A Groundedness Predicate for Kripke's Theory of Truth

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Abstract

The object language of Kripke's 1975 semantic theory of truth, based on the Strong Kleene valuation scheme, cannot contain a predicate that express the notion of "ungroundedness" that Kripke provides an analysis of. This is unfortunate: it means that, in the object language of Kripke's theory, there is no obvious way to express Kripke's diagnostic insight about what causes semantic pathology. This paper shows how to introduce a "groundedness" predicate, G, to a Kripkean theory of truth that can fill this expressive gap. In the fixed-point construction that determines an interpretation for G, G's anti-extension tracks networks of sentences that, due to predications of truth, result in non-terminating trees of semantic dependence. In the fixed-point models that provide a class of intended interpretations for G: (i) every sentence with a classical semantic value is in the extension of G, (ii) every sentence in the anti-extension of G receives the value $\frac{1}{2}$, and (iii) the anti-extension of G includes *all* the sentences that receive $\frac{1}{2}$ in the corresponding K3 model of Kripke's original theory. The language $\mathcal{L}$[T,G] has the resources for describing Kripke's diagnostic insight as it applies to $\mathcal{L}$[T,G].

It is well-known that the object language described by Kripke's 1975 semantic theory of truth, based on the Strong Kleene valuation scheme, cannot contain a predicate/operator that expresses "ungroundedness."[1] This would be an object language predicate that is true of all sentences that do not receive a classical truth-value in the fixed-point model one is taking as a semantic theory. The introduction of such a predicate would undermine the monotonicity of the jump operation used to construct Kripke's fixed-point models. The simplest way to see that is to note that the resulting language would contain Strengthened Liar sentences like (1):

[1]These points hold as well regarding "exclusion negation."

(1) (1) is not true or (1) is ungrounded.

There is no fixed-point model based on the Strong Kleene scheme in which (1) is in the extension of "ungrounded" if and only if it does not receive a classical value. Given the intended interpretation of "ungrounded," is inconsistent to suppose that (1) doesn't have a classical value, and any model in which (1) does have a classical value cannot be a fixed-point.

This aspect of Kripke's theory is unfortunate. As Hartry Field puts it, we would like to be able to express in the object language, in an assertive way, that pathological sentences like the Liar (2008 pg. 94-96) are to be *rejected*. If there is no predicate in the object language that expresses ungroundedness, it is not clear how we can do this. Moreover, Kripke's semantic theory has struck many as offering an insightful *diagnosis* of Liar-like pathology, and it would nice to be able to express this in the object language as well. That diagnosis, roughly, is that sentences that predicate truth semantically *depend* on the sentences they predicate truth of, and Liar-like pathology is caused by being part of a non-well-founded network of semantic dependence. Since the object language of Kripke's theory cannot contain a predicate that holds just of the sentences that are semantically ungrounded in this way, it would appear that there are no resources in the object language for expressing Kripke's diagnostic insight. Part of the attraction of Kripke's theory rests on the fact that it makes intelligible how we can talk, in the object language, about the truth of of object language sentences, so it is, *prima facie*, a defect if so much of what is philosophically interesting regarding Kripke's response to the semantic *paradoxes* can only be expressed once we make the reflective ascent to a metalanguage.

I'll suggest in this paper that we can develop a Kripkean theory of truth that goes a long way to satisfying these desiderata if we relax our standard for what counts as a ungroundedness predicate. Arguably, the intended interpretation of a "ungroundedness" predicate described above—a predicate that is true of all sentences that do not receive a classical value—is stronger than it is reasonable to demand. It would not be so surprising if a groundedness/ungroundedness predicate is susceptible to semantic failure for similar reasons that a truth/falsity predicate is. And if so, we might expect that, for some objects that are ungrounded, applying the groundedness/ungroundednes predicate to those objects just results in semantic failure rather than a claim with a truth value.

Here I develop a fixed-point semantics for a language $\mathcal{L}$[T,G] that extends the usual language for a Kripkean theory of truth, $\mathcal{L}$[T], by adding a *Groundedness* predicate G. In the fixed-point models that will be my focus, G has the following properties:

- Every sentence with a classical truth value that model is in the extension of G.
- Every sentence in the anti-extension of G receives the value $\frac{1}{2}$ in that model.

- The anti-extension of G in that model includes *all* the sentences that receive $\frac{1}{2}$ in the corresponding K3 model (of Kripke's original theory).

I begin, in Section 1, by introducing the language $\mathcal{L}$[T,G] and defining the notions relevant to the jump operation by which I construct fixed-point models. In Section 2, following Kripke, I define the sequences that lead to the construction of such models. In Section 3, I compare my $\mathcal{L}[T, G]$ models to the corresponding K3 models of Kripke's original semantics. The main result of this section is Theorem 8: for all sentences s of $\mathcal{L}$[T], s receives the value $\frac{1}{2}$ in a K3 model if and only if s's Gödel code is in the anti-extension of "G," in the corresponding $\mathcal{L}$[T,G] model. I close, in Section 4, by elaborating on my suggestion that, by introducing the G predicate, we end up with a language in which we can affirmatively express rejection of semantically pathological sentences, and can express the core diagnostic insight of Kripke's theory of truth. I also compare my Groundedness predicate to Roy Cook's 2007 hierarchy of "pathology" predicates, which are introduced to solve a similar problem.

1 Definitions

Let $\mathcal{L}$ be the language of Peano Arithmetic. I'll suppose that $\mathcal{L}$ contains only $\neg, \vee$ and the existential quantifier as logical operators. Let $\mathcal{L}$[T] be $\mathcal{L}$ with an additional unary predicate, T, the truth predicate; let $\mathcal{L}$[T,G] be $\mathcal{L}$[T] with an additional unary predicate G, the groundedness predicate. A **non-alethic** sentence of $\mathcal{L}$[T,G] is any sentence that lacks an occurrence of T.[2] I assume throughout that in $\mathcal{L}$[T,G] there is some suitable Gödel-coding of the sentences of $\mathcal{L}$[T,G] (and I'll sometimes identify sentences with their codes).

