Counting Many as One: the Ontological Insignificance of Grammatical Number

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1 Introduction

There are many intuitive cases where it appropriate to characterize one and the same thing as both one F and many Gs, for different count nouns F and G.

(1) This is both one copse and five trees.

(2) These two women are one couple.

These claims are witnesses to a more general thesis I'll call **One-Many**:

One-Many Something can be one F and m Gs, where $m > 1$.

From antiquity to the present, many philosophers have endorsed the One-Many thesis. Plato and Frege provide two illustrative examples:

> "If someone should demonstrate that I am one thing and many, what is astonishing about that? He will say, when he wants to show that I'm many, that my right side is different from my left. . . . But when he wants to show that I'm one, he will say I'm one person among the seven of us."
>
> Plato 1997, *Parmenides*, 129c-d

> "One pair of boots may the same visible and tangible phenomenon as two boots."
>
> Frege 1884/1950

One-Many is a special case of a broader thesis we can call **Distinct-Manys**, which has intuitive witnesses in statements like (3).

Distinct-Manys Something can be n F(s) and m Gs, where $n \neq m$.

(3) These fifteen tourists are four families from Argentina.

In what follows, I'll tend to focus on the One-Many case since this is the case with the most intuitive metaphysical import.

The One-Many thesis is significant for a variety of metaphysical debates. By itself, it has interesting consequences for the relationship between ontology and counting: it implies there is no univocal answer to the question "How many things are there?" But perhaps One-Many's greatest interest is the fact that it is presupposed by Composition As Identity (CAI). This is the ancient mereological view that, when some objects are parts that compose a whole, the whole is identical with the parts.[1] On the most straightforward understanding of this view, it implies that something can be both *one* whole and *many* parts. CAI offers attractive solutions to a variety of mereological problems. For instance, if CAI is correct, then one can endorse Classical Extensional Mereology without worrying about any massive/bizarre proliferation of ontological commitments that many have argued follows from the endorsement of unrestricted mereological composition. Given CAI, ontological commitment to a sum does not go beyond being ontologically committed to the the things that compose it.[2]

However, in recent years, many authors have dismissed the One-Many thesis as a non-starter. The arguments that have done the most to discredit it are due largely to Yi (1999, 2005, 2006), who argued that the One-Many thesis is logically false. According to Yi, one can derive a contradiction from the supposition that one thing is identical to many things using only purely logical principles regarding the plural term forming operator $\oplus$ ("and") and the logical constant $\preccurlyeq$ ("is one of"). Though there have been some notable defenses of CAI and One-Many, many subsequent discussions have taken Yi's arguments as decisive.[3]

In my view this verdict is overhasty. The key elements of plural logic that Yi relies on to demonstrate the logical falsity of the One-Many Thesis do not faithfully regiment the natural language phenomena that plural logic takes as its motivation. There are semantics with independent empirical motivation according to which instances of the One-Many Thesis can be true.

The plan of the paper is as follows. In Section 2 I summarize two arguments, drawn from Yi's work, that purport to show, from plural logical principles, that no One-Many claims are true and no singular expression corefers with any proper plural expression. In Section 3, I argue that plural logic, considered as a regimentation of grammatical number marking, distorts natural language some radical ways. Specifically, I argue that plural logic ignores the compositional aspects of grammatical number marking, and that the axioms characterizing the inclusion predicate $\preccurlyeq$ do not hold of its supposed natural language counterpart, "is one of." Section 4 provides an example, drawing on Landman 2016, 2020, of an independently

[1]For a survey of historical figures who have affirmed CAI, see Normore and Brown 2014

[2]This, anyway, is what proponents of CAI have traditionally maintained. For critical discussion see Cameron 2011, Trueman and Thunder 2025, Loss 2021, Payton 2022a.

