34	0.52	0.22
ψ	–	0.989	0.989	0.989	1.055	1.055
γ	–	1.061	1.061	1.061	0.997	0.998

Notes: "RE" evaluates the full-information model at the same (ψ, γ) as the Baseline specification, with $(\theta, \theta^*, d) = (1, 1, 1)$. "RI+Inattentiveness" columns allow the inattentiveness interval d to be estimated freely; "RI Only" columns fix $d = 1$. Baseline uses the same weighting and parameter estimates as Table 4; FX-Weight GMM uses $w = [50, 50, 1, 1, 1, 1]$, emphasizing exchange rate moments.

7 Online Appendix (Not for Publication)

7.1 Online Appendix A: Rational-inattentive Investors with Concerns for Long-run Consumption Risk

If an investor has limited information-processing ability but does not take the induced long-run consumption risk into account, the standard consumption-based Euler equation applies:

$$\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1-\gamma}{\psi-1}} R_{p,t+1}^{\frac{1/\psi-\gamma}{1-1/\psi}}\left(R_{e,t+1}-R_f\right)\right]=0,$$

where $M_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\rho}{\psi}} R_{p,t+1}^{\rho-1}$ is the stochastic discount factor. Taking logs of the original variables and rearranging, we have

$$\mu - r_f + 0.5\omega^2 = -\operatorname{Cov}_t(m_{t+1}, r_{e,t+1}),$$

where the covariance of the log stochastic discount factor and the log equity return can be written as:

$$\begin{aligned} -\operatorname{Cov}_t(m_{t+1}, r_{e,t+1}) &= \operatorname{Cov}_t\left(\frac{\rho}{\psi}\Delta c_{t+1} + (1-\rho)\, r_{p,t+1+j}, r_{e,t+1}\right) \qquad (61)\\ &= \frac{\rho\theta}{\psi}\cdot\alpha\omega^2 + (1-\rho)\cdot\alpha\omega^2. \end{aligned}$$

Then the optimal portfolio and consumption rule becomes

$$\alpha_{RI} = \left(\frac{(1-\gamma)}{(\psi-1)}\theta + \frac{\gamma-1/\psi}{1-1/\psi}\right)^{-1}\frac{\mu-r_f+0.5\omega^2}{\gamma\omega^2}, \qquad (62)$$

$$\Delta c_{t+1}^{RI} = \theta\left[\frac{\alpha_{RI}u_{t+1}}{1-((1-\theta)/\phi)\cdot L} + \left(\xi_{t+1} - \frac{(\theta/\phi)\,\xi_t}{1-((1-\theta)/\phi)\cdot L}\right)\right]. \qquad (63)$$

In contrast, when the investor takes the induced long-run consumption risk into account, as argued in Section 3, we need to use the long-run Euler equation to derive the optimal decisions, i.e., the future consumption growth matters. Considering the consumption Euler equation between period t and $t+1+S$ for the risky asset, the standard Euler equation can be revised and solved forward to be

$$\mathbb{E}_t\left[R_f^S\left(\frac{C_{t+1+S}}{C_t}\right)^{-\frac{\rho}{\psi}} R_{p,t+1}^{\rho-1}R_{p,t+1}^{\rho-1}\cdots R_{p,t+1+S}^{\rho-1}R_{e,t+1}\right]=1,$$

Rearranging the above equation and we get

$$\mathbb{E}_t\left[\left(R_f^S\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\rho}{\psi}}\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\frac{\rho}{\psi}}\cdots\left(\frac{C_{t+1+S}}{C_{t+S}}\right)^{-\frac{\rho}{\psi}} R_{p,t+1}^{\rho-1}R_{p,t+1}^{\rho-1}\cdots R_{p,t+1+S}^{\rho-1}\right)R_{e,t+1}\right]=1, \qquad (64)$$