A $\mathcal{L}[T, G]$ **model** is a triple $\mathcal{M} = \langle N, \langle T_+, T_- \rangle, \langle G_+, G_- \rangle \rangle$, where N is a standard classical interpretation of $\mathcal{L}$, T_+ is the extension of T, T_- is the anti-extension of T, G_+ is the extension of G, and G_- is the anti-extension of G. Naturally, removing the interpretation of G from an $\mathcal{L}[T, G]$ results in a K3 model for $\mathcal{L}[T]$. A valuation v of an $\mathcal{L}[T, G]$ model is a function from $\mathcal{L}[T, G]$ sentences to $\{0, \frac{1}{2}, 1\}$, defined by the Strong Kleene Scheme.[3] A simplified representation of a model $\mathcal{M}$ for $\mathcal{L}$[T,G] is $\mathcal{M}$[T,G], where T abbreviates $\langle \mathrm{T}_+, \mathrm{T}_- \rangle$, the "truth concept" of $\mathcal{M}$, and G abbreviates $\langle \mathrm{G}_+, \mathrm{G}_- \rangle$, the "groundedness concept" of $\mathcal{M}$.

A truth concept T$'$ **extends** a truth concept T iff $\mathrm{T}_+ \subseteq \mathrm{T}'_+$ and $\mathrm{T}_- \subseteq \mathrm{T}'_-$. Likewise, a groundedness concept G$'$ extends a groundedness concept G iff $\mathrm{G}_+ \subseteq \mathrm{G}'_+$ and $\mathrm{G}_- \subseteq \mathrm{G}'_-$. A model $\mathcal{L}[\mathrm{T}', \mathrm{G}']$ extends a model $\mathcal{L}$[T,G] iff T$'$ extends T and G$'$ extends G.

[2]So sentences like "G$\ulcorner 0 = 0 \urcorner$" count as non-alethic—I discuss this further in Section 4.

[3]That is: $v(\ulcorner \neg\phi \urcorner) = 1 - v(\ulcorner \phi \urcorner)$ and $v(\ulcorner \phi \vee \psi \urcorner) = \mathrm{Max}(v(\ulcorner \phi \urcorner), v(\ulcorner \psi \urcorner)$. Existential generalizations are understood as infinite disjunctions.

Our ultimate goal is to describe a G predicate whose anti-extension tracks sentences that can be shown to be members of an illegitimate semantic dependence structure. This sort of illegitimate structure is given an analysis in the definition of an **ungrounded set**, Defintion 7. I'll start with explaining some notions that are presupposed by that definition—beginning with the relevant relation of semantic dependence: direct calling.[4]

Definition 1 *For any sentences x and y of $\mathcal{L}[T,G]$, x* ***directly calls*** *y iff:*

(i) x is an atomic sentence of the form $\ulcorner Tc \urcorner$ and c refers to the (the Gödel number of) y.

(ii) x is a sentence $\ulcorner \neg\phi \urcorner$ and y is ϕ.

(iii) x is $\ulcorner \phi \vee \psi \urcorner$ and y is ϕ or ψ.

(iv) x is $\ulcorner \exists v \phi \urcorner$ and y is $\ulcorner \phi[v/c] \urcorner$ for some constant c, where $\ulcorner \phi[v/c] \urcorner$ is ϕ with every free occurence of v replaced with c.

A sequence of $\mathcal{L}$[T,G] sentences $p_0, \dots, p_n$ is a **calling path** iff in for every consecutive pair in the sequence $< x, y >$, x directly calls y.

Definition 2 *Let N be a set of indexed sentences $\{y_i\}$ for any natural number i. Then a directed graph $R(N)$, where R is a binary relation on N, is a* ***partial dependence tree*** *under the sentence p iff:*

(i) $p_0 \in N$.

(ii) No member of N bears R to p_0.

(iii) For every y_k in N, y_k bears R to some z_i only if y directly calls z.

(iv) For any n and m in N such that $n \neq m$, there is no q such that n and m both bear R to q.

(v) $N = \{x \mid$ there is an R path from p_0 to $x\}$.[5]

Definition 3 *A dependence tree $R(N)$ is* ***drawn from*** *a set B of $\mathcal{L}[T,G]$ sentences iff every member of N is y_k for some $y \in B$ (for some index k) and for every $y \in B$ there is a node y_k in N (for some index k).*

Definition 4 *If p is an $\mathcal{L}[T,G]$ sentence, then q is an* ***anchor*** *of p iff:*

(i) p has the form $\ulcorner Tc \urcorner$ and q is the sentence whose Gödel number is c.

(ii) p is a complex sentence containing a subsentence of the form $\ulcorner Tc \urcorner$, and q is the sentence whose Gödel number is c.

The set of all p's anchors is the **sufficiency set** of p.

[4]I borrow this terminology from Gaifman 1988, 1992, 2000. For similar graph-theoretic treatments of semantic dependence, see Yablo 1982 and Cook 2004.

[5]This definition is based on the characterization of dependence trees in Yablo 1982.

Definition 5 *We inductively define* ***Settled-as-True*** *and* ***Settled-as-False*** *for sentences of* $\mathcal{L}[T,G]$*:*

If p is an atomic sentence, then p is settled-as-true relative to a model $\mathcal{M}$ *iff:*
(i) p is a non-alethic sentence and p $v(\mathcal{M})(p) = 1$
(ii) p is a sentence of the form $\ulcorner Tc \urcorner$ *and* $v(\mathcal{M})(c) = 1$.

If p is an atomic sentence, then p is settled-as-false relative to a model $\mathcal{M}$ *iff:*
(i) p is a non-alethic sentence and p $v(\mathcal{M})(p) = 0$.
(ii) p is a sentence of the form $\ulcorner Tc \urcorner$ *and* $v(\mathcal{M})(c) = 0$

If p is a complex sentence, then p is settled-as-true relative to a model $\mathcal{M}$ *iff:*
(i) p is $\ulcorner \neg\phi \urcorner$ *and* ϕ *is settled-as-false in* $\mathcal{M}$.
(ii) p is $\ulcorner \phi \vee \psi \urcorner$ *and either* ϕ *or* ψ *is settled-as-true on* $\mathcal{M}$.
(iii) p is $\ulcorner \exists v\phi \urcorner$ *and some sentence* $\ulcorner \phi[v/c] \urcorner$ *is settled-as-true in* $\mathcal{M}$.

If p is a complex sentence, then p is settled-as-false relative to a model $\mathcal{M}$ *iff:*
(i) p is $\ulcorner \neg\phi \urcorner$ *and* ϕ *is settled-as-true in* $\mathcal{M}$.
(ii) p is $\ulcorner \phi \vee \psi \urcorner$ *and both* ϕ *and* ψ *are settled-as-false on* $\mathcal{M}$.
(iii) p is $\ulcorner \exists v\phi \urcorner$ *and some sentence* $\ulcorner \phi[v/c] \urcorner$ *is settled-as-false in* $\mathcal{M}$.

Definition 6 *For any sentence x of* $\mathcal{L}[T,G]$*, x is* ***settled*** *by a model* $\mathcal{M}$ *iff x is settled-as-true on* $\mathcal{M}$ *or settled-as-false on* $\mathcal{M}$.