[3]Notable exceptions are Wallace 2011a, 2011, 2023; Payton 2019, 2021, 2022, Bohn 2011, 2014, 2019; Loss 2019, 2021. For authors who have endorsed Yi's arguments, or similar arguments deriving from plural logic, see Varzi 2014, Lewis 1991 p.87, Sider 2007; Calosi 2016, 2018.

empirically motivated semantics in which One-Many claims can be true. In Section 5, I use that semantics to provide a rebuttal of the arguments in Section 2, which I argue turn on fallacious applications of Leibniz's Law within non-extensional contexts. I also compare my rebuttal of these arguments with similar responses by authors writing in defense of CAI, in particular responses by Jonathan D. Payton (2019, 2021, 2022) and A.J. Cotnoir (2008). I close, in Section 6, by considering what my verdict—that standard plural logic fails as a regimentation of natural language singular/plural marking—implies for the metaphysical debates that plural logic regularly figures in.

2 Two Arguments Against One-Many

To present Yi's arguments, I'll first describe a standard language for plural logic.[4] This language extends the language of first-order logic by adding plural terms—plural constants $(aa, bb, cc...)$ as well as plural plural variables $(xx, bb, cc...)$—the term forming operator $\oplus$, and the constant $\preccurlyeq$.

A singular term (relative to an assignment) refers to an object, whereas a plural term (relative to an assignment) refers to one or more objects. Here, "refers" should be read collectively—to say that *aa* refers to Mia and Glenn does not imply that *aa* refers to Mia or to Glenn individually.[5] I'll call a plural expression that refers to several objects a *proper plural* and a plural expression that refers to one object an *improper* plural.

Any terms τ and σ can serve as argument to $\oplus$ to form a complex plural $\tau \oplus \sigma$. $\tau \oplus \sigma$ refers to the objects that either τ or σ refers to. $\oplus$ is meant to regiment the use of "and" as a coordinating conjunction in noun phrases, i.e. "Mia and Glenn met." This sentence can be translated as $M(m \oplus g)$, where $m \oplus g$ is a plural term referring to Mia and Glenn.

Finally, $\preccurlyeq$ is a binary relation that takes a singular terms as its first argument and a singular or plural term as its second argument. This is meant to regiment the phrase "is one of" and it holds when the object referred to by its first argument is among the things referred to by its second argument.

Yi argues that no singular term can have the same referent as any *proper* plural term.[6] I'll call one argument he has made in a number of places the *Term-Forming Argument*.[7] I'll first present the argument in English, and then show how it is regimented in plural logic. Suppose

[4]For general introductions to plural logic, see Yi 2005, 2006; McKay 2006; Oliver and Smiley 2016; Florio and Linnebo 2021.

[5]This choice is arbitrary—one can just as well develop standard plural logic under "refers" distributively (Oliver and Smiley 2016 Ch. 6), though the choice forces slightly different formulations of the definition of truth/satisfaction. I choose the collective reading because it allows for a simpler characterization of the difference between proper and improper plurals.

[6]For improper plurals, it is easy to see how this might happen. For instance, "Cicero $\oplus$ Tully" refers to all the things referred to by "Cicero" and by "Tully," but since these both refer just to the one Roman personage, all three terms have the same referent.

[7]This argument appears in Yi 1999 pg. 146-148, 2005 pg. 471-472,

the singular term "Mash" has the same referent as the proper plural term "Quine and Kripke," which refers to Quine and Kripke.