By exploiting lognormality, the above equation can be written further as

$$
\begin{aligned}
0 = & -\frac{\rho}{\psi}\mathbb{E}_t\left[\sum_{j=0}^{S}\Delta c_{t+1+S}\right] + (\rho-1)\,\mathbb{E}_t\left[\sum_{j=0}^{S} r_{p,t+1+S}\right] + S\cdot r_f + \mu \\
& + \frac{1}{2}\operatorname{Var}_t\left[r_{e,t+1}\right] + \frac{1}{2}\operatorname{Var}_t\left[-\frac{\rho}{\psi}\sum_{j=0}^{S}\Delta c_{t+1+S} + (\rho-1)\sum_{j=0}^{S} r_{p,t+1+S}\right] \\
& + \operatorname{Cov}_t\left(-\frac{\rho}{\psi}\sum_{j=0}^{S}\Delta c_{t+1+S} + (\rho-1)\sum_{j=0}^{S} r_{p,t+1+S}, r_{e,t+1}\right)
\end{aligned}
$$

Replacing $R_{e,t+1}$ with R_f in Equation (64) we can derive another similar equation. Taking the difference of the two equations, we can get Equations (16) and (17) in the main text.

7.2 Online Appendix B: Deriving the Expressions for Volatilities and Correlations

Using the expression for Δe_{t+1}, the volatility of the RER growth is:

$$
\begin{aligned}
\operatorname{Var}\left(\Delta e_{t+1}\right) = & \left(\frac{1-\gamma}{\psi-1}\right)^2 \operatorname{Var}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right) + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2 \operatorname{Var}\left(r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right) \\
& - 2\frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\operatorname{Cov}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}, r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right),
\end{aligned} \tag{65}
$$

and the correlation of the real FX growth and consumption growth differential is

$$
\operatorname{Corr}(\Delta e, \Delta c - \Delta c^*) = \frac{\frac{1-\gamma}{\psi-1}\operatorname{Var}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right) - \frac{1/\psi-\gamma}{1-1/\psi}\operatorname{Cov}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}, r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right)}{\sqrt{\operatorname{Var}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right)\operatorname{Var}\left(\Delta e^{RI}\right)}}, \tag{66}
$$

where

$$
\begin{aligned}
\operatorname{Var}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right) = & \frac{\theta^2}{1-\left(\frac{1-\theta}{\phi}\right)^2}\alpha_{RI}^2\omega^2 + \frac{\theta^{*2}}{1-\left(\frac{1-\theta^*}{\phi}\right)^2}\alpha_{RI}^{*2}\omega^{*2} \\
& - 2\frac{\theta\theta^*}{1-\frac{1-\theta}{\phi}\frac{1-\theta^*}{\phi}}\alpha_{RI}\alpha_{RI}^*\rho_u\omega\omega^*, \\
\operatorname{Var}\left(r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right) = & \alpha_{RI}^2\omega^2 + \alpha_{RI}^{*2}\omega^{*2} - 2\rho_u\alpha_{RI}\alpha_{RI}^*\omega\omega^*, \\
\operatorname{Cov}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}, r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right) = & \theta\alpha_{RI}^2\omega^2 + \theta^*\alpha_{RI}^{*2}\omega^{*2} - \rho_u(\theta+\theta^*)\alpha_{RI}\alpha_{RI}^*\omega\omega^*.
\end{aligned}
$$

When $\theta = \theta^*$,

$$\begin{aligned}
\mathrm{Var}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}\right) &= \frac{\theta^2}{1-\left(\frac{1-\theta}{\phi}\right)^2}\alpha_{RI}^2\omega^2 + \frac{\theta^2}{1-\left(\frac{1-\theta}{\phi^*}\right)^2}\alpha_{RI}^{*2}\omega^{*2} \\
&\quad - 2\frac{\theta^2}{1-\left(\frac{1-\theta}{\phi}\right)\left(\frac{1-\theta}{\phi^*}\right)}\alpha_{RI}\alpha_{RI}^*\rho_u\omega\omega^*, \\
\mathrm{Var}\left(r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right) &= \alpha_{RI}^2\omega^2 + \alpha_{RI}^{*2}\omega^{*2} - 2\rho_u\alpha_{RI}\alpha_{RI}^*\omega\omega^*, \\
\mathrm{Cov}\left(\Delta c_{t+1}^{RI*} - \Delta c_{t+1}^{RI}, r_{p,t+1}^{RI*} - r_{p,t+1}^{RI}\right) &= \theta\alpha_{RI}^2\omega^2 + \theta\alpha_{RI}^{*2}\omega^{*2} - 2\rho_u\theta\alpha_{RI}\alpha_{RI}^*\omega\omega^*.
\end{aligned}$$

