“Gaiyi Soccer” Board Game Project

More than An Original Board Game Set - It Contains Math and Statistics!

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I. Introduction

Gaiyi Soccer was originally designed as a tribute to 2014 FIFA World Cup in Brazil. The original Chinese version was released on June 12, 2014. The board game mixes deployment and tactics, introduces specialized knowledge and rules in modern soccer, combines the essentials of card game and chess game, and therefore, it provides ultimate interactive gaming experiences.

II. Basics

The game has 110 cards, including 67 “Footballer Cards” (60 players + 7 goalkeepers), 13 “Formation Cards”, 2 “Marginal Decision Cards”, 22 “Functional Cards”, plus 6 “Penalty Cards”.
The game has 17 chess pieces, including 8 player pieces for both sides marking players 1-8, plus a piece as soccer.
The game has two dice.
The game consists of 20 rounds with no half-time break.

III. Pre-Game Settings

3.1 Two sides of players (a referee is welcome) choose one “Formation Card” from the pile randomly. The card is for reference and restrictions for each side to decide starting 8

players, and their numbers.

3.2 Two sides of players choose two (2) goalkeepers randomly as their goalkeepers of the game.

3.3 21 footballers are assigned randomly to both sides of players randomly and alternately. Together with two (2) goalkeepers already chosen, the 23 players form respective team squad for both players. During the entire game, each team has three (3) chances to substitute players.

3.4 The “Formation Card” indicates 8 players of certain position, as indicated on the bottom-right of the “Footballer Card” (For reasons of simplification, “Gaiyi-Soccer” designs positions restricted to GK: Goalkeeper, LB: Left Back, CB: Center Back, RB: Right Back, CM: Center Midfielder, LW: Left Winger, RW: Right Winger, and CF: Center Forward), to be selected as starting players out of the team squad of 23. If no player of such position on the squad for the team as the “Formation Card” requires, he can be replaced by a player of any position. An X

on the “Formation Card” indicates that a player of any position could be picked there.

3.4.1 Before the game starts, the chess pieces with numbers 1-8 indicating footballers on both sides should be placed on the chess board according to the “Formation Card”. The rule for placing pieces for footballers is to place them at the intersection of white lines on the chess board, all the way throughout the whole game, while the piece indicating soccer ball should be placed at the center of the kickoff circle, while moving within grids during the entire game.

3.4.2 By tossing a die, the side with bigger number on the die has the right to kick-off. The team to start the game select two players entering the kickoff circle to start the game, while the other team must have all players outside of the kickoff circle.

3.4.3 Meanwhile, both sides are required to choose four (4) “Functional Cards” randomly and alternatively, as can be used in any duel during the game.

IV. Gameplay Settings

4.1 The game starts with the kickoff of the team with the ball. This marks the game entering round 1.

4.2 Each round (up to 20), includes two half rounds during which both team alternatively move their footballers on the chess board (including duels). Keep in mind how many rounds the game has played!

All rounds are marked started with the team with the ball moving all players. Within his of her own half round, the team with the ball can move every footballer on his or her team within the maximum range each footballer can go. One and only one pass can be done for the whole team.

4.3 The other team not having ball in control can move every footballer within the maximum range each footballer can go after the team with the ball ends its own half round. The round ends with both half rounds are completed. A new round will start.

4.4 Duels can be generated in every half round. Upon the duel, both teams can pick randomly and alternately two (2) “Functional Cards” with the side causing the duel picking first. Both teams can use as many “Functional Cards” as possible during the duel, in order to increase the winning probability. The teams cannot pick “Functional Cards” until upon the next duel. If the duel ends with a team holding more than four (4) “Functional Cards”, the team should abandon some “Functional Cards” in order to hold no more than four (4) “Functional Cards”.

4.5 The game enters a new round immediately when possession of the ball alters.

4.6 The game restarts an old round when possession of the ball does not alter while the game is dead for reasons including but not restricted to: foul by the defensing team and free-kick or penalty is awarded, corner kick, etc.

4.7 Both teams can make substitutions of players (up to three (3) for the entire game) when the ball is dead. The cases of dead balls include but are not restricted to: fouls resulting in free-kicks or penalties, goal-kicks, throw-ins, goals scored, etc. Players who are replaced during the game cannot be re-introduced to play.

4.8 After the 12^{th} round, RWs (Right Wingers) and LWs (Left Wingers) with speed 3 must reduce to 2.

4.9 The game ends as the 20^{th} round ends, and only one additional round can be awarded when the attacking team has a shooting opportunity that can alter the game result. The attacking team must take a legal shot immediately at the $(20+1)^{th}$ round.

4.10 The game ends with the team with more scores winning. If the game is drawn, teams are to play penalty shoot-out upon the will of both teams.

V.Game Rules

Note: This instruction of game rules are designed for better understanding for players during the gameplay. Since a soccer game can be unpredictable, there are cases that are not included in this instruction. Upon these cases, we require both players to compromise. If a referee joins the game, they must have a high level of understanding of the following rules.

5.1 Tests

A test should be performed before the attacking team finish their following actions: Long passes, Shooting (using either foot or head), and tackling.

There are two types of tests: tests for turnovers and tests for scenarios.

(1) Tests for turnovers: A toss of die-When the attacking team performs long passes or shooting (using either foot or head). This action is to include the cases where players may fail in these cases. When functional cards are applied, if the footballer's (with the ball) attribute of passes or shooting, or heading (for head shooting) is 9 or above, the action will succeed if the toss is not 4. If his attribute is between 7 and 8, the action will succeed if the toss is not 2 or 4. If his attribute is 6 or below, the action will succeed if the toss is not 2, 4, or 6.