Now we can give an analysis of the sort of semantic pathology that the anti-extension of the G predicate is intended to track:

Definition 7 *A set of* $\mathcal{L}[T,G]$ *sentences B is an* ***ungrounded set*** *relative to a model* $\mathcal{M}$ *iff:*

(i) every member of B has the value $\frac{1}{2}$ *on* $\mathcal{M}$.
(ii) no member of B is settled by $\mathcal{M}$.
(iii) for all x, x is unevaluated on $\mathcal{M}$ *and some member of B directly calls x, then* $x \in B$. *(i.e. B is closed under direct call among the sentences that are unevaluated on* $\mathcal{M}$.*)*
(iv) there is a partial dependence tree $R(N)$ *drawn from B such that every* $n \in N$ *bears R to something.*

If x is a member of an ungrounded set relative to a model $\mathcal{M}$, then all of the terminating limbs of x's semantic dependence tree have been evaluated, and there is still no basis for determining x's truth-value—we are left with an infinite descending tree of semantic dependence. What I'm calling Kripke's *diagnostic insight* is the claim that being a member of such an ungrounded set causes semantic pathology.

Next, we define the operation by which sentences get "added" to the extension and anti-extension of G. This is a function J from $\mathcal{L}$[T,G] models to $\mathcal{L}$[T,G] models, based on Kripke's jump operator. For the following three definitions, let $Q = \{x \in \mathbb{N} \mid x \text{ is not a code of a sentence}\}$.

Definition 8 *J is a function from $\mathcal{L}[T,G]$ models to $\mathcal{L}[T,G]$ models. $J(\mathcal{M}[T,G]) = \mathcal{M}[T',G']$, where:*

$T'_+ = \{x \mid v(\mathcal{M})(x) = 1\}$
$T'_- = \{x \mid v(\mathcal{M})(x) = 0\} \cup Q$
$G'_+ = \{x \mid v(\mathcal{M})(x) = 1 \textit{ or } v(\mathcal{M})(x) = 0\}$
$G'_- = G_- \cup \{x \mid x \textit{ is a member of an ungrounded set in } \mathcal{M}\} \cup Q$

Again following Kripke, I will be considering fixed-point models that are generated by a sequence starting with a certain base model. The construction can only be carried out if we make certain assumptions about that base model—namely that it is in **good standing**, in the sense defined below.[6]

Definition 9 *An $\mathcal{L}[T]$ model $\mathcal{M}$ is in **good standing** iff for every sentence ϕ of $\mathcal{L}[T]$, if $v(\mathcal{M})\phi \in \{0,1\}$, then iff $v(\mathcal{M})\phi = v(\mathcal{M})(T\ulcorner\phi\urcorner)$.*

*An $\mathcal{L}[T,G]$ model $\mathcal{M}$ is in **good standing** iff for every sentence ϕ of $\mathcal{L}[T]$:*

(i) if $v(\mathcal{M})\phi \in \{0,1\}$, then, $v(\mathcal{M})\phi = v(\mathcal{M})(T\ulcorner\phi\urcorner)$ and $v(\mathcal{M})(G\ulcorner\phi\urcorner) = 1$.
(ii) $G_- = Q$.[7]

Finally, since I want to compare $\mathcal{L}$[T,G] models with K3 models, we need to characterize the sense in which two such models can be said to correspond to each other.

Definition 10 *An $\mathcal{L}[T]$ model $\mathcal{M}[T]$ and an $\mathcal{L}[T,G]$ model $\mathcal{M}[T', G']$ **correspond to each other** iff $T = T'$ and $G'_+ = \{T_+ \cup T_-\} \setminus Q$.*

2 The construction

The next definition describes the construction that yields fixed-point models for $\mathcal{L}$[T,G].[8]

Definition 11 *Suppose that $\mathcal{M}[T,G]$ is an $\mathcal{L}[T,G]$ model that is in good standing. We define a sequence of $\mathcal{L}[/T,G]$ models as follows:*

[6]The first part of this definition is just the condition that a base model must meet for Kripke's original construction to yield a fixed-point.

[7]Alternatively, (ii) could stipulate that $G_- = \varnothing$. (Though we would have to appropriately modify Definition 11). Perhaps a weaker condition then either of these will do, but I won't explore the possibility.

[8]The organization of the proofs in this section draws on unpublished notes by **Redacted for Blind review**.

(i) $\mathcal{M}[T,G]_0 = \mathcal{M}[T,G]$
(ii) $\mathcal{M}[T,G]_{\alpha+1} = J(\mathcal{M}[T,G])$
(iii) Where α *is a limit ordinal,* $\mathcal{M}[T,G]_\alpha = \mathcal{M}[\langle \bigcup_{\beta<\alpha} T^\beta_+, \bigcup_{\beta<\alpha} T^\beta_- \rangle, \langle \bigcup_{\beta<\alpha} G^\beta_+, \bigcup_{\beta<\alpha} G^\beta_- \rangle]$

(In the last clause T^β_+ refers to the T_+ of $\mathcal{M}$[T,G]$_\beta$.)

Theorem 1 *For any* $\mathcal{L}[T,G]$ *sentence* p *and any* $\mathcal{L}[T,G]$ *models* $\mathcal{M}[T,G]$ *and* $\mathcal{M}[T', G']$, *if* T' *extends* T *and* G' *extends* G, *then* $v(\mathcal{M}[T', G'])(p) = v(\mathcal{M}[T,G](p)$.

The proof of Theorem 1 is a simple induction on formula complexity.

Theorem 2 (Monotonicity Conditions) *For all ordinals* α *and* β*:*

- *If* $\alpha > \beta$, *then* $\mathcal{M}[T,G]_\alpha$ *extends* $\mathcal{M}[T,G]_\beta$.
- *If If* $\alpha > \beta$, *then for all* $\mathcal{L}[T,G]$ *sentences* p, $v(\mathcal{M}[T,G]_\alpha)(p) = v(\mathcal{M}[T,G]_\beta)(p)$.

Proof: The first Monotonicy Condition implies the second, given Theorem 1, so it is sufficient to prove the first. If α is 0 or a limit ordinal the condition holds trivially. So it suffices to show that the first condition holds for an ordinal $\alpha + 1$ on the assumption that both Monotonicty Conditions hold for α. We do this by showing that $\mathrm{T}^\alpha_+ \subseteq \mathrm{T}^{\alpha+1}_+$, $\mathrm{T}^\alpha_- \subseteq \mathrm{T}^{\alpha+1}_-$, $\mathrm{G}^\alpha_+ \subseteq \mathrm{G}^{\alpha+1}_+$, and $\mathrm{G}^\alpha_- \subseteq \mathrm{G}^{\alpha+1}_-$.