When $\theta = \theta^*$ and $\phi = \phi^*$, by rearranging the terms we get the expressions in Proposition 2. The expressions for the volatilities of consumption growth and their correlation naturally follow from the dynamics of aggregate consumption. Otherwise, when the information rigidities differ in the two countries, $\theta \neq \theta^*$, adjustments of consumption would differ more, $\frac{\theta\theta^*}{1-\frac{1-\theta}{\phi}\frac{1-\theta^*}{\phi}}$ becomes smaller holding other terms constant, leading to a larger variance of consumption differential hence a more volatile RER.

When $\theta = \theta^* = 1$, the volatility of the RER becomes $\mathrm{Var}(\Delta e) = \gamma^2(\alpha^2\omega^2 + \alpha^{*2}\omega^{*2} - 2\alpha\alpha^*\rho_u\omega\omega^*)$, which is the same as what we derived under RE, and $\mathrm{Corr}(\Delta e, \Delta c - \Delta c^*)$ is reduced to 1.

7.3 Online Appendix C: Deriving the Expressions for Autocorrelations and Other Moments

Using the expression for Δc, the autocovariance of consumption growth can be derived as follows:

$$\begin{aligned}
\mathrm{AC}(\Delta c) &= \mathrm{Cov}\left(\theta\left(\frac{\alpha_{RI}u_{t+1}}{1-((1-\theta)/\phi)\cdot L}\right), \theta\left[\frac{\alpha_{RI}u_t}{1-((1-\theta)/\phi)\cdot L}\right]\right) \\
&= \frac{\theta^2\alpha_{RI}^2(1-\theta)/\phi}{1-((1-\theta)/\phi)^2}\omega^2.
\end{aligned}$$

The autocovariance of the consumption growth differential can then be written as:

$$\begin{aligned}
&\mathrm{AC}(\Delta c^* - \Delta c) \\
&= \mathrm{AC}(\Delta c) + \mathrm{AC}(\Delta c^*) - \mathrm{Cov}(\Delta c_{t+1}, \Delta c_t^*) - \mathrm{Cov}(\Delta c_{t+1}^*, \Delta c_t) \\
&= \mathrm{AC}(\Delta c) + \mathrm{AC}(\Delta c^*) - \frac{\theta\theta^*\alpha_{RI}\alpha_{RI}^*}{1-\frac{1-\theta^*}{\phi^*}\frac{1-\theta}{\phi}}\rho_u\omega\omega^*\left(\frac{1-\theta}{\phi} + \frac{1-\theta^*}{\phi^*}\right),
\end{aligned}$$

and we can calculate the autocovariance of the RER growth using the above moments:

$$\mathrm{AC}(\Delta e) = \left(\frac{1-\gamma}{\psi-1}\right)^2 \mathrm{AC}(\Delta c^* - \Delta c) - \frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\mathrm{Cov}(\Delta c_{t+1}^* - \Delta c_{t+1}, r_{p,t}^* - r_{p,t}).$$

The variance and covariance of the SDF can then be written as:

$$\mathrm{Var}(m)=\left(\frac{1-\gamma}{\psi-1}\right)^2\mathrm{Var}(\Delta c)+\left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2\mathrm{Var}(r_p)-\frac{1-\gamma}{\psi-1}\frac{1/\psi-\gamma}{1-1/\psi}\mathrm{Cov}(\Delta c,r_p),$$

and

$$\begin{aligned}\mathrm{Cov}(m,m^*)&=\left(\frac{1-\gamma}{\psi-1}\right)^2\mathrm{Cov}(\Delta c,\Delta c^*)+\left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2\mathrm{Cov}(r_p,r_p^*)\\&=\left(\frac{1-\gamma}{\psi-1}\right)^2\frac{\theta\theta^*\alpha_{RI}\alpha_{RI}^*\rho_u\omega\omega^*}{1-((1-\theta)/\phi)((1-\theta^*)/\phi^*}+\left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2\alpha_{RI}\alpha_{RI}^*\rho_u\omega\omega^*,\end{aligned}$$

respectively.

