Hohmann Transfer Orbit: Calculating the Optimal Mars Mission

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ABSTRACT

The purpose of this model is to simulate a mission to Mars using the Hohmann Transfer with varying starting velocities in order to find the optimal starting velocity. The Hohmann Transfer is a two-impulse elliptical transfer between two coplanar circular orbits; in this case, the orbits of Mars and Earth. The Hohmann Transfer is the best method for a space mission to Mars, because it is the most efficient. It requires the least change in velocity which means less fuel is needed and it's quick -- far quicker than alternative methods, given fuel constraints. The model shows that when the starting velocity of the ship is too low, the ship never manages to reach Mars' orbit, whereas when the starting velocity is too high, the ship goes beyond Mars' orbit. There is only a small range of initial velocities for which the ship manages to reach Mars' orbit at the right time and place, coming close enough to Mars to be caught in its sphere of influence (SOI), wherein we assume the ship begins to orbit Mars and the mission is considered a success.

I. Elliptical Interplanetary Transfer

The Hohmann Transfer, developed in 1925 by German scientist Walter Hohmann, is a two-impulse elliptical trajectory between two co-planar circular orbits about a single body. By necessitating the least change in velocity of any known interplanetary transfer, the Hohmann Transfer is praised for requiring the least amount of fuel. However, this transfer makes the fundamental assumption that just one body exerts gravitational force on the body of interest.

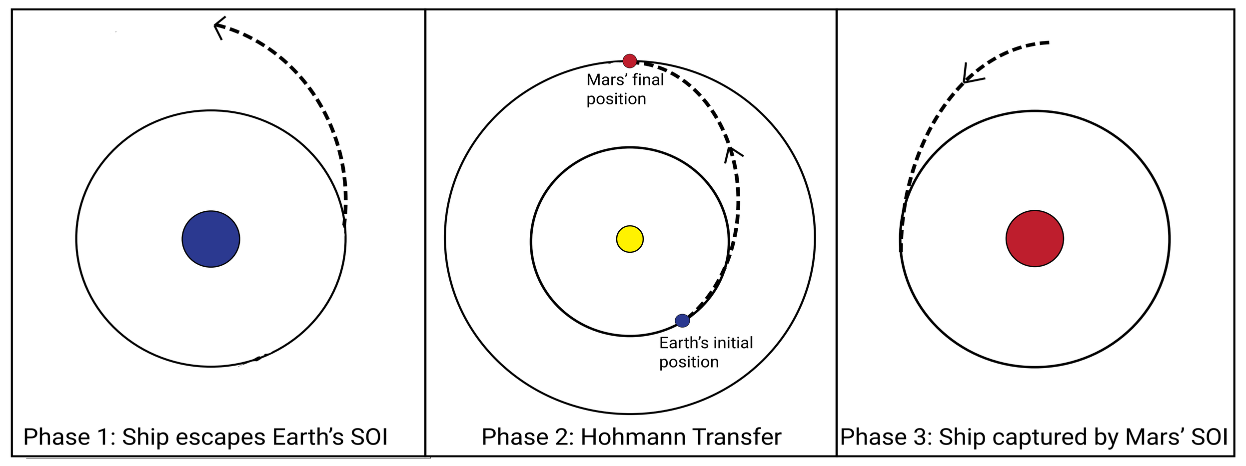

Figure 1

In order to model the entire trip, a patched conic approach would be used to model all three phases: the mission leaving Earth's gravity well, the Hohmann Trajectory between planets (based on the sun's gravitational field), and the mission's capture by Mars' gravitational pull.

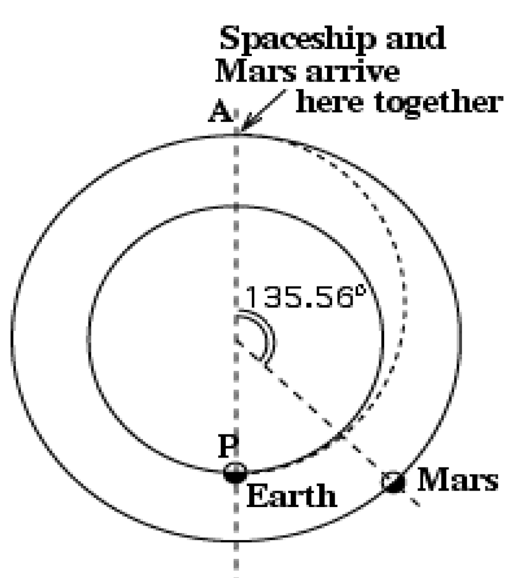

Figure 2 *Courtesy of http://www.phy6.org/stargaze/Smars1.htm*

In modeling the Hohmann Transfer (the mission's second phase), the question of starting conditions is implied. The objective of this model is to determine the answer to the following question: *How does the initial ship velocity at the edge of Earth's sphere of influence (SOI) impact the distance between the ship and Mars at the completion of the Hohmann Transfer?*

II. Possible Cases of Motion

Through basic laws of physics, the following three possibilities can be hypothesized. Success can only be characterized by meeting the conditions of Case 2.