- Suppose that c is an element of T^α_+. Let γ be the least ordinal such that $c \in \mathrm{T}^\gamma_+$. γ cannot be a limit ordinal, so γ is either 0 or a successor. Suppose $\gamma = 0$. By the assumption that $\mathcal{M}$ is in good standing, $v(\mathcal{M}[\mathrm{T,G,}]_\gamma)(c) = 1$. Since we assume that both Monotonicity conditions hold for α, and $\gamma \leq \alpha$, it follows that $v(\mathcal{M}[\mathrm{T,G,}]_\alpha)(c) = 1$. Therefore, by definition of the sequence, $c \in \mathrm{T}^{\alpha+1}_+$. Suppose that γ is a successor. Then there is some ordinal that is $\gamma - 1$. By definition of the sequence, $v(\mathcal{M}[\mathrm{T,G}]_{\gamma-1})(c) = 1$. Since we assume that both Monotonicity conditions hold for α, and $\gamma - 1 \leq \alpha$, it follows that $v(\mathcal{M}[\mathrm{T,G}]_\alpha)(c) = 1$. Therefore, by definition of the sequence $c \in \mathrm{T}^{\alpha+1}_+$.
- The proofs for $\mathrm{T}^{\alpha+1}_-$ and $\mathrm{G}^{\alpha+1}_+$ have the same form. The proof of $\mathrm{G}^{\alpha+1}_-$ is simpler because, by the definition of good standing, G^0_- is guaranteed to contain no sentences, and jump always preserves the contents of G_-. Therefore $\mathrm{T}^\alpha_+ \subseteq \mathrm{T}^{\alpha+1}_+$, $\mathrm{T}^\alpha_- \subseteq \mathrm{T}^{\alpha+1}_-$, $\mathrm{G}^\alpha_+ \subseteq \mathrm{G}^{\alpha+1}_+$, and $\mathrm{G}^\alpha_- \subseteq \mathrm{G}^{\alpha+1}_+$.
- So both Monotonicity Conditions hold for all ordinals α and β.

Theorem 3 (Fixed Point) *There is an ordinal* β *such that* $\mathcal{M}[T,G]_\beta = \mathcal{M}[T,G]_{\beta+1}$.

This follows since there are ordinals that exceed the cardinality of any set of $\mathcal{L}$[T,G] sentences.

3 Comparison with K3 Fixed-Points

Suppose that we have a $\mathcal{L}$[T] model $\mathcal{M}_0^*$ and a $\mathcal{L}$[T,G] model $\mathcal{M}_0^+$ that corresponds to $\mathcal{M}_0^*$, and that both are in good standing. Let $\mathcal{M}^*$ refer to the fixed point model generated by the standard Kripkean Jump-sequence construction on $\mathcal{M}_0^*$, and let $\mathcal{M}^+$ refer to the fixed point model generated by the Jump-sequence construction described in Definition 12 on $\mathcal{M}_0^+$.

Theorem 4 *For any ordinal α and any $\mathcal{L}[T]$ sentence p, $p \in T_+^{*\alpha}$ iff $p \in T_+^{+\alpha}$ and $p \in T_-^{*\alpha}$ iff $p \in T_-^{+\alpha}$.*

Proof: Proof by transfinite induction. For $\alpha = 0$, the condition is trivial on the assumption that $\mathcal{M}_0^*$ and $\mathcal{M}_0^+$ correspond. For $\alpha = \beta + 1$, suppose that for any $\mathcal{L}$[T] sentence p the truth concepts T_β^* and T_β^+ agree about p. Then, for every sentence p of $\mathcal{L}$[T], $v(\mathcal{M}_\beta^*)(p) = v(\mathcal{M}_\beta^+)(p)$. Therefore, by the definition of the jump operations for the respective sequences, for every sentence p of $\mathcal{L}$[T], $p \in T_+^{*\alpha}$ iff $p \in T_+^{+\alpha}$ and $p \in T_-^{*\alpha}$ iff $p \in T_-^{+\alpha}$. Now we prove that the condition holds for a limit ordinal α, on the assumption that it holds for all $\beta < \alpha$. This case is trivial too, since the Jump operation takes the union of T_+ and T_-, respectively, for all prior steps.

Corollary 4.1 *The truth concept of $\mathcal{M}^+$ extends the truth concept of $\mathcal{M}^*$.*

Corollary 4.2 *For any $\mathcal{L}[T]$ sentence p, $v(\mathcal{M}^+)(p) = v(\mathcal{M}^*)(p)$.*

Lemma 5 and 6 are helpful preliminaries for the proof of Theorem 7.

Lemma 5 *If p is an $\mathcal{L}[T]$ sentence with the sufficiency set H, and p receives a classical value (0 or 1) on a model $\mathcal{M}[T,G]$, then, for any model $\mathcal{M}[T', G']$ if T'_+ is such that $(H \cap T_+) \subseteq T'_+$, and T'_- is such that $(H \cap T_-) \subseteq T'_-$, then $v(\mathcal{M}[T])(p) = v(\mathcal{M}[T'])(p)$.*

Proof: Induction on formula complexity:

- Suppose p is a non-alethic atomic. The condition holds trivially because, if p is non-alethic, its interpretation is invariant with respect to changes in the interpretation of T.

- Suppose p is an atomic of the form $\ulcorner \mathrm{T}c \urcorner$. The sufficiency set of p is just $\{c\}$. Suppose p has the value 1 on $\mathcal{M}$[T,G]. Then $H \cap \mathrm{T}_+ = \{c\}$. Now suppose that $\{c\} \subseteq \mathrm{T}'_+$. Then p has the value 1 on $\mathcal{M}[\mathrm{T}', \mathrm{G}']$. Suppose, alternatively, p gets 0 on $\mathcal{M}$[T,G]. Then $H \cap \mathrm{T}_- = \{c\}$. Now suppose that $\{c\} \subseteq \mathrm{T}'_-$. Then p also gets 0 on $\mathcal{M}[\mathrm{T}', \mathrm{G}']$. So the condition holds if p is a alethic atomic.

- Suppose p is $\ulcorner\neg\phi\urcorner$. We assume the inductive hypothesis holds for ϕ. Suppose p gets the classical value x on $\mathcal{M}$[T,G]. Then, by the Strong Kleene Scheme, ϕ receives the value $|x-1|$ on $\mathcal{M}$[T,G]. By inductive hypothesis, $v(\mathcal{M}[\mathrm{T}', \mathrm{G}'])(\phi) = |x-1|$. So $v(\mathcal{M}[\mathrm{T}', \mathrm{G}'])(p) = x$. So the condition holds if p is $\ulcorner\neg\phi\urcorner$.