Note: If there is a turnover in lobbing passes, the ball will goes out of the play where the

extension line decided by the point where the ball is passed and the point where the receiver of the pass should be, meets the border of the field. Upon this case, possession changes (either throw-ins or goal kicks), and the round restarts.

(2) Tests for scenarios: We use marginal decision cards. When tests for turnovers is performed and the attacking team is successful in the demanding action, the team should immediately pick randomly a marginal decision card from the pile of 2. If "+1" is picked, then if the attacking team losses the following duel, then the scenario will be of the benefit of the attacking team. On the other hand, if "-1" is picked, then the scenario will be of the benefit of the defending team.

5.2 Duels

Duels are independent of rounds. A duel is performed with comparing the sums (corresponding attributes of players involved in the action + functional cards added on + a toss of die) Here we use "SA" to denote the sum for the aggressive team (A) that starts the duel, and "SP" for the sum of the passive team (P) that have to accept the duel. The team whichever has its players involved with a bigger number will win the duel. If the numbers are equal, we restart the duel until there is a winner. The team that creates this duel tosses the die first.

There are 10 common cases we need duels. They might be not exhaustive.

(1) Duel for Shot on Foot: SA in shooting vs. SP in saving

(2) Duel for Shot on Head: SA in heading vs. SP in header-saving

(3) Duel for Penalty Shot: If in the same direction, SA in shooting vs. SP in penalty-saving

(4) Duel for Breakthrough: SA in dribbling vs. SP in tackling

(5) Duel for tackling: SA in tackling vs. SP in dribbling

Note: If multiple defenders to tackle are involved, please refer to Section 5.6.3

(6) Duel for interception of ordinary pass: SA in tackling vs. SP in passing

(7) Duel for interception of lobbing pass: SA in heading vs. SP in passing

(8) Duel for heading: SA in heading vs. SP in heading

(9) Duel for heading+shot on head: (8) (if SA higher) + SA in heading vs. SP in saving headers.

(10) Duel for heading + Duel for heading + ….. : (8) (if SA wins) + (8)…..

In the above 10 cases, each case is associated with a test of scenario, and the results are as followings (The numbers in the brackets are corresponding to the duel cases above), and we assume that A passes test for turnover, and has lost the duel:

(1) If A picks "+1", the shot is saved by the goalkeeper of P but out of his reach, A gets a corner. If A picks "-1", the shot is saved and ball is held by the goalkeeper of P. A new round is to start with possession change.

(2) As in (1).

(3) As in (1).

(4) Possession change absolutely, no test of scenarios is to be performed.

(5) If A picks "+1", a foul is committed and a card should be rewarded. The ball is dead, and P has a free-kick. If A picks "-1", the tackling player of A is immediately beaten by the dribbling player of P. Game plays on.

Note: If multiple defenders to tackle are involved, please refer to Section 5.6.3. Card is awarded to the player with (i): greatest attribute in tackling, (ii) closest to the mid-point of the goal of A.

(6) Possession change absolutely, no test of scenarios is to be performed.

(7) As in (6)

(8) No test of scenarios is to be performed.

(9) As in (8) + As in (1)

(10) As in (8) + As in (8) + …..

Duels and rounds:

If the duel ends, and no foul committed and no corner-kick, then the round ends, and a new round starts with the team with possession.

If the duel ends with a foul that leads to free-kick or penalty, or there is a corner-kick, then the round restarts with the team with possession.

5.3 Dead-balls, Re-formations, and Substitutions

There are for cases in this game that result in dead-balls. They are:

(1) Kick-out of sidelines.

(2) Kick-out of goal lines

(3) Fouls

(4) Goal-scored

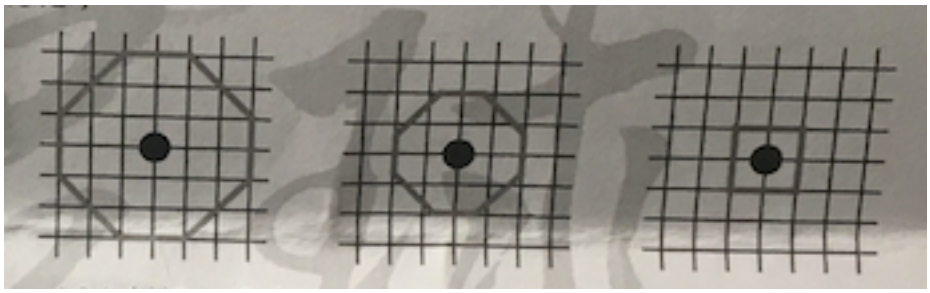
When any of these cases happens, both teams are allowed to re-formation their players, say, any players on the chess board can be moved freely to another place. Additionally, when the ball is dead, both teams are allowed to make substitutions (Please refer to 4.7). When the goalkeeper is sent off, the team is allowed to use the goalkeeper on the bench to replace a player on the field. Whenever any adjustment on squad is made, the players' numbers should always be consistent with the card order of rosters shown next to the chess board, and with the numbers appear on formation cards.

About calculating rounds, please refer to 5.1 and 5.2.