7.4 Online Appendix D: Deriving the Wealth Transition, Aggregate Consumption and Key Moments under Inattentiveness and RI

We first derive the perceived wealth transition under both inattentiveness and RI in Section 5.1. For the adjusting consumer in time $t+1$, we combine the wealth transition equation during these periods and get

$$\phi^{d-1}a_{t+d}-\phi^{-1}a_t=\left(1-\phi^{-1}\right)\sum_{i=0}^{d-1}\phi^ic_{t+i}+\sum_{i=0}^{d-1}\phi^i\left(\Lambda+r_{p,t+1+i}\right).$$

Since t and $t+d$ are the two subsequent adjusting periods, consumption between the periods follows the deterministic path (42):

$$\phi^{d-1}a_{t+d}=\phi^{-1}a_t+\frac{\phi^d-1}{\phi}c_t^o+\left[\frac{\phi\left(1-\phi^{d-1}\right)}{(1-\phi)^2}-\frac{\phi^d}{1-\phi}(d-1)\right]A+\sum_{i=0}^{d-1}\phi^i\left(\Lambda+r_{p,t+1+i}\right).$$

At planning dates the consumer is subject to incomplete information $\widetilde{a}_{t+d}=a_{t+d}+\xi_{t+d}$, and he uses the Kalman filter to update his or her perceived state and infer the variance of the observational noise ξ_{t+d}. The perceived state can be written as:

$$a_{t+d}=(1-\theta)\left(\frac{1}{\phi^d}a_t+\frac{\phi^d-1}{\phi^d}c_t^o+\left[\frac{\phi\left(\phi^{1-d}-1\right)}{(1-\phi)^2}-\frac{\phi}{1-\phi}(d-1)\right]A+\frac{\left(1-\phi^d\right)}{\phi^{d-1}(1-\phi)}\Lambda-\log(1-K)\right)+\theta\widetilde{a}_{t+d} \tag{67}$$

Combining the above equation with the true wealth transition equation, we get

$$\widehat{a}_{t+d}=\frac{1}{\phi^d}\widehat{a}_t+\frac{\left(\phi^d-1\right)}{\phi^d}c_t^o+\left[\frac{\phi\left(\phi^{1-d}-1\right)}{(1-\phi)^2}-\frac{\phi}{1-\phi}(d-1)\right]A+\frac{\left(1-\phi^d\right)}{\phi^{d-1}(1-\phi)}\Lambda-\log(1-K)+\eta_{t+d},$$

where

$$\eta_{t+d}=\frac{\theta}{\phi^d}\left(a_t-\widehat{a}_t\right)+\theta\left(\sum_{i=0}^{d-1}\phi^ir_{p,t+1+i}+\xi_{t+d}\right),$$

and by employing Equations (67) and (43), we have

$$a_{t+d}-\widehat{a}_{t+d}=\frac{(1-\theta)\sum_{i=0}^{d-1}\phi^{i-d+1}r_{p,t+1+i}}{1-\left[(1-\theta)/\phi^{d}\right]\cdot\mathbb{L}^{d}}-\frac{\theta\xi_{t+d}}{1-\left[(1-\theta)/\phi^{d}\right]\cdot\mathbb{L}^{d}}.$$

In what follows, we derive the aggregate dynamics and key model moments. For the adjusting consumers in time $t+1$:

$$c_{t+1}^{o}=b_1+\widehat{a}_{t+1}, \tag{68}$$

where b_1 is determined in the same manner as b_0, and these consumers' consumption change is given by:

$$\begin{aligned}\Delta c_{t+1}^{adj}&=c_{t+1}^{o}-c_t\\&=b_0+\widehat{a}_{t+1}-\left[A\left(d-1\right)+c_{t+1-d}^{o}\right]\\&=\left(\widehat{a}_{t+1}-\widehat{a}_{t+1-d}\right)-A\left(d-1\right).\end{aligned}$$