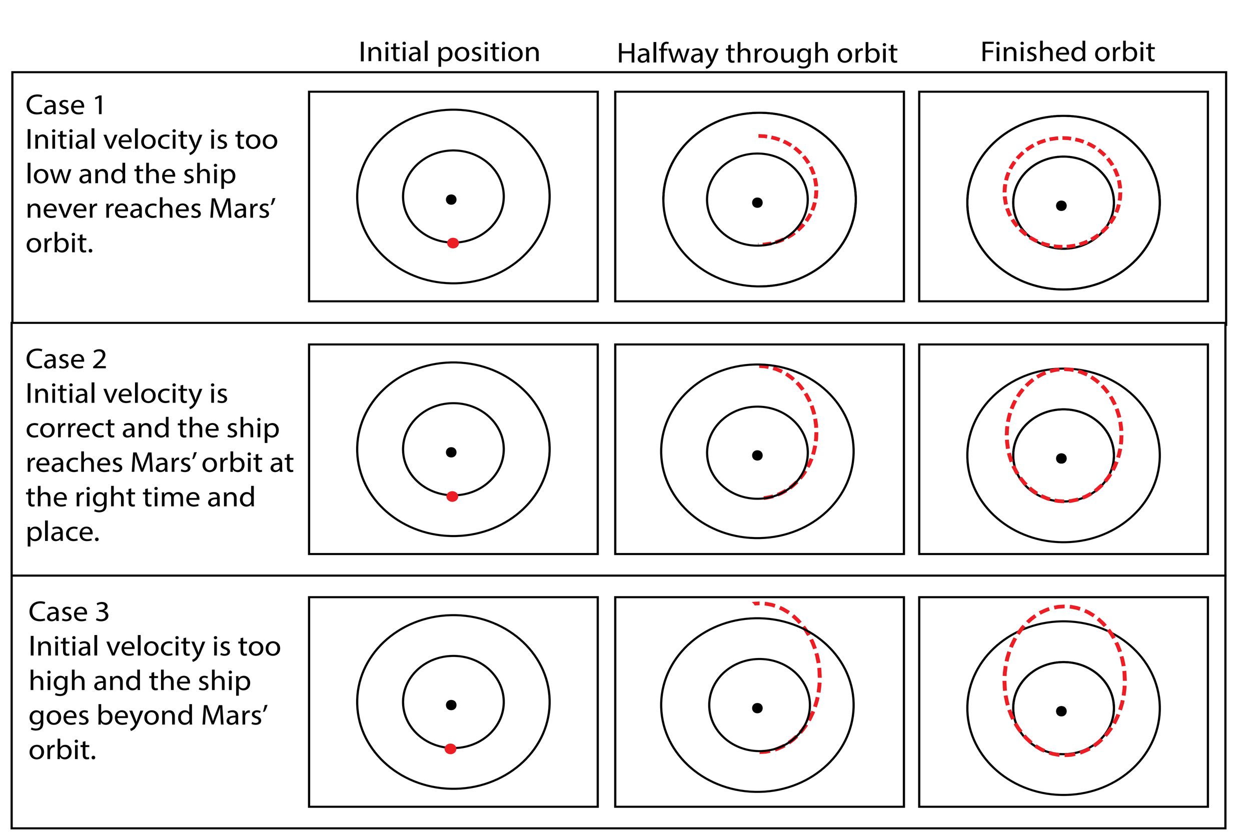

Figure 3

III. Simple Orbital Model

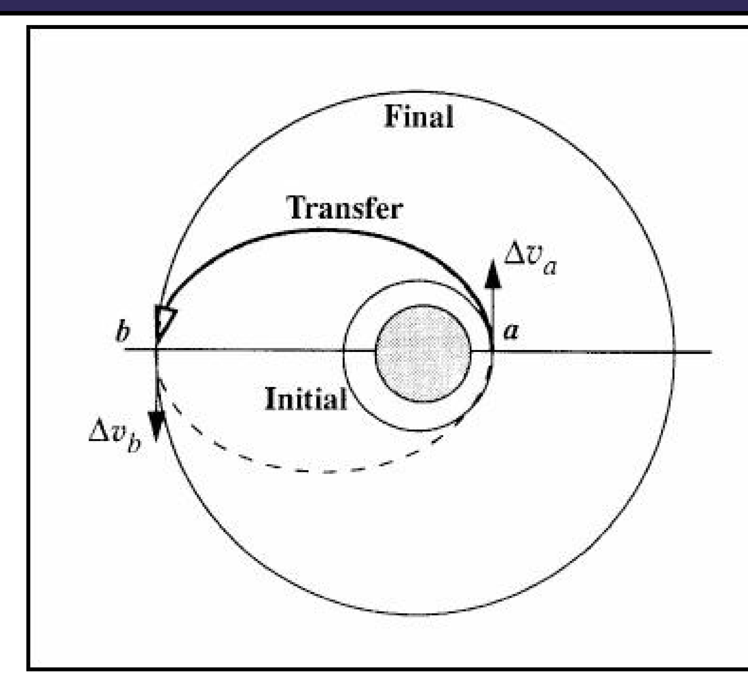

Figure 4 *Courtesy of http://rantonels.github.io/capq/q/OM2.html*

The only forces included in the orbital model are the gravitational forces of the Sun on the three bodies: Mars, Earth, and the ship. There are only two force equations (the force in the x direction and the force in the y direction) for every body. The equations look like this:

$$F_{sun} = \frac{G(M_{sun})(M_{body})}{r^2}\hat{r}$$

$$A_{body} = \frac{F_{body}}{M_{body}}$$

The bracketed part of the force equation is the directional component. Using these two equations for the gravitational force being exerted by the Sun onto Mars, Earth, and the ship, and data from NASA for the initial velocities of Mars and Earth, the model is able to simulate Earth and Mars' orbits around the Sun.

IV. Model Limitations

The model...

- Assumes only the Sun is exerting a gravitational force on the ship
- Assumes the ship will begin to orbit around Mars if the ship enters Mars' sphere of influence, regardless of the ship's velocity at that time
- Simulates in two dimensions instead of three
- Does not incorporate thrust or aerobraking -- the ship gaining/losing speed or changing direction on its own
- Assumes that both Earth and Mars' orbits are circular for the Hohmann Transfer -- Mars is affected by Jupiter, so it does not have a perfectly circular orbit
- Does not take fuel into account