5.4 About Players' Attributes

Every players (except goalkeepers) has six (6) attributes: speed, shooting, heading, tackling, passing and dribbling. For goalkeepers, three (3) more attributes are added: saving, header-saving, and penalty-saving. Except speed (ranging from 1 to 3), all other attributes range from 1 to 10. Where the larger the number is, the stronger the corresponding attribute is. Here we have a detailed break-down of these attributes:

(1) Speed: The speed of a player, ranging from 1 to 3, reflects the corresponding maximum boundary that he can go within the half-round he is allowed to make a move. Players are allowed to move within the range but never go beyond.



The picture on the left shows the maximum boundary of players with speed 3, 2, and 1 respectively.

(2) Shooting: Scaled from 1 to 10, it represents the ability of a player to shoot within the penalty area. And for long-range shots outside the box, every grid from the closest edge of the box contributes -1 to PA in duel for shooting, but this disadvantage resulting from range can be entirely offset by the functional card "Cannon". Players are not allowed to take shots beyond or equal to 5 grids from the box.

(3) Heading: Scaled from 1 to 10, it represents the ability of a player to perform headers, including heading, and shooting with head (within the box). Players are not allowed to use head to shoot outside of the box. After a successful heading, the player can use "heading" attribute to pass the ball to his teammates instead of using "passing".

(4) Tackling: Scaled from 1 to 10, it represents the ability of a player to perform tackles from a dribbling player on the opposite team, or intercepting a pass from the opposite team.

(5) Dribbling: Scaled from 1 to 10, it represents the ability of a player to perform dribbling to make breakthrough of the defense of the opposite team.

5.5 About Functional Cards

(1) Types: There are four (4) "Spears" cards, each add 1 point to the shooting attribute of any specific player.

There are four (4) "Fortification" cards, each add 1 point to the tackling attribute of any specific player.

There are four (4) "Airpower" cards, each add 1 point to the heading attribute of any specific player.

There are four (4) "Dribbling-mania" cards, each add 1 point to the dribbling attribute of any specific player.

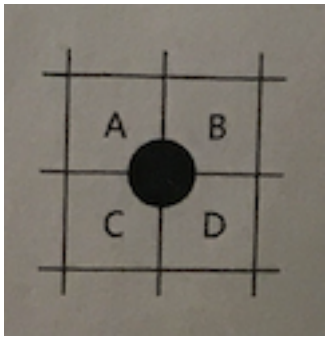
There are two (2) "Cannon" cards, which makes a specific player's shooting attribute outside the box equal to that inside the box. Adding a second "Cannon" card makes no distinction with the case of adding just one "Cannon" card.

(2) Effects: Except the "Cannon" cards, one can overlie as many as possible one same type out of the other four types of functional cards. Although we cannot overlie the "Cannon" cards, we can add as many as possible "Spears" to "Cannon" cards to boost the shooting attribute.

(3) About the mechanic of using functional cards during the game, please refer to 4.4.

5.6 Game Rules Details

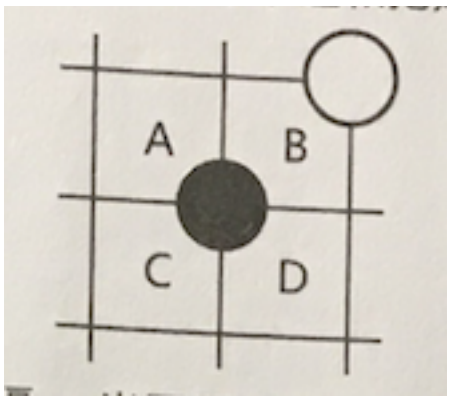
This detailed part of game rules, in comply with rules of modern soccer, elaborates finesse technique in forming tactics during the game play. Here we still adopt to the terms "A", "SA", "P" and "SP" in 5.2. Additionally, we will frequently analyze on a mini perspective of the chess board, as shown in the picture below, the "ruling zone (aka. RZ)" where any specific player can take actions. Here, an RZ is divided into four (4) parts, labeled as "Aa (to be different from the "A" defined before)", "B", "C", and "D".

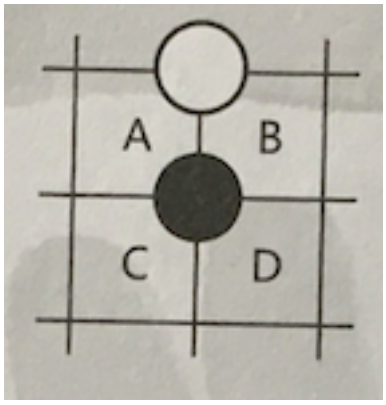


5.6.1 Position of players: Every players (including goalkeepers) must always stay on the intersection point (IP) between the "latitudes" and "longitudes" on the chess board. For any IP, it is allowed to stand only one player. No overlapping in positions is allowed. Except only a few special cases (kick-offs, corners, throw-ins, penalties) where the ball is placed on special IPs (The kickoff points, the 12-yard spots, IPs on the sidelines, and corner-kick spots), the balls should be places in the square grids on the chess board. When a player is with the ball, he can possess the ball within his RZ, on Aa, B, C or D as shown in the picture above.

5.6.2 Moving with barricading: If there are players on the opposite team barricading on the desired moving route of a player, his speed will be reduced: from 3 to 2 and from 2 to 1.