If the consumer is inattentive in time $t+1$, we have:

$$\Delta c_{t+1}^{ina}=c_{t+1}-c_t=A. \tag{69}$$

The dynamics of aggregate consumption can thus be written as:

$$\begin{aligned}\Delta c_{t+1}&=\frac{1}{d}\left[\left(\widehat{a}_{t+1}-\widehat{a}_{t+1-d}\right)-A\left(d-1\right)\right]+\left(1-\frac{1}{d}\right)A\\&=\frac{1}{d}\left(\widehat{a}_{t+1}-\widehat{a}_{t+1-d}\right)\\&=\frac{1-\theta}{\phi^{d}}\left(a_{t+1-d}-\widehat{a}_{t+1-d}\right)+\theta\left(\sum_{i=0}^{d-1}\phi^{i}r_{p,t+2-d+i}+\xi_{t+1}\right)\\&=\frac{\theta}{d}\left[\frac{\sum_{i=0}^{d-1}\phi^{i-d+1}r_{p,t+2-d+i}}{1-\left[(1-\theta)/\phi^{d}\right]}+\xi_{t+1}-\frac{\theta/\phi^{d}\cdot\xi_{t+1}}{1-\left[(1-\theta)/\phi^{d}\right]}.\right]\end{aligned} \tag{70}$$

When the observational noise is idiosyncratic, the change in aggregate consumption reduces to:

$$\begin{aligned}\Delta c_{t+1}=\frac{\theta}{d}&\left[\phi^{-d+1}r_{p,t+2-d}+\phi^{-d+2}r_{p,t+3-d}+...r_{p,t+1}+\frac{1-\theta}{\phi^{d}}\left(\phi^{-d+1}r_{p,t+2-2d}+\phi^{-d+2}r_{p,t+3-2d}+...r_{p,t+1-d}\right)\right.\\&\left.+\left(\frac{1-\theta}{\phi^{d}}\right)^{2}\left(\phi^{-d+1}r_{p,t+2-3d}+\phi^{-d+2}r_{p,t+3-3d}+...r_{p,t+1-2d}\right)+...\right]\end{aligned} \tag{71}$$

Note that in the RI model without sticky information, aggregate consumption evolves according to:

$$\Delta c_{t+1}=\theta\left(r_{p,t+1}+\frac{1-\theta}{\phi}r_{p,t}+\left(\frac{1-\theta}{\phi}\right)^{2}r_{p,t-1}+...\right)$$

Optimal portfolio share. The ultimate consumption risk under RI can be expressed as:

$$
\begin{aligned}
-\operatorname{Cov}_t(m^s_{t+1}, r_{e,t+1}) &= -\operatorname{Cov}_t\left(-\frac{1-\gamma}{\psi-1}\sum_{j=0}^{s}\Delta c_{t+1+j} + \frac{1/\psi-\gamma}{1-1/\psi}\sum_{j=0}^{s} r_{p,t+1+j}, r_{e,t+1}\right) \\
&= \frac{\theta(1-\gamma)}{d(\psi-1)}\left[\phi^{-d+1}+\phi^{-d}+...1+\frac{1-\theta}{\phi^d}\left(\phi^{-d+1}+\phi^{-d}+...1\right)+...\right]\cdot\alpha_{RI}\omega^2 \\
&\quad -\frac{1/\psi-\gamma}{1-1/\psi}\cdot\alpha_{RI}\omega^2.
\end{aligned}
$$

When $s\to\infty$, this expression reduces to:

$$
\lim_{s\to\infty} -\operatorname{Cov}_t\left(-\frac{1-\gamma}{\psi-1}\sum_{j=0}^{s}\Delta c_{t+1+j} + \frac{1/\psi-\gamma}{1-1/\psi}\sum_{j=0}^{s} r_{p,t+1+j}, r_{e,t+1}\right) = \left(\frac{1-\gamma}{\psi-1}\zeta - \frac{1/\psi-\gamma}{1-1/\psi}\right)\alpha_{RI}\omega^2,
$$

where $\zeta = \frac{\theta\left(\phi^{-d+1}-\phi\right)}{d\left[1-(1-\theta)/\phi^d\right](1-\phi)}$, and $\phi = 1-\exp(c-a)$ is the steady state saving rate.