Term-Forming Argument

1	Mash is Quine and Kripke	Assumption
2	Quine is not Kripke	Assumption
3	Mash is one of Mash and Kripke	Plural Logical Truth
4	Mash is one of Quine and Kripke and Kripke	Leibniz's Law, 1, 3
5	Mash is Quine or Mash is Kripke or Mash is Kripke	Plural Logic, 4

The argument is a *reductio*—(5) cannot be true. "Mash" is supposed to refer to something that is Quine and Kripke together, not either of them individually. In fact, it is simple to derive a contradiction from 1, 2, and 5 using Leibniz's Law. Premise (1) is justified on the assumption that "Mash" and "Quine $\oplus$ Kripke" corefer. Premise (2) must be true if "Quine $\oplus$ Kripke" is to be a proper plural. (3), which would be symbolized as "Mash $\preccurlyeq$ Mash $\oplus$ Kripke" pronounced "Mash is one of Mash and Kripke," Yi regards as a logical truth. The referent of "Mash" is guaranteed to be one of the things referred to by "Mash and Kripke." (4) follows from (1) and (3) by Leibniz's law. Yi takes (5) to follow from (4) by the logic of plurals. Intuitively, we are applying the same sort of principle here that we apply in a valid argument like (4):

(4) a. The culprit is one of the dogs.
 b. Bella and Desmond are the dogs.
 c. Bella is the culprit or Desmond is the culprit.

In Yi's logic, this reasoning is captured by three axioms that concern $\preccurlyeq$ and $\oplus$, and singular terms: Axioms 9, 10, and 11. In the presentation below, x and y are singular variables, and τ and σ are metavariables ranging over terms of any kind, plural or singular.[8]

(**Axiom 9**)
$$\exists x(x \preccurlyeq \tau)$$

(**Axiom 10**)
$$\forall x \forall y(x \preccurlyeq y \rightarrow x = y)$$

(**Axiom 11**)
$$\forall x(x \preccurlyeq \tau \oplus \sigma \leftrightarrow (x \preccurlyeq \tau \vee x \preccurlyeq \sigma))$$

[8] To avoid introducing too many metavariables, I'm presenting Yi's Axioms in terms of their (singular) universal closure.

Using these axioms, we can regiment the the Term-Forming Argument in a way that shows its deductive validity. The lines in the original argument, (3) and (5), that were given a merely intuitive justification are formally derived below, in lines 3-10 and 12-18, respectively.

Term-Forming Argument

1	$\text{Mash} = \text{Quine} \oplus \text{Kripke}$	
2	$\text{Quine} \neq \text{Kripke}$	
3	$\text{Mash} \preccurlyeq \text{Mash} \oplus \text{Kripke} \leftrightarrow$	
	$(\text{Mash} \preccurlyeq \text{Mash} \vee \text{Mash} \preccurlyeq \text{Kripke})$	Axiom 11
4	$\exists x(x \preccurlyeq \text{Mash})$	Axiom 9
5	v | $v \preccurlyeq \text{Mash}$	Ass. for $\exists$ E
6	| $v \preccurlyeq \text{Mash} \rightarrow v = \text{Mash}$	Axiom 10
7	| $v = \text{Mash}$	$\rightarrow$E, 3, 4
8	| $\text{Mash} \preccurlyeq \text{Mash}$	Leibniz's Law, 3, 5
9	| $\text{Mash} \preccurlyeq \text{Mash} \vee \text{Mash} \preccurlyeq \text{Kripke}$	$\vee$I, 8
10	| $\text{Mash} \preccurlyeq \text{Mash} \oplus \text{Kripke}$	$\leftrightarrow$E, 3, 9
11	$\text{Mash} \preccurlyeq \text{Mash} \oplus \text{Kripke}$	$\exists$E, 3–9
12	$\text{Mash} \preccurlyeq (\text{Quine} \oplus \text{Kripke}) \oplus \text{Kripke}$	Leibniz's Law, 1, 11
13	$\text{Mash} \preccurlyeq (\text{Quine} \oplus \text{Kripke}) \oplus \text{Kripke} \leftrightarrow$	
	$(\text{Mash} \preccurlyeq (\text{Quine} \oplus \text{Kripke}) \vee \text{Mash} \preccurlyeq \text{Kripke})$	Axiom 11
14	$\text{Mash} \preccurlyeq (\text{Quine} \oplus \text{Kripke}) \vee \text{Mash} \preccurlyeq \text{Kripke}$	$\leftrightarrow$E, 12, 13
15	$\text{Mash} \preccurlyeq \text{Quine} \oplus \text{Kripke} \leftrightarrow \text{Mash} \preccurlyeq \text{Quine} \vee \text{Mash} \preccurlyeq \text{Kripke}$	Axiom 11
16	$(\text{Mash} \preccurlyeq \text{Quine} \vee \text{Mash} \preccurlyeq \text{Kripke}) \vee \text{Mash} \preccurlyeq \text{Kripke}$	Const. Dilemma, 14, 16
17	$\text{Mash} \preccurlyeq \text{Quine} \rightarrow \text{Mash} = \text{Quine}$	Axiom 10
18	$\text{Mash} \preccurlyeq \text{Kripke} \rightarrow \text{Mash} = \text{Kripke}$	Axiom 10
19	$(\text{Mash} = \text{Quine} \vee \text{Mash} = \text{Kripke}) \vee \text{Mash} = \text{Kripke}$	Const. Dilemma, 17, 18, 19