V. Model Validation: NASA HORIZONS

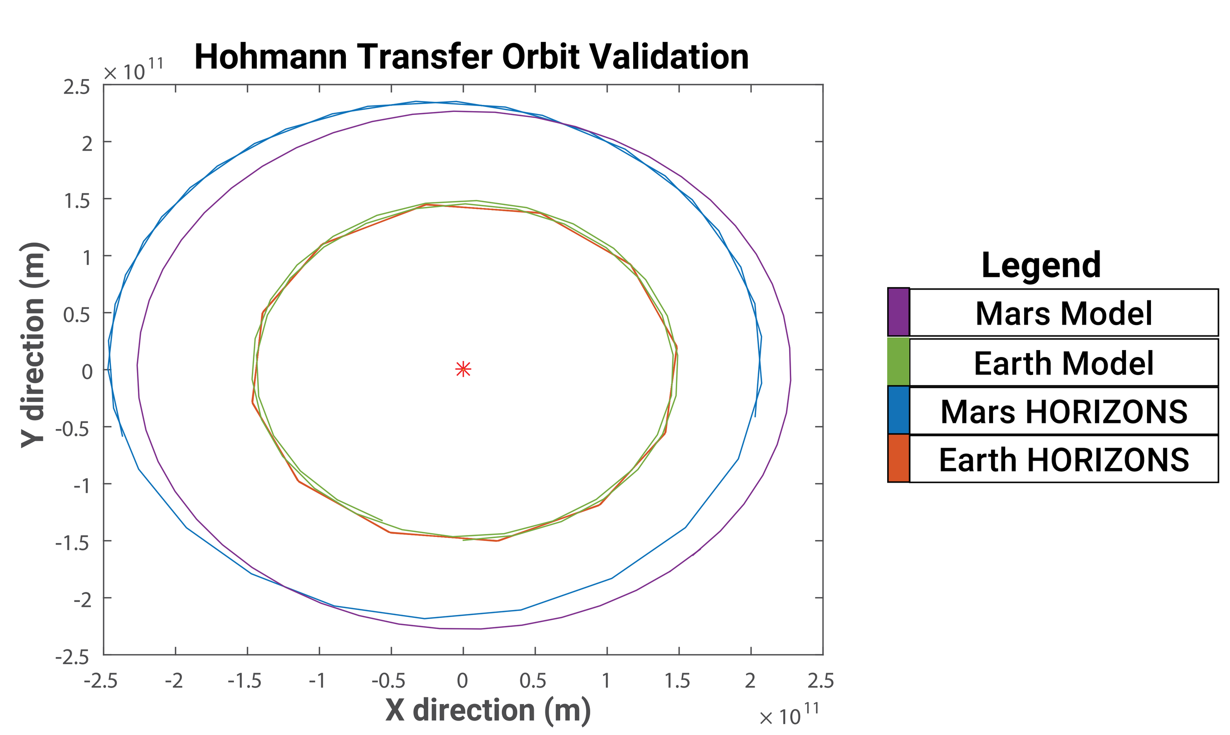

Figure 5

In order to validate our model, NASA HORIZONS positions data was generated for the January 1, 2015 to January 1, 2018 time-period with a step size of 1 calendar month. This data was then overlaid with our model's findings in Figure 5.

Our model of Earth's orbit was very similar to the Earth HORIZONS data. However, Mars' orbit from HORIZONS data is shifted slightly left and up.This variation was anticipated due to the fact that we did not include the gravitational effects of planets (i.e. Jupiter) in our model of Mars. This shift means that a ship following our model will be slightly further away from the actual position of Mars.

The Hohmann Transfer was validated in that it reached Mars in our "best case model" from a reasonable launch velocity from Earth. This launch velocity was around the order of magnitude of Earth's known escape velocity, with additional velocity added to create the Hohmann Transfer's elliptical orbit.

VI. Hohmann Transfer: Velocities

The model simulated the ship's orbit with varying starting velocities based on the Earth's starting velocity. The ship has to be moving faster than Earth to break out of the Earth's orbit around the Sun, but not so much faster that it goes beyond Mars' orbit. The optimal starting velocity (given to the green orbit) for the ship is roughly 1.093 x the Earth's starting velocity, about 2.6e9 m/day.