5.6.3 Pressure, tackle and breakthrough:

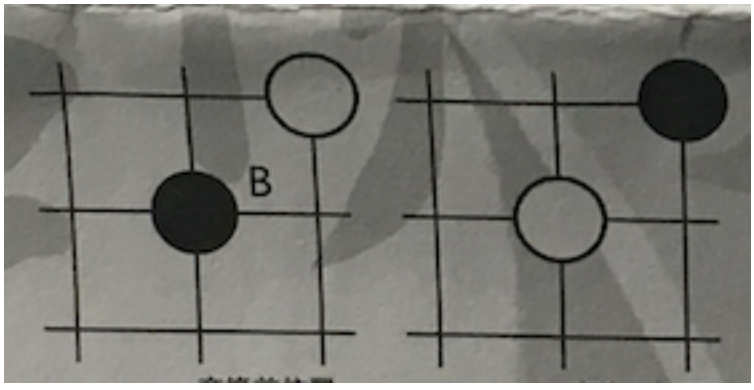




In this game, "Pressuring and Tackling" are two separate concepts, just as in real-world soccer.

For example, in the picture on the left, if the black player controls the ball in Aa, C or D, then the white player can put pressure on the black player, but no duel for tackling. Pressures are beneficial to the defending team since if the black player wish to pass in the direction to the top-right, duel for interception will be activated.

But on the other hand, if the black player controls the ball in B (in the top-left picture), or in A or B (in the top-right picture) and the white player, moves to the position shown in either picture in his following step, then duel for tackling is activated. After the duel, the two players switch their positions, with whoever wins the duel has the ball possession (the player is allowed to



place the ball in any grids except in the former place).

Similarly, this mechanic works for breakthroughs. If the black player has the ball in B, wishes to breakthrough the defense of the white player whose position is shown above, then duel for breakthrough is activated.

After the duel, the two players should switch their positions, with whoever wins the duel has the ball possession (the player is allowed to place the ball in any grids except B). Note that, if the black player wins the duel, then the round restarts and the black team continue with the move.

If the tackle involves multiple tacklers, then the SA in the duel should be (the highest attribute of players in A involved in the action + functional cards added on + a toss of die + n-1) where n stands for the number of players in A involved in the tackling action. If successful in tackling, the ball goes to (i) has the highest attribute in tackling, and (ii) the closest to the mid-point of the goal of A. If unsuccessful in tackling, and the test for scenario shows +1, the card should be awarded to (i) has the highest attribute in tackling, and (ii) the closest to the mid-point of the goal of A.

About calculating rounds, please refer to 5.1 and 5.2.

5.6.4 Passing: Passings are classified into ordinary ground passes and lobbing passes. During the half-round the team with ball possession is allowed to take a pass, by any means. When executing passes, a team can choose to pass first and then the player receives the pass move with the ball (breakthrough is allowed), or let the player to receive the pass move first and then complete the pass. Before any pass, the team with the ball should declare a grid (G) into which the ball is desired to pass. If any player on the opposite team blocks in the route of the ordinary ground pass, then the duel for interception of ordinary pass should be involved. In this case, any additional blocking players contributes +1 in the calculation of SA. If A wins the duel, then the ball is intercepted by whoever (i) has the highest attribute in tackling, and (ii) farthest from the grid where the ball is passed from. If no players on the opposite team blocks in the route of the ordinary ground pass, then we can conclude with the ball being successfully passed whatever the passer's attribute in passing is. If lobbing passes are to performed, we should involve test for turnover first (Please refer to 5.1). If the test fails, please refer to 5.1 for test for turnover and 5.3 for dead-balls. If the test does not fail, and there is not any other player on the opposite team blocking within one grid from the place where the lobbing pass is to be performed, then the pass is considered success, and if receiver of the pass shared the same G with other players from the opposite team, then the duel for heading is to be involved. Any additional players on the same contributes +1 for either SA or SP, and whoever (i) on the team that wins the duel, (ii) has the highest attribute in heading, and (iii) closest to the place where the ball is passed. (iv) closest to the mid-point of the goal of the opposite team.

Note that heading passes are no farther than 6 grids from the starting point. Use Pythagorean Theorem where applicable.

About calculating rounds, please refer to 5.1 and 5.2.

5.6.5 Shots: Shots are classified into four categories: ordinary shots inside the box, header shots inside the box, long-range shots outside of the box (including direct free-kicks), and penalty shots. If there are players on the defensive team blocking on the route of the shots, each player contributes -1 to SA in duel for shooting. And for long-range shots outside the box, every grid from the closest edge of the box contributes -1 to SA in duel for shooting, but this disadvantage resulting from range can be entirely offset by the functional card "Cannon". Players are not allowed to take shots beyond or equal to 5 grids from the box. For direct free-kicks, no matter how many defensive players are on the shooting route, they altogether contribute -1 to SA.

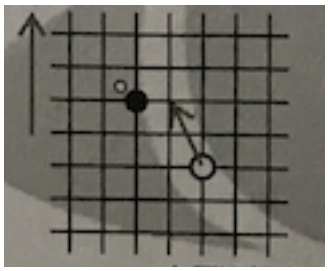
For all shots, tests for turnover should be performed before the shooting action (Please refer to 5.1). For respective duels afterwords, please refer to 5.2. Subsequently, do tests for scenario (Please refer to 5.1).

For penalty shots, the mechanic is different. After passing the test for turnover (all players are scaled to the case if the die does not show "4" they will pass the test), both teams pick a direction (from "left, right or middle") they wish to kick (for the attacking team), and to make a save (for the defensing team). If the two directions differ, then a goal is scored. If the two directions are the same, then duel for penalty shot is called. If the penalty is saved, there should be a subsequent test for scenario (Please refer to 5.1 and 5.2). Note that for penalties taken during the normal gameplay, both teams can add functional cards to their players, while for penalties taken during the penalty shootout, no functional cards can be added to players.