The only premises in this argument, (1) and (2), are justified by the assumptions that "Mash" corefers with "Quine $\oplus$ Kripke" and that "Quine $\oplus$ Kripke" is a proper plural term. The rest follows by plural logic. Since the terms in this argument are arbitrary we can conclude, according to Yi, that no singular term corefers with any proper plural term of the form "$\tau \oplus \sigma$," where τ and σ are terms of any kind. And, since it is provable in Yi's system that every plural expression corefers with some expression of the form "$\tau \oplus \sigma$," this is tantamount to proving that no singular term corefers with *any* proper plural term.

In Yi's logic, we can also frame a version of this argument without using the $\oplus$ operator, which Yi regards as dispensable.[9] If "aa" is a proper plural term then the sentence "$\exists x \exists y((x \preccurlyeq aa \wedge y \preccurlyeq aa) \wedge x \neq y)$" will be true. If we suppose that "b" co-denotes with such a proper plural terms, then we can frame the following *reductio* argument.

Pure-$\preccurlyeq$ Argument[10]

1	$aa = b$	
2	$\exists x \exists y((x \preccurlyeq aa \wedge y \preccurlyeq aa) \wedge x \neq y)$	
3	$\exists x \exists y((x \preccurlyeq b \wedge y \preccurlyeq b) \wedge x \neq y)$	Leibniz's Law, 1, 2
4	$\forall x(x \preccurlyeq b \rightarrow x = b)$	Yi Axiom 4
5	$\perp$	FOL, 3–4

(1) holds on the assumption that aa and b corefer. (2) amounts to the assumption that aa is a proper plural term. (3) follows from (1) and (2) by Leibniz's Law. (4) is an instance of Axiom 10. It's elementary to derive a contradiction (e.g. "$b \neq b$") from (3) and (4) using First-Order Logic. So we must reject the assumption that "b" or any other singular term co-denotes with any proper plural term.

I'll take it for granted that these arguments, if they are successful, also show that no One-Many claims are true. For if there was a true One-Many claim, i.e. "This is one F and several Gs," then we could, in the right context, refer to thing in question as both "The F" and "The Gs".[11] But if no singular expression can corefer with any proper plural expression, then the former condition cannot hold. Singular/plural coreference is One-Many identity approached by semantic ascent.

These, then, are two suggestive arguments that purport to show that singular and proper plural coreference and One-Many claims are logically incoherent. The force of these arguments, however, depends on the suggestion that Axioms 9-10 regiment ordinary principles of reasoning with plurals. But, I'll argue in the next section, there is reason to think that these axioms do not adequately reflect the semantics of plural referring expressions, "is one of," and their interaction.