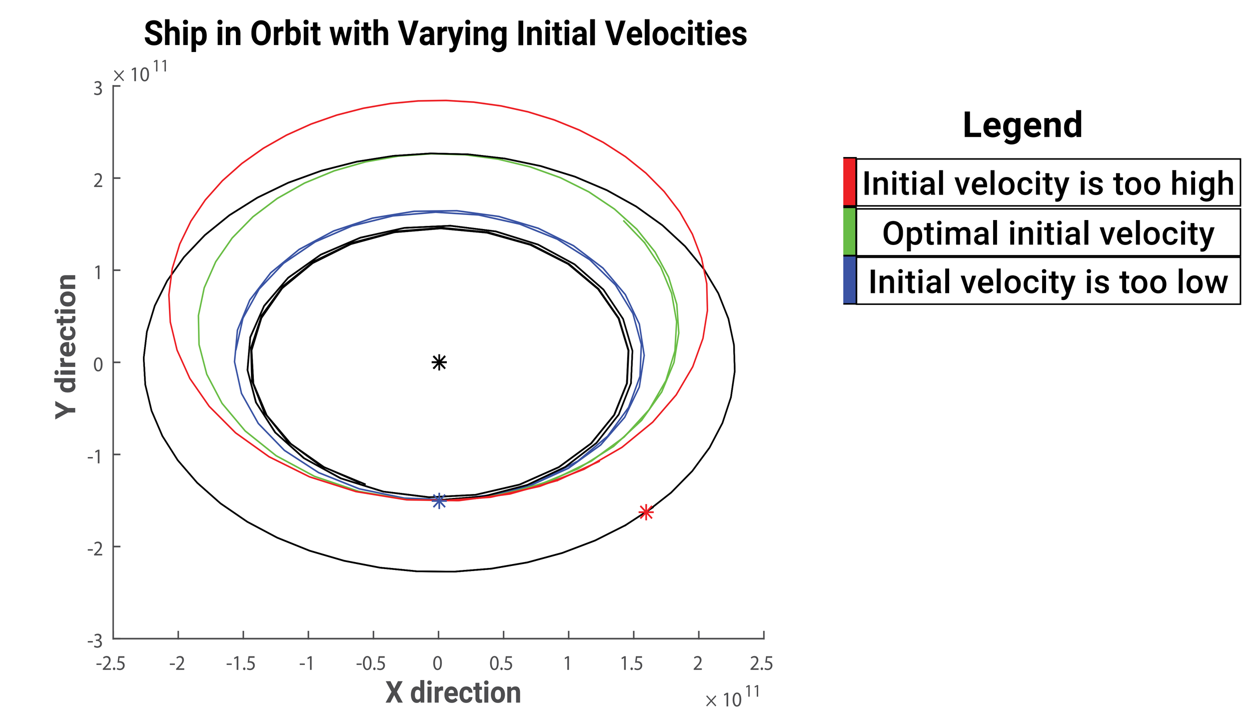

Figure 6

VII. Impact of Velocity on Distance

There is a very narrow range of initial velocities that allow the ship to enter Mars' SOI. Initial velocities lower than about 2.82e9 m/day result in increasingly greater final distances from Mars and vice versa for velocities higher than roughly 2.85e9. Between those two numbers, there is a small range of velocities that will achieve the desired result -- the ship orbiting Mars.

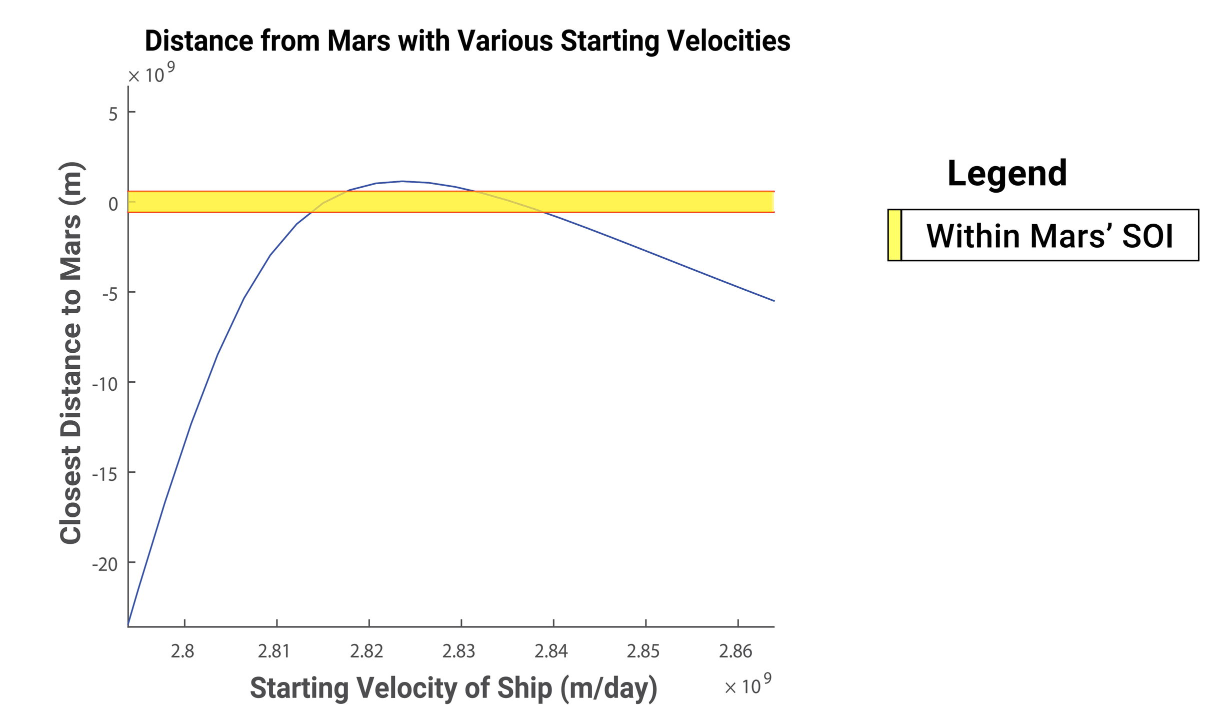

Figure 7

VIII. Future Work and Conclusions

Gravitational pull of other bodies
The model should incorporate the gravitational forces between planets, such as the attraction between Mars and Jupiter, as well as the forces of Mars and Earth on the ship. This will alter the ship's path slightly and likely affect the range of initial velocities that let it get close enough to Mars.

Phases 1 and 2
The ship's escape from Earth's sphere of influence and hyperbolic capture by Mars' gravitational pull (phases one and two of the patched conic approach) could be included in the model in the future.

Final velocity
The ship's velocity when it reaches Mars matters, because if it's too high, the ship won't stay in orbit around Mars and if it's too low, the ship will crash into the planet. Right now, only the ship's distance from Mars is being examined, not its velocity at that distance.

Fuel
The ship will need to gain speed to break out of its orbit around Earth and the Sun -- this would require fuel, which could be incorporated in the model. A real Mars' mission will have fuel restraints, so this is an important factor.