About calculating rounds, please refer to 5.1 and 5.2.

5.6.6 Corners: Corners have relations with passing and heading (5.6.4).
And about calculating rounds, please refer to 5.1 and 5.2.

5.6.7 Fouls and bookings: Every foul except offside calls are related to booking of at least a yellow card. Fouls includes but not limited to: tactical fouls, marginal decision fouls, and offside calls. For tactical fouls, they are usually to be considered seriously. For example in the illustration below, where the arrow on the left shows the attacking direction of the black team. In this case, the black team has a potential one-on-one situation, while the white defensive player has speed of 2, which means that he could not catch up the black player or he could not get a chance to start a duel for tackling, and in order to stop him, the white team can run to the RZ of the black player and commit a foul to gain his team advantage to re-formation its defense. This kind of fouls will results in dead-ball, and the attacking team with a direct free-kick if the foul is outside the box, and a penalty if the foul is inside the box. If the white player is the last defender other than the goalkeeper, a red card should be awarded to him.



For fouls that are related to duels for tackling with marginal decisions (tests), please refer to (5.6.3). For fouls involving goalkeepers, if the goalkeeper is on team A while fouls, a red card is given. If the goalkeeper is on team P while fouls, a yellow card is given.
When there is an offside call, the ball possession should change, and an indirect free-kick is awarded to the opposite team at the spot the foul is given.
About calculating rounds, please refer to 5.1 and 5.2.

5.6.8 Throw-ins and goal-kicks: Their mechanics are the same as “passing”, please refer to 5.6.3.

5.6.9 Added-on times and end of the game. Please refer to 4.9.

5.6.10 Surrender and immediate losses: When a team has been down for goals, that the team is subjectively considered unlikely to comeback, the losing team can appeal for surrender. But this appeal should be valid only if the opposite team shows consent.
Cases for forced losses: (i) When a third player on the same team is sent off, the game is immediately over with score 0-3. (ii) When a second goalkeeper on the same team is sent off, the game is immediately over with score 0-3.

5.6.11 Penalty shootouts: If the game is drawn, and both teams agree to play penalty shoot-out, then both teams toss a die, and whichever with a higher outcome choose to kick first or to defend first. The penalty shootout has five (5) rounds with both teams take penalties alternatively. The duel for penalty shootings are the same as in 5.6.5, but no functional cards are allowed. If the score is a draw when the fifth (5th) round ends, an additional round is to be added, and this iterates until there is a winner. The penalties can only be taken by the players on the field, and they cannot betaken by players on the bench, or players that had been sent off. The player can only take his second penalty after all eligible players on the same team finish taking their first penalties. If the eligible players are not equal on both teams, the team with more eligible penalty takers are to pick m players to take penalties where m equals to the number of eligible penalty takers on the team with less eligible players, but both teams should send a goalkeeper to be on the roster of penalty takers.

VI. Probability analysis

6.1 We will discuss the probability models that are used in this game.

	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Firstly, we start with the probability of tossing two dice. This model fits because in this game, team A and team B are allowed to toss independently and respectively a die when they are battling in a duel. In the table above, each grids denotes the probability of getting the outcome as the pair (row, column), where the rows denote the outcome for team A, and the columns denote the outcome for team B.

Table A:

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
-4	0	0	0	0	0	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34
-3	0	0	0	0	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1
-2	0	0	0	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1
-1	0	0	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1
level	0	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1	1
+1	0	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1	1	1
+2	0	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1	1	1	1
+3	0	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1	1	1	1	1
+4	1/34	1/11	3/16	10/31	1/2	21/31	13/16	10/11	33/34	1	1	1	1	1	1	1	1

Take a look at “table A” above: The columns stands for the differences between team A and team B in certain attributes that are on the cards, and the rows stands for the differences between team A and team B in functional cards that are helpful in the duel. The probabilities shown in the table are conditional probabilities that given the both conditions (differences in attributes, and differences in numbers of functional cards), the probability of team A to win the duel, by an additional simultaneous toss of die with the opposition. These probabilities are obtained by summing up geometric series because when SA equals SP, they are to re-toss a game of die simultaneously. For example, when we have (level, -3), the probability for team A to win is: (the probability of getting (5,1), (6,1), or (6,2) in the long run)=3/36*(1+3/36*3/36+(3/36)^2*3/36+……)=3/36*36/33=1/11. This is obtained because from our first table in this section, we can see that the winning chance for team A in 1 toss-of-dice is 3/36 (as shaded in red), and for team A to win in 2 tosses-of-dice is (3/36)*(3/36), because this case can only happen when the first toss results in SA=SP, the outcomes of which is as shaded in brown, and team A has 3/36 winning chance of the second toss, since the two tosses are independent of each other. Therefore, for team A to win after n tosses-of-dies is (3/36)^(n-1)*(3/36). When n goes to infinity, the total probability for team A to win becomes an infinite sum of geometric series which is convergent, and we get the result as above.