- The proofs for $\ulcorner\phi \vee \psi\urcorner$ and $\ulcorner\exists v\phi\urcorner$ follow the same pattern as $\neg\phi$.

- QED

This lemma holds for all K3 models as well, since it only concerns $\mathcal{L}$[T] sentences. In a slogan this lemma is: for $\mathcal{L}$[T] sentences, *only anchors matter.*

Lemma 6 (Sufficiency Principle) *For all $\mathcal{L}[T]$ sentences p and any model $\mathcal{M}$, $J(\mathcal{M})$ gives p a classical value iff $\mathcal{M}$ settles p.*

The proof of Lemma 6 is a simple induction on formula complexity and it is left to the reader.

Theorem 7 is crucial for establishing the main result of this paper: that for all $\mathcal{L}[T]$ sentences p, $v(\mathcal{M}^*)(p) = \frac{1}{2}$ iff p('s code) is a member of G_-^+ (Theorem 8). But it is also of independent interest in characterizing constructions that lead to $\mathcal{L}$[T,G] fixed-point models. It establishes that, once a sentence('s code) is added to G_-, it will never receive a classical value at a subsequent stage in the construction.

Theorem 7 *For any ordinal τ, if p('s code) is a member of G_-^τ, then, for all $\beta \geq \tau$, $v(\mathcal{M}[T,G]_\beta)(p) = \frac{1}{2}$.*

Proof: Induction on formula complexity.

- Suppose p is an atomic that does not feature the predicate T or G. Suppose $p \in \mathrm{G}_-^\tau$. Let α be the least ordinal such that $p \in G_-^\alpha$. α cannot be 0 or a limit ordinal, so there is an ordinal $\alpha - 1$. By the definition of the sequence, p must be a member of an ungrounded set in $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$, and therefore $v(\mathcal{M}[\mathrm{T,G}]_{\alpha-1})(p) = \frac{1}{2}$. But, since p doens't contain T or G, p receives a classical value at every model in the sequence. Contradiction. So the condition holds trivially of every atomic sentence lacking T and G.

- Suppose p is an atomic of the form $\ulcorner \mathrm{G}c\urcorner$. Suppose $p \in \mathrm{G}_-^\tau$. Let α be the least ordinal such that $p \in G_-^\alpha$. α cannot be 0 or a limit ordinal, so there is an ordinal $\alpha - 1$. By the definition of the sequence, p must be a member of an ungrounded set in $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$. Let that set be B. By the definition of ungrounded set, there is a partial dependence tree drawn from B that has no terminal nodes. But, if p is $\ulcorner \mathrm{G}c\urcorner$, then it does not directly call any sentence. Therefore, any dependence tree drawn from B will have at least one terminal node p_i. Contradiction. So the condition holds trivially of every atomic sentence of the form $\ulcorner \mathrm{G}c\urcorner$.

- Suppose p is an atomic of the form $\ulcorner \mathrm{T}c \urcorner$. Let α be the least ordinal such that $p \in G^{\alpha}_{-}$. Again α cannot be 0 or a limit ordinal, so there is an ordinal $\alpha - 1$. By the definition of the sequence, p must be a member of an ungrounded set in $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$. Call that set B. Now, suppose for contradiction that there is an ordinal greater than α such that p receives a classical value in the model corresponding to that ordinal. Let β be the least such ordinal. β cannot be 0 or a limit ordinal, so there is an ordinal $\beta - 1$. By the definition of the sequence, if $\ulcorner \mathrm{T}c \urcorner$ has a classical value on $\mathcal{M}[\mathrm{T,G}]_{\beta}$, then c has a classical value on $\mathcal{M}[\mathrm{T,G}]_{\beta-1}$. We can show that c is a member of B. Suppose not for *reductio*. p directly calls c, and B is closed under direct call among sentences with the value $\frac{1}{2}$ in $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$, so if $c \notin B$, then c has a classical value on $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$. But if c has a classical value on $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$, then p is settled on $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$. This contradicts the assumption that B is an ungrounded set. Therefore, $c \in B$. In order to yield a dependence tree without terminating nodes, c must be an alethic sentence. Let W be the smallest set containing c and all of c's anchors, and which is closed under anchoring. Let $H = B \cap W$. Since anchoring is a special case of a calling path and B is closed under anchoring, for every $a \in H$, if b is an anchor of a and $v(\mathcal{M}[\mathrm{T,G}]_{\alpha-1})(b) = \frac{1}{2}$, then $b \in H$.

  - We can show (Helper Lemma) that, for any model $\mathcal{M}'$ extending $\mathcal{M}[\mathrm{T,G}]_{\alpha-1}$ such that no member of H is a member of T'_{+} or T'$_{-}$, no member of H receives a classical value in $\mathcal{M}'$. For, suppose that no member of H is a member of T'_{+} or T'_{-} in $\mathcal{M}'$. Suppose now for contradiction that $o \in H$ has a classical value on $\mathcal{M}'$. Let $F = \mathrm{T}'_{+} \cap \{x \mid x \text{ is an anchor of } o\}$ and $E = \mathrm{T}'_{-} \cap \{x \mid x \text{ is an anchor of } o\}$. Since we are supposing that F and E are both disjoint from H, it follows that every member of F or E receives a classical value in $\mathcal{M}_{\alpha-1}$, because H includes all of the anchors of o that receive the value $\frac{1}{2}$ in $\mathcal{M}_{\alpha-1}$. By Lemma 5, o must receive a classical value in $\mathcal{M}_{\alpha}$ ($J(\mathcal{M}_{\alpha-1})$), since o has a classical value in $\mathcal{M}'$, and by the definition of the sequence $F \subseteq \mathrm{T}^{\alpha}_{+}$ and $E \subseteq \mathrm{T}^{\alpha}_{-}$. But, by the Sufficiency Principle, this implies that o is settled by $\mathcal{M}_{\alpha-1}$. But this yields a contradiction, since o is assumed to be a member of B, which is an ungrounded set.