[9] It can be given a contextual definition in terms of $\preccurlyeq$ along the following lines: $P^n(t_i, .., \tau \oplus \sigma, ...t_n) \coloneqq \exists xx(\forall y(y \preccurlyeq xx \leftrightarrow (y \preccurlyeq \tau \vee y \preccurlyeq \sigma)) \wedge P^n(t_i, .., xx, ...t_n))$.

[10] This is a variation on the the argument Yi gives in 2014 pg. 179. In that work, Yi's aim is to show that nothing is both *One* and *Many*, where these are predicates defined in terms of $\preccurlyeq$. My line 2 amounts to the assumption that aa is *Many* and my line 4 amounts to the assumption that b is *One*. To my knowledge Yi never presents the Pure-$\preccurlyeq$ Argument exactly as I do here.

[11] On a Russellian analysis of definite descriptions, of course, neither "the F" or "the Gs" is a referring expression. But this doesn't undermine the point that things satisfying "Gs" are apt to be referred to plurally.

3 Simplifications In Plural Logic

3.1 What is grammatical number?

What is the difference between a singular referring expression and a plural referring expression? Empirical linguistics and plural logic suggest radically different answers to this question. They give different pictures of what the singular/plural distinction in nominal expressions *is*.

To begin with, as a natural language phenomenon, the singular/plural distinction is a distinction within the broader category of *grammatical number*. Grammatical number is a feature that nominals,[12] in some languages, have—it is a correlation between morphosyntactic marking (e.g. *cow* v.s. *cow-s*) and meanings having to do with quantity. Typologically, there is wide variation in how grammatical number is realized across human languages.[13] There are languages that lack any number morphology on nominals, like Classical Chinese and Aymara (Harbour 2014). There are languages that, in addition to singular have other "exact" number markers like *dual* (Syrian Arabic) and *trial* (Larike). There are also languages that morphologically mark "approximative" number values like *paucal*, *greater paucal* and *greater plural*—roughly, "a few," "a bit more than a few," and "very many."[14] The singular/plural dichotomy found in, for example, English is a common system of number marking, but it is only one of many.

By itself, the fact that there is such variety does not suggest a radical disconnect between natural language and plural logic. One might think that the basic semantic distinction is between singular and plural, and that values like, e.g., paucal and dual are to be analyzed as restricted versions of plural. But one of the striking things about the cross-linguistic variety of grammatical number systems is that it is constrained: the sorts of number values that are present or absent in a given language are not independent of each other. Figure 1, below, summarizes the relevant dependencies, which were first noted by Greenberg 1966 and cataloged extensively by Corbett 2000.[15]

So, for instance, there are no languages have trial but not dual marking, or have *paucal* but not plural. One of the central phenomena to be explained by an empirical account of grammatical number is why these dependencies exist: why certain combinations of number markings are available but others are not. The most influential account that provides such an explanation is due to Harbour 2011, 2014. According to Harbour: (a) grammatical number is the head of a Number Phrase, (b) grammatical number comprises three optional and independent number features—$\pm$ Atomic, $\pm$ Minimal, and $\pm$ Additive and (c) Number Phrases are a

[12]There is also grammatical number in the verbal domain (see Corbett 2000 Chapter 8, Cabredo Hofherr and Doetjes 2021 Part III.). This doesn't describe verbal agreement with nominal number marking (as in, *the dog sits* v.s. *the dogs sit*) but morphological marking on verb phrases expressing the number of events characterized by the verb phrase.

[13]For an overview of this variation see Corbett 2000.

[14]This typological information is drawn from Harbour 2014. See the citations therein for details regarding the linguistic studies that are the source of this data.

[15]This table reproduces Table 1 in Harbour 2014.

NOTATION	IMPLICATION
TR → DU	Trial requires dual.
DU → SG	Dual requires singular.
SG → PL	Singular requires plural.
PL → SG/MIN	Plural requires singular or minimal.
U.AUG → AUG	Unit augmented requires augmented.
MIN → AUG/PL	Minimal requires augmented or plural.
AUG → MIN	Augmented requires minimal.
GR.PC → PC	Greater paucal requires (lesser) paucal.
PC → PL	Paucal requires plural.
GR.PL → PL/AUG	Greater (and global) plural requires plural or augmented.