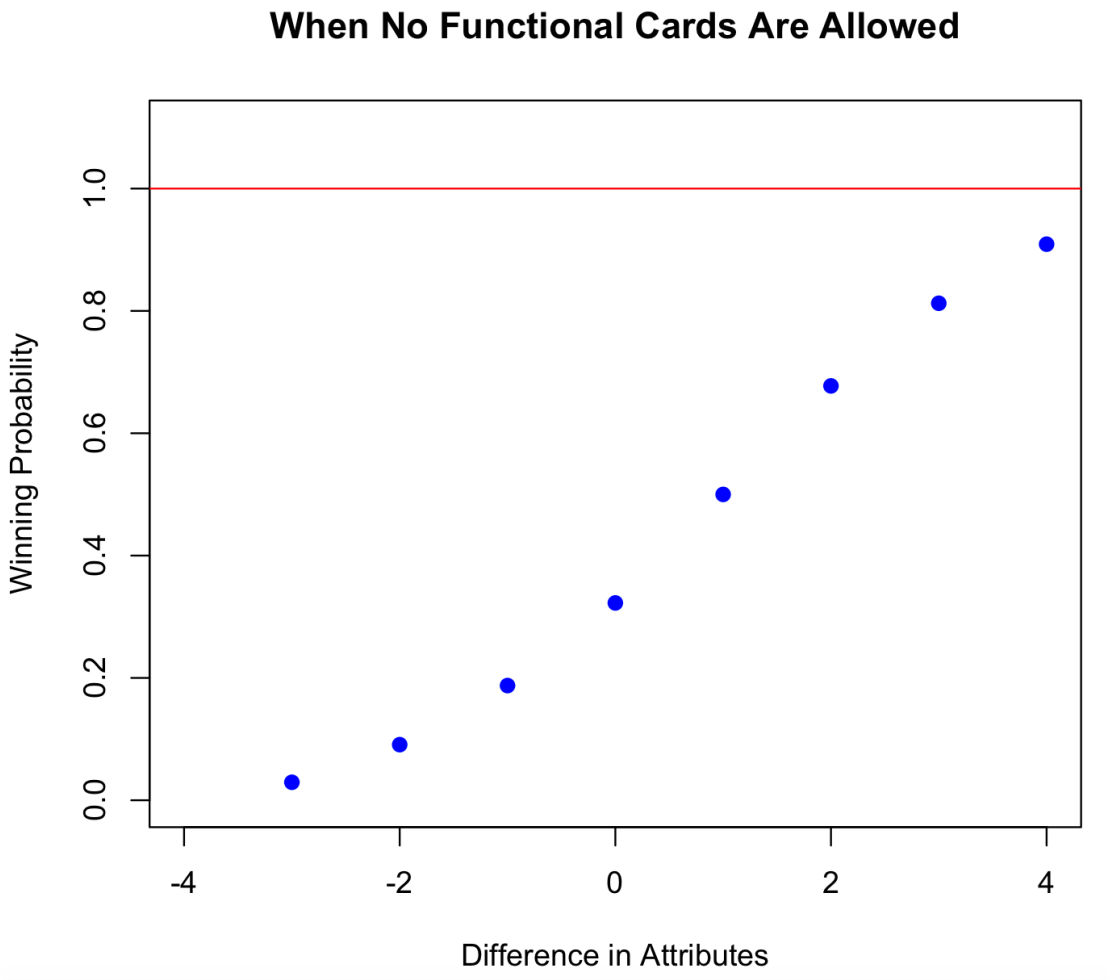
Table A (Probability In Decimals):

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
-4	NA	NA	NA	NA	NA	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706
-3	NA	NA	NA	NA	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA
-2	NA	NA	NA	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA
-1	NA	NA	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA
level	NA	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA	NA
+1	NA	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA	NA	NA
+2	NA	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA	NA	NA	NA
+3	NA	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA	NA	NA	NA	NA
+4	0.0294	0.0909	0.1875	0.3226	0.5000	0.6774	0.8125	0.9091	0.9706	NA	NA	NA	NA	NA	NA	NA	NA

And more obviously we can observe that if we change fractions into decimals, and discard these situations where a toss of die would not help.

The plot below shows the winning probability in duels of player A when functional cards are not allowed for both sides against the differences in attributes (Please refer to the area of data that are circled-red in the table above, each blue dot in the plot below denotes one figure in the table above, from left to right, respectively). More specifically, the x-axis shows differences between the attribute player A uses in the duel, and the attribute player B uses in the duel. The probability increases more aggressively in the middle, while less on

both ends.



We know that there are some duels involving tests for turnover. In the following three tables, we show the probability of winning the test+the duel based on the cases where PA is greater than or equal to 9, PA is either 7 or 8, PA is less than or equal to 6, respectively:

Table B (1, 2, 3):

	−8	−7	−6	−5	−4	−3	−2	−1	level	+1	+2	+3	+4	+5	+6	+7	+8
−4	NA	NA	NA	NA	NA	NA	NA	NA	0.0245000	0.0757500	0.1562500	0.2688333	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333
−3	NA	NA	NA	NA	NA	NA	NA	0.0245000	0.0757500	0.1562500	0.2688333	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333	NA
−2	NA	NA	NA	NA	NA	NA	NA	0.0757500	0.1562500	0.2688333	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333	NA	NA
−1	NA	NA	NA	NA	NA	NA	NA	0.1562500	0.2688333	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333	NA	NA	NA
level	NA	NA	NA	NA	NA	NA	NA	0.2688333	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333	NA	NA	NA	NA
+1	NA	NA	NA	NA	NA	NA	NA	0.4166667	0.5645000	0.6770833	0.7575833	0.8088333	NA	NA	NA	NA	NA
+2	NA	NA	NA	NA	NA	NA	NA	0.5645000	0.6770833	0.7575833	0.8088333	NA	NA	NA	NA	NA	NA
+3	NA	NA	NA	NA	NA	NA	NA	0.6770833	0.7575833	0.8088333	NA	NA	NA	NA	NA	NA	NA
+4	NA	NA	NA	NA	NA	NA	NA	0.7575833	0.8088333	NA	NA	NA	NA	NA	NA	NA	NA