Now we do a transfinite induction to show: for every ordinal $\epsilon \geq \alpha - 1$, no member of H has a classical value on $\mathcal{M}_{\epsilon}$. For $\epsilon = \alpha - 1$ this is trivial given the assumptions above. Suppose $\epsilon = \gamma + 1$, where $\gamma > \alpha - 1$. By inductive hypothesis every member of H has the value $\frac{1}{2}$ on $\mathcal{M}[\mathrm{T,G}]_{\gamma}$. Suppose for *reductio*, that some $o \in H$ has a classical value on $\mathcal{M}[\mathrm{T,G}]_{\epsilon}$. By the Helper Lemma, this implies that some member of H is a member of $\mathrm{T}^{\epsilon}_{+}$ or $\mathrm{T}^{\epsilon}_{-}$. But, given the definition of the sequence, this implies that that some member of H has a classical value in $\mathcal{M}[\mathrm{T,G}]_{\gamma}$—contrary to hypothesis. Suppose ϵ is a limit ordinal, and suppose that the inductive hypothesis holds for all $\gamma < \epsilon$. Since nothing is receives a classical value at a limit ordinal that does not receive it at some predecessor ordinal, no member of H receives a classical value in $\mathcal{M}[\text{T,G}]_\epsilon$.

Since c is a member of H this implies that there is no ordinal $\epsilon \geq \alpha - 1$ such that c receives a classical value in $\mathcal{M}[\text{T,G}]_\epsilon$. Therefore, contrary to assumption, c does not receive a classical value in $\mathcal{M}[\text{T,G}]_{\beta-1}$. This concludes the *reductio* of the assumption that there is an ordinal β such that p has classical value on $\mathcal{M}[\text{T,G}]_\beta$ Therefore, for all $\beta \geq \alpha$, $v(\mathcal{M}[\text{T,G}]_\beta)(p) = \frac{1}{2}$. Thus the condition holds if p is $\ulcorner \text{T}c \urcorner$.

- Suppose p is a sentence $\neg\phi$. Suppose the inductive hypothesis holds for ϕ. Suppose $p \in \text{G}^\tau_-$. Let α be the least ordinal such that $p \in G^\alpha_-$. α cannot be 0 or a limit ordinal, so there is an ordinal $\alpha - 1$. By the definition of the sequence, p must be a member of an ungrounded set in $\mathcal{M}[\text{T,G}]_{\alpha-1}$. Let that set be B. ϕ must be a member of B. For, suppose not for *reductio*. Then, since, B is closed under direct call among sentences evaluated as $\frac{1}{2}$ in $\mathcal{M}[\text{T,G}]_{\alpha-1}$ and p calls ϕ, then ϕ would have a classical value in $\mathcal{M}[\text{T,G}]_{\alpha-1}$. This in turn implies that $\mathcal{M}[\text{T,G}]_{\alpha-1}$ settles p, contradicting the assumption that B is an ungrounded set. So, $\phi \in B$, which, by definiton of the sequence, implies that $\phi \in \text{G}^\alpha_-$. By inductive hypothesis, for all $\epsilon \geq \alpha$ $v(\mathcal{M}[\text{T,G}]_\epsilon)(\phi) = \frac{1}{2}$. By the Strong Kleene Scheme, for all $\epsilon \geq \alpha$ $v(\mathcal{M}[\text{T,G}]_\epsilon)(p) = \frac{1}{2}$. Since, $\alpha \leq \tau$ the condition holds for if p is $\ulcorner \neg\phi \urcorner$.
- The proofs for $\ulcorner \phi \vee \psi \urcorner$ and $\ulcorner \exists v \phi \urcorner$ follow the same pattern as $\ulcorner \neg\phi \urcorner$.
- QED

Theorem 8 *For all $\mathcal{L}[T]$ sentences p, $v(\mathcal{M}^*)(p) = \frac{1}{2}$ iff p('s code) is a member of G^+_-, i.e. the G_- of $\mathcal{M}^+$.*

Proof:
Left to Right: For all $\mathcal{L}[T]$ sentences p, if $v(\mathcal{M}^*)(p) = \frac{1}{2}$, then p('s code) is a member of G^+_-. Induction on formula complexity.

- Suppose p is a non-alethic atomic. Trivial, since p has a classical value in $\mathcal{M}^*$.
- Suppose p has the form $\ulcorner \text{T}c \urcorner$. Suppose that $v(\mathcal{M}^*)(p) = \frac{1}{2}$. There is no ordinal α such that $\mathcal{M}^+_\alpha$ gives p a classical value, because if it did p would have a classical value on $\mathcal{M}^+$, and, by Corollary 4.2, p would have a classical value on $\mathcal{M}^*$ contrary to assumption. It follows too that there is no ordinal α such that $\mathcal{M}^+_\alpha$ settles p. Let B be the smallest set including p that is closed under direct call. Let β be the least ordinal such that every member of B that receives a classical value in $\mathcal{M}^+$ receives a classical value in $\mathcal{M}^+_\beta$. Let $B' = B \backslash \{x \mid x \text{ receives a classical value in } \mathcal{M}^+_\beta\}$. We

can prove that B' is an ungrounded set in $\mathcal{M}^+_\beta$. By hypothesis, no member of B' receives a classical value in $\mathcal{M}^+_\beta$ and it is closed under direct call among sentences that receive $\frac{1}{2}$ in $\mathcal{M}^+_\beta$. Every member $o \in B'$ must be unsettled on $\mathcal{M}^+_\beta$, because if o were settled, then o would receive a classical value in $J(\mathcal{M}^+_\beta)$, and therefore in $\mathcal{M}^+$, contrary to hypothesis. Finally, there is a partial dependence tree under p drawn from B' that has no terminal nodes. Since every non-alethic sentence of $\mathcal{L}$[T] receives a classical value in $\mathcal{M}^+_\beta$, every member of B' is an alethic sentence, and therefore directly calls some sentence. Further, for every $o \in B'$, o must directly call some sentence that does not receive a classical value in $\mathcal{M}^+_\beta$, because if every sentence called by o had a classical value in $\mathcal{M}^+_\beta$, o would be settled by $\mathcal{M}^+_\beta$—contradiction. Since B' is closed under direct call among sentences that do not receive classical values in $\mathcal{M}^+_\beta$, every element of B' calls some element of B'. Since p calls every element of B' and every member of B' is a potential non-terminal node in a dependence tree, there is a partial dependence tree under p that has no terminal nodes. Therefore p is a member of an ungrounded set in $\mathcal{M}^+_\beta$. Therefore, $p \in \mathrm{G}^{\beta+1}_-$, and by Monotonicity, $p \in \mathrm{G}^+_-$. So the condition holds if p is $\ulcorner \mathrm{T}c \urcorner$.