Figure 1: From Harbour 2014

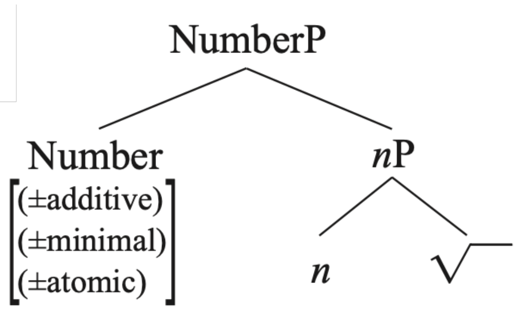


Figure 2: From Harbour 2014

recursive category. Figure 2 displays Harbour's account of the syntax of a Number Phrase. In this tree, $\surd$ indicates a lexical root and n is a category-forming head.[16]

I'll give a few examples to demonstrate the fruitfulness of this theory. (The semantic details of these explanations matter less than their general form.) Consider again the fact that trial marking requires dual marking. If these markers simply expressed additional restrictions on the denotation of expression that was fundamentally plural, there is no in-principle reason why a language could not mark trial without marking dual. On Harbour's account dual and trial both arise from the interaction of the features $\pm$ Atomic and $\pm$ Minimal, which he assigns the denotations below. In (6) $\sqsubset$ expresses *is a proper part of.*

(5) a. $[\![\text{Atomic+}]\!] = \lambda P.\lambda x.Px \wedge \text{atom}(x)$

b. $[\![\text{Atomic-}]\!] = \lambda P.\lambda x.Px \wedge \neg\text{atom}(x)$

(6) a. $[\![\text{Minimal+}]\!] = \lambda P.\lambda x.Px \wedge \neg\exists y(Py \wedge y \sqsubset x)$

b. $[\![\text{Minimal-}]\!] = \lambda P.\lambda x.Px \wedge \exists y(Py \wedge y \sqsubset x)$

With these interpretations, dual can be analyzed as +Minimal(-Atomic). This will take a property P and return all the Ps that are non-atoms and that are have no non-atomic Ps as parts, i.e. pairs of P atoms. Trial is +Minimal(-Minimal((-Atomic)). This will take a property

[16]Harbour adopts the standard assumption from Distributive Morphology that roots are themselves acategorical and that they are turned into a noun by combining with the category forming head n (see Harley 2014).

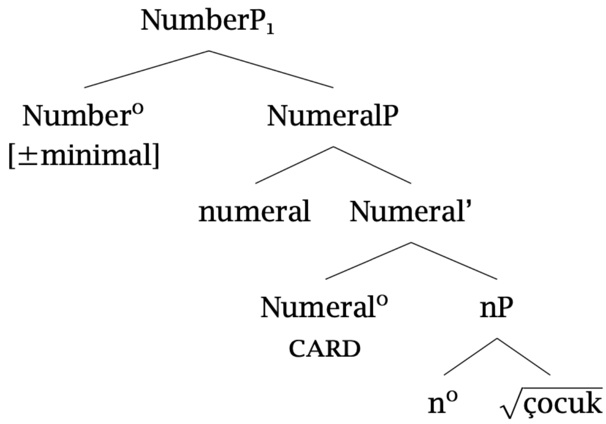


Figure 3: From Martí 2020

Ps and return all the non-atomic Ps that *have* non-atomic Ps as parts but do not have as parts any non-atomic Ps that themselves have non-atomic Ps as parts, i.e. triples of P atoms.[17]

This framework also provides an elegant explanation of the fact that in some languages that do have singular/plural marking, e.g. Turkish, numerals obligatorily combine with *singular* nouns.