	−8	−7	−6	−5	−4	−3	−2	−1	level	+1	+2	+3	+4	+5	+6	+7	+8
−4	NA	NA	NA	NA	NA	NA	NA	NA	0.0196000	0.0606000	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	NA
−3	NA	NA	NA	NA	NA	NA	NA	0.0196000	0.0606000	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA
−2	NA	NA	NA	NA	NA	NA	NA	0.0196000	0.0606000	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA
−1	NA	NA	NA	NA	NA	0.0196000	0.0606000	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA
level	NA	NA	NA	NA	NA	0.0606000	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA	NA
+1	NA	NA	NA	NA	NA	0.1250000	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA	NA	NA
+2	NA	NA	NA	NA	NA	0.2150667	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA	NA	NA	NA
+3	NA	NA	NA	NA	NA	0.3333333	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA	NA	NA	NA	NA
+4	NA	NA	NA	NA	NA	0.4516000	0.5416667	0.6060667	0.6470667	NA	NA	NA	NA	NA	NA	NA	NA

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
-4	NA	NA	NA	NA	NA	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	NA	NA	NA
-3	NA	NA	NA	NA	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	NA	NA	NA
-2	NA	NA	NA	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	NA	NA	NA
-1	NA	NA	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA
level	NA	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA	NA
+1	NA	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA	NA	NA
+2	NA	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA	NA	NA	NA
+3	NA	0.01470	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA	NA	NA	NA	NA
+4	0.0147	0.04545	0.09375	0.16130	0.25000	0.33870	0.40625	0.45455	0.48530	NA	NA	NA	NA	NA	NA	NA	NA

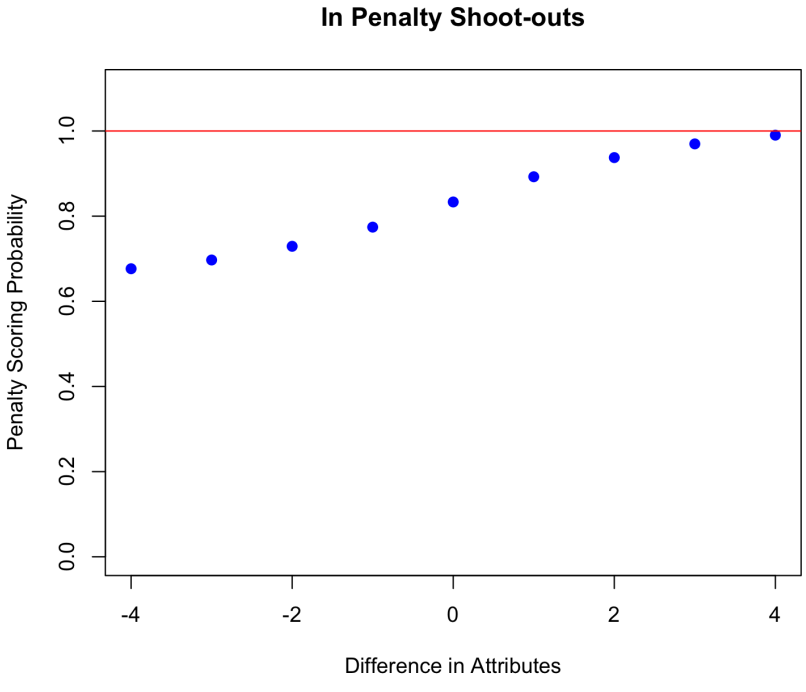
6.2 Analysis of penalty scoring probability

Please refer to (5.6.5) for penalty-kicking rule in this board game. Let us first simplify the model by omitting tests for turnover (we will definitely introduce them back to our model later). Thus, we start from both teams picking their own direction cards (directions to kick or to save) strategically. Therefore, if the kicker pass the test for turnover, he has 2/3 chance to score directly (the chance for both teams to pick different direction is (3*2)/(3*3)=2/3). And when it comes to the rest 1/3 chance to have both team picking the same direction, a duel for penalty is called. Therefore, the probability table becomes 2/3+(1/3)*Table A (Probability in Decimals) for penalties happening during the gameplay where teams can use functional cards, and when it comes to penalties in shootouts where functional cards are not allowed, the scoring probability becomes 2/3+(1/3)*(the area that is circled red in Table A(Probability in Decimals)). The two tables are displayed below as Table C (1, 2) plus a Probability plot for penalty shoot-outs:

Table C (1, 2):

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
-4	NA	NA	NA	NA	NA	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902
-3	NA	NA	NA	NA	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA
-2	NA	NA	NA	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA
-1	NA	NA	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA
level	NA	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA
+1	NA	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA	NA
+2	NA	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA	NA	NA
+3	NA	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA	NA	NA	NA
+4	0.6765	0.6970	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA	NA	NA	NA	NA

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
level	NA	NA	NA	NA	0.6765	0.697	0.7292	0.7742	0.8333	0.8925	0.9375	0.9697	0.9902	NA	NA	NA	NA



Now, our discussion should introduce the test for turnover (Please refer to 5.6.5). In all cases, we multiply the probability by 5/6. Then the overall table will become:

Table D:

	-8	-7	-6	-5	-4	-3	-2	-1	level	+1	+2	+3	+4	+5	+6	+7	+8
-4	NA	NA	NA	NA	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252
-3	NA	NA	NA	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA
-2	NA	NA	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA
-1	NA	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA
level	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA
+1	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA	NA
+2	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA	NA	NA
+3	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA	NA	NA	NA
+4	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA	NA	NA	NA	NA

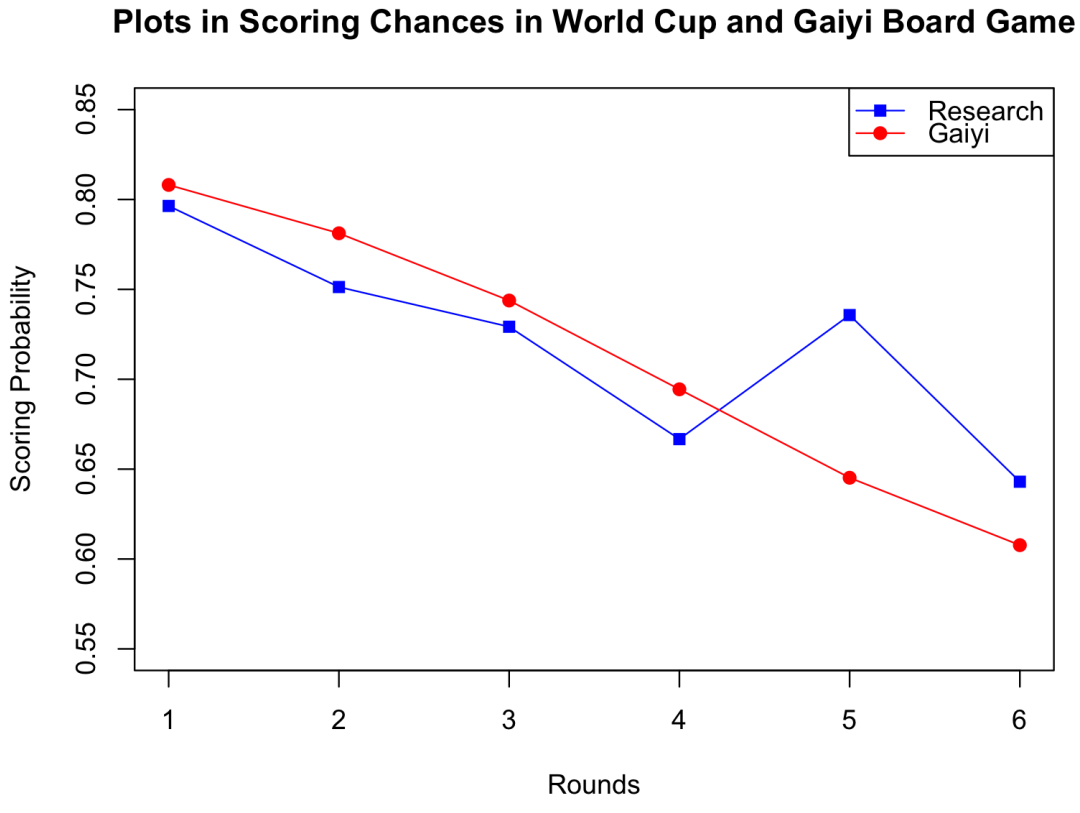
And the probability of scoring in penalty shoot-outs in general, is:

Table E:

level	NA	NA	NA	NA	0.5638	0.5808	0.6077	0.6452	0.6944	0.7438	0.7812	0.8081	0.8252	NA	NA	NA	NA

According to a research on penalty shoot-outs in World Cups that is conducted by Jordet et. al. in 2006, "in all competitions including World Cup, European Championship, and Copa America, the scoring success rates are: first kick 86.6%, second kick 81.7%, third kick 79.3%, fourth kick 72.5%, fifth kick 80%, and "Sudden death' kicks 64.3%.(*)" The researchers also mentioned that: "the percentage success rate in the World Cup is 71.2% compared to 82.7% in Copa America and 84.6% in the European Championship." If we slightly modify (*) by multiplying a multiplier beta=71.2%/((71.2%*2+82.7%+84.6%)/4)=0.9196, then the scoring success rates for World Cup (*) will become: "first kick 79.64%, second kick 75.13%, third kick 72.92%, fourth kick 66.67%, fifth kick 73.57%, and "Sudden death' kicks 59.13% (**)".

We then plot (**) against the entries 7-12 from Table E, we can see similar chances and trends.



And if we conduct paired t-test with null hypothesis H0: True difference in means is equal to 0, and alternative hypothesis: True difference in means is not equal to 0. From the t-test below, we can see that since p-value is 0.732>0.05, at 5% significance level, we do not reject the null hypothesis that "True difference in means is equal to 0". Therefore, there is another way to demonstrate that our penalty kick model (for penalty shoot-outs) fits the real-world situation very well.

```
        Paired t-test

data:  pair$Research and pair$Gaiyi
t = 0.36215, df = 5, p-value = 0.732
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.04258534  0.05655201
sample estimates:
mean of the differences
            0.006983333
```

VII. Acknowledgments

Jordet, G., Hartman, E., Visscher, C. and Lemmink, K. A. P. M. "Kicks from the penalty mark in soccer: The roles of stress, skill, and fatigue for kick outcomes." *Journal of Sports Sciences, 1-9*, Preview article.