- Suppose p is $\neg\phi$. Suppose the inductive hypothesis holds for ϕ. Suppose that $v(\mathcal{M}^*)(p) = \frac{1}{2}$. By the Strong Kleene semantics, $v(\mathcal{M}^*)(\phi) = \frac{1}{2}$. By inductive hypothesis, $\phi \in \mathrm{G}^+_-$. Let α be the least ordinal such that $\phi \in G^\alpha_-$. α cannot be 0 or a limit ordinal, so there is an ordinal $\alpha - 1$. By the definition of the sequence ϕ is a member of an ungrounded set in $\mathcal{M}_{\alpha-1}$. Call that set B. The set $B \cup \{p\}$ is also an ungrounded set. Therefore, $p \in \mathrm{G}^{\alpha+1}_-$, and by Monotonicity $p \in \mathrm{G}^+_-$. So the condition holds if p is $\neg\phi$.

- The proofs for $\ulcorner \phi \vee \psi \urcorner$ and $\ulcorner \exists v \phi \urcorner$ follow the same pattern as $\ulcorner \neg\phi \urcorner$.

Right to Left: For all $\mathcal{L}[T]$ sentences p, if p('s code) is a member of G^+_-, then $v(\mathcal{M}^*)(p) = \frac{1}{2}$.

- Suppose p('s code) is a member of G^+_-. Now suppose for contradiction that p receives a classical value in $\mathcal{M}^*$. By Corollary 4.2, p has a classical value on $\mathcal{M}^+$. But by Theorem 7, $v(\mathcal{M}^+)(p) = \frac{1}{2}$. Contradiction. So $v(\mathcal{M}^*)(p) = \frac{1}{2}$.

- QED

There is, then, a straightforward parallelism between $\mathcal{L}$[T,G] fixed-point models and corresponding K3 models: the $\mathcal{L}$[T] sentences in the anti-extension of G will be *exactly* the sentences that are ungrounded in the corresponding K3 model. Since the parallelism holds for all fixed-points, the standard axiomatization of T, PFK, is also sound for all $\mathcal{L}$[T,G] fixed-point models. (I won't pursue in this paper whether or not there is a natural axiomatization of G.) The parallelism is also modular in the sense that it doesn't depend specifically on the Strong Kleene semantics—claims analogous to Theorems 4-8 can be proved regarding corresponding constructions for $\mathcal{L}$[T,G] and $\mathcal{L}$[T] models based on a supervaluationist semantics.

4 Discussion

For an intuitive sense of how the G predicate works, it is helpful to think about the construction starting with the base-model $\mathcal{M}[\mathrm{T,G}]_0 = \mathcal{M}[\langle\varnothing, Q\rangle, \langle\varnothing, Q\rangle]$. (Where, again, $Q = \{x \in \mathbb{N} \mid x \text{ is not a code of a sentence}\}$.) The fixed-point generated by this construction corresponds to Kripke's Minimal Fixed Point. (In what follows I will use "[p]" to refer to the Gödel code of p.)

Liar sentences like $a = [\neg \mathrm{T}a]$ and truth-tellers like $b = [\mathrm{T}b]$ both get added to G_- in the first jump since they are members of ungrounded sets in the base model. (This is also true of any Liar cycles or Yablo-paradoxical chains consisting of atomic truth-predications.) But there are other sentences that only get added to G_- at later stages—after their grounding possibilities "run out." Consider e.g. $c = [\mathrm{T}\ulcorner 0 \neq 0 \urcorner \vee \neg \mathrm{T}c]$. Since c's first disjunct receives 0 in the base model, c can only receive $\frac{1}{2}$ in the fixed-point. But c is *not* a member of an ungrounded set in the base model. In any base model, the sentences in the following set all receive $\frac{1}{2}$: $\{c, \neg\mathrm{T}c, \mathrm{T}c, \mathrm{T}[0 \neq 0]\}$. This set is closed under direct call among unevaluated sentences, but it is not an ungrounded set because $\mathrm{T}[0 \neq 0]$ is settled (as false) in the base model. However, $\{c, \neg\mathrm{T}c, \mathrm{T}c\}$ will be an ungrounded set in the second stage, once $\mathrm{T}[0 \neq 0]$ receives the value 0.

In the fixed point generated from $\mathcal{M}[\langle\varnothing, Q\rangle, \langle\varnothing, Q\rangle]$:

- $c = [\neg\mathrm{T}c]$ receives $\frac{1}{2}$
- $\neg Gc$ and $\mathrm{T}[\neg\mathrm{G}c]$ both receive 1
- $d = [\neg\mathrm{T}d \vee \neg Gd]$, a strengthened-Liar, receives $\frac{1}{2}$.
- $e = [Ge]$ receives $\frac{1}{2}$, as does $f = [\neg Gf]$.

My suggestion is that G expresses groundedness in the same way that T expresses truth in standard Kripke constructions. T's extension contains everything true, though its anti-extension doesn't include everything untrue. G's extension contains everything grounded, though its anti-extension doesn't include everything ungrounded. None-the-less, Theorem 8 shows that ¬G can be used to pick out all the things things that are ungrounded in the fragment $\mathcal{L}$[T]. In that sense, I think it can serve to express in the object language the diagnostic insight of Kripke's original construction: that predicating truth induces semantic dependence, and that, to be be grounded, you must be part of a well-founded semantic dependence structure.

One might object: the fact that "¬G" does not apply to certain sentences that are are ungrounded disqualifies it from expressing ungroundedness in a fully general way. After all, a Strengthened Liar like b, where $b = [\neg \mathrm{T}b \vee \neg G]$ is ungrounded in the sense that it receives $\frac{1}{2}$ in any $\mathcal{L}$[T,G] fixed-point model, but that isn't captured by "G." b is not in the extension or anti-extension of G in any fixed point. Moreover, the scope of G_{-} is narrowed in a way that might seem unprincipled. In my introduction, I suggested that G should arguably be susceptible to semantic failure precisely because it is a semantic predicate like T. But the characterization of ungrounded sets in Definition 7 is based on a analysis of semantic dependence (direct calling) according to which predications of "T" initiate semantic dependence links but predications of "G" do not. In other words, G_{-} is not tracking ungroundedness *per se*, just ungroundedness induced by predictions of truth. This threatens to make G uninteresting. We already knew how to express "ungrounded in $\mathcal{L}$[T]" in a richer language that extends $\mathcal{L}$[T]—how does the G predicate improve on this?

In addressing these objections, we should be clear about the problems "G" is (and is not) intended to solve. It is not intended to be a general response to "Revenge Paradoxes" or to remove all philosophically puzzling expressive limitations present in Kripke's theory of truth. As I said in the introduction, I'm forswearing the project of introducing a "ungroundedness" predicate to that correctly applies to all sentences of that language that are ungrounded. Whatever pernicious Revenge objections can be raised against Kripke's theory of truth can equally be raised against my semantics for $\mathcal{L}$[T,G].[9] But I do think the inclusion of the G predicate allows us to express, in the object language, what I have characterized as Kripke's main diagnostic insight.