Exchange rate volatility. We first derive the variance of consumption growth differential and the covariance between the differential and the equity return::

$$
\begin{aligned}
\operatorname{Var}\left(\Delta c^{RI*}_{t+1}-\Delta c^{RI}_{t+1}\right) &= \frac{1}{d^2}\frac{\theta^2}{1-\left(\frac{1-\theta}{\phi^d}\right)^2}\frac{\phi^{2-2d}-\phi^2}{1-\phi^2}\alpha^2_{RI}\omega^2 + \frac{1}{d^2}\frac{\theta^{*2}}{1-\left(\frac{1-\theta^{*2}}{\phi^{*d}}\right)^2}\frac{(\phi^*)^{2-2d}-\phi^2}{1-\phi^{*2}}\alpha^{*2}_{RI}\omega^{*2} \\
&-\frac{2}{d^2}\frac{\theta\theta^*}{1-\frac{1-\theta}{\phi^d}\frac{1-\theta^*}{\phi^{*d}}}\frac{(\phi\phi^*)^{1-d}-\phi\phi^*}{1-\phi\phi^*}\alpha_{RI}\alpha^*_{RI}\rho_u\omega\omega^*,
\end{aligned}
$$

$$
\operatorname{Cov}\left(\Delta c^{RI*}_{t+1}-\Delta c^{RI}_{t+1}, r^{RI*}_{p,t+1}-r^{RI}_{p,t+1}\right) = \frac{\theta}{d}\alpha^2_{RI}\omega^2 + \frac{\theta^*}{d}\alpha^{*2}_{RI}\omega^{*2} - \frac{\rho_u(\theta+\theta^*)}{d}\alpha_{RI}\alpha^*_{RI}\omega\omega^*.
$$

From equation (10), the RER change is:

$$
\Delta e_{t+1} = -\frac{1-\gamma}{\psi-1}(\Delta c^*_{t+1}-\Delta c_{t+1}) + \frac{1/\psi-\gamma}{1-1/\psi}(r^*_{p,t+1}-r_{p,t+1}).
$$

Using this expression, the variance of the RER change is:

$$
\begin{aligned}
\operatorname{Var}(\Delta e_{t+1}) &= \left(\frac{1-\gamma}{\psi-1}\right)^2\operatorname{Var}(\Delta c^*_{t+1}-\Delta c_{t+1}) + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right)^2\operatorname{Var}(r^*_{p,t+1}-r_{p,t+1}) \\
&+2\left(\frac{1-\gamma}{\psi-1}\right)\left(\frac{1/\psi-\gamma}{1-1/\psi}\right)\operatorname{Cov}(\Delta c^*_{t+1}-\Delta c_{t+1}, r^*_{p,t+1}-r_{p,t+1}) \qquad (72)
\end{aligned}
$$

where

$$
\operatorname{Var}(r^*_{p,t+1}-r_{p,t+1}) = \alpha^{*2}\omega^{*2}+\alpha^2\omega^2-2\alpha\alpha^*\rho_u\omega\omega^*,
$$

$$
\operatorname{Corr}(\Delta e, \Delta c-\Delta c^*) = \frac{\frac{1-\gamma}{\psi-1}\operatorname{Var}(\Delta c^*-\Delta c) - \frac{1/\psi-\gamma}{1-1/\psi}\operatorname{Cov}(\Delta c^*-\Delta c, r^*_p-r_p)}{\sqrt{\operatorname{Var}(\Delta e)}\cdot\sqrt{\operatorname{Var}(\Delta c^*-\Delta c)}}.
$$