Let me elaborate on that point a bit. The objection above is quite right that I have described a groundedness predicate whose anti-extension is specifically sensitive to networks of semantic dependence induced by predications of *truth.* This a restriction—it doesn't take account of the semantic dependence induced by predications of "G"—but it is not an unmotivated restriction. Arguably, truth and falsity are *basic* semantic categories and the dependence among claims attributing truth or falsity is the *basic* sort of semantic dependence. It is the kind of semantic dependence that Kripke's diagnosis is concerned with. And although "¬G" does specifically track ungroundedness due to predications of truth, it is *not* an ungroundedness predicate specifically *for* the language $\mathcal{L}$[T]. Infinitely

[9]It's also worth noting that, although one can truly assert "¬G[λ]", where λ is a Liar sentence, one cannot truly assert "¬T[λ]". So the resulting system does not allow for the expression of the Chrysippus Intuition—the suggestion that we can truly say of pathological sentences that they are not true.

many sentences containing occurrences of "G" will end up in G_- in the fixed-point models I've described—including sentences commenting on the *truth* of sentences containing "G." For instance, let $h = [\neg \mathrm{T}h \vee \neg \mathrm{G}[0 = 0]]$. "$\neg \mathrm{G}h$" will get the value 1 in any $\mathcal{L}$[T,G] fixed-point model.[10] So the "¬G" is not merely just another device for expressing semantic failures occurring in the language $\mathcal{L}$[T] in a richer metalanguage—it is a resource for expressing *in* $\mathcal{L}$[T,G] Kripke's diagnostic insight as it applies to $\mathcal{L}$[T,G].

I'll close by comparing my G predicate to the pathology predicates in Cook 2007, which are introduced to accomplish a similar purpose. Cook 2007 describes an indefinitely extensible sequence of languages—$\mathcal{L}_0, \mathcal{L}_\infty ... \mathcal{L}_\alpha, ...$—where $\mathcal{L}_0$ is a a truth-free language, $\mathcal{L}_1$ extends $\mathcal{L}_0$ with the addition of a truth predicate, and each successor language $\mathcal{L}_{\beta+1\geq 2}$ contains a pathology predicate "P_β" suitable for describing the sort of pathology that arises in $\mathcal{L}_\beta$.[11] (If α is a limit ordinal, then $\mathcal{L}_\alpha$ contains all "$\mathrm{P}_{\beta<\alpha}$.") Each language $\mathcal{L}_\alpha$ determines a fixed-point model generated from a base, and, if $\mathcal{L}_{\alpha+1}$ is a successor language, the fixed-point for $\mathcal{L}_{\alpha+1}$ is generated in such a way that "P_α" is true of all and only the the sentences that that are pathological$_\alpha$ where this is the pathological value required to give a semantics for $\mathcal{L}_\alpha$.[12]

The most direct parallel obtains between $\mathcal{L}$[T,G] and Cook's $\mathcal{L}_2$—the language with just one pathology predicate, "P_1." One difference in the semantics provided for these languages is particularly salient: unlike G_-, Cook's "P_1" doesn't characterize semantic pathology induced by predications of truth generally—it specifically captures semantic pathology that arises due to truth being predicated of $\mathcal{L}_1$ sentences. For instance, in the fixed-point for $\mathcal{L}_2$, the sentence $a = [\neg \mathrm{T}a \vee \mathrm{P}_1[0 = 0]]$ will not be in the extension of P_1. Both a and "$\mathrm{P}_1 a$" receive the value pathological$_2$ in the fixed-point for $\mathcal{L}_2$. a is, of course, the analogue in Cook's system of the sentence h, described two paragraphs above, which, as we saw, *is* in the anti-extension of G in every fixed-point model for $\mathcal{L}$[T,G]. I would suggest that G_- succeeds in picking out the more natural semantic category. If Kripke's diagnostic insight is apt, then a semantically fails for the *same reason* that simple Liar sentences or sentences like $d = [\neg \mathrm{T}d \vee 0 \neq 0]$ fail—each of these sentences are part of a network that can be judged ungrounded simply on the basis of semantic dependence relations induced by predications of truth. It seems irrelevant that a involves predicating truth of a sentence not in $\mathcal{L}_1$. Since this sort of semantic failure is expressed by "$\neg G$" but not by "P_1," in my view, the semantics I have offered for $\mathcal{L}$[T,G] does a better job of providing us with an object-language resource capable of expressing Kripke's diagnostic

[10] Once "0=0" is added to G_+, "$\neg \mathrm{T}h \vee \neg \mathrm{G}[0 = 0]$" will be a member of the ungrounded set $\{\neg \mathrm{T}h \vee \neg \mathrm{G}[0 = 0], \neg \mathrm{T}h, \mathrm{T}h\}$ and it will get added to G_- in the next step of the sequence.

[11] Technically, Cook considers this a sequence of stages in which a single language is *expanded*.

[12] For the sake of simplicity, I focus on the proposal in Cook 2007, but Cook has since elaborated and modified this basic account in work with Nicholas Tourville (Tourville and Cook 2016, 2020). The critical remarks I make in the next paragraph hold, *modulo* some technical adjustments, for these more recent accounts as well.

insight as it applies to that very language.

That said, the general response to Revenge Paradoxes offered in Cook 2007 looks like it could be naturally pursued in a setting featuring languages with my G predicate. On this view, which Cook calls "Embracing Revenge,"[13] the perpetual availability of Revenge Paradoxes is due the fact that the concept of a truth-value is indefinitely extensible. On such a picture, Revenge paradoxes are not a "problem to be overcome," they are an natural consequence of the fact that no language can have a fully exhaustive classification of truth-values. I think the reasons Cook offers for thinking that the notion of a *truth-value* is indefinitley extensible equally support the idea that we have an indefinitely extensible notion of *semantic dependence*. I won't pursue the suggestion here, but there is no obvious obstacle to the project of defining a sequence of languages $\mathcal{L}[\mathrm{T},\mathrm{G}_\alpha]$, such that every successor language contains a Groundedness predicate, modeled on G, that is suitable for describing networks of dependence induced by the semantic vocabulary in the prior language. If one is compelled by the "Embracing Revenge" response, one might fruitfully develop it in the setting of a hierarchy of languages based on $\mathcal{L}[\mathrm{T},\mathrm{G}]$.

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[13] The view is also defended in Tourville and Cook 2016, 2020 and Schlenker 2010.

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