(7) çocuk

boys

(8) çocuklar

boys

(9) iki çocuk/*lar

*two boy/*s*

"Lar" is typically described as a plural morphological marker in Turkish. But it is unacceptable in Turkish to use plural marking in any numeral + noun construction. One must say "iki çocuk" (two boy) rather than "iki çocuklar" (two boys).

Martí 2020 provides an explanation of this fact by proposing that the singular morpheme in languages like Turkish expresses the number feature +Minimal, and that Numeral scopes *under* Number in a Number Phrase, as in Figure 3.[18]

By the time the feature $\pm$ Minimal enters into the semantic composition of the noun phrase "iki çocuk," the root $\sqrt{\text{çocuk}}$ has already been modified by the numeral "iki". +Minimal(**two boy**) (iki çocuk) picks out the boy-pairs that have no boy-pairs as proper parts (i.e all of them). On the other hand, -Minimal(**two boy**) (*iki çocuklar) trivially applies to nothing, since nothing is a boy-pair that has boy-pairs as proper parts. And, on that basis, this construction

[17] Harbour's treatment of descriptive categories like *dual* as internally complex is prefigured by Hale 1973 and Silverstein 1986.

[18] This syntax follows Scontras 2014, 2022.

is filtered out by the grammar as unacceptable. The puzzling difference between the pattern of singular marking in Turkish as opposed to, e.g. English, is explained by the fact that, in Turkish, the singular marking expresses +Minimal, whereas in English it expresses +Atomic.

To summarize these points: the empirical study of grammatical number suggests that singular/plural marking in nominals emerges as one of several possible configurations of number values, and number values semantically operate on denotations provided by other parts of nominal expressions.[19] Both of these characteristics are absent from treatment of singular/plural distinction in plural logic. In plural logic, the distinction is primitive and exhaustive—terms are either singular or plural, and there's no deeper explanation of that fact. And it is a distinction among semantically *atomic* expressions: names and variables. But, as the discussion above illustrates, explanations regarding number marking in natural language often turn on the fact that grammatical number values take other parts of Number phrases as arguments. The idea that singular/plural marking corresponds to a fundamental dichotomy among atomic vehicles of reference has no real basis in the natural language phenomenon of grammatical number.

3.2 Counting and the Partitive Constraint

The Term Forming argument and the Pure $\preccurlyeq$ argument crucially depend on assumptions about the logical constant $\preccurlyeq$, which is meant to regiment "is one of," as in "Bruce is one of the dogs." Natural language phrases like "one of the dogs" are called cardinal partitive constructions since they feature the partitive preposition *of*. Partitive constructions in general are subject to what has been called "the Partitive Constraint": they are constrained in terms of what DP they can embed.

(10) a. I saw one of the dogs
b. #I saw one of most dogs.
c. #I saw one of the dog.
d. #I saw one of dogs.
e. #I saw one of both dogs.

The nature of this constraint has been a matter of debate since it was first discussed by Jackendoff 1977.[20] Significantly, the arguments we considered in Section Two crucially depend on premises that, when paraphrased into English, yield sentences that have been cited as *violations* of the Partitive Constraint. Consider (11) (12).

(11) # There is something that is one of Jon.

[19] The idea that number features interact compositionally with some other element of a nominal is elaborated in a particularly interesting way in Harbour's account, but it is common ground for linguists whether they adopt his framework or not. Any account needs to explain how $-s$ contributes to the meaning of "cows" by composing with "cow."

[20] Abbott 1996 has argued that many of the anomalies cited as "violations of the Partitive Constraint" should be given a pragmatic rather than semantic analysis, but the consensus seems to be that most the core examples do illustrate a semantic constraint (see e.g. de Hoop 1997, Ionin et al. 2006). In any case, Abbott's analysis of partitives still judges (11) and (13) below as ungrammatical for semantic reasons regarding countability.

(12) # Harris needs to win one of Nevada and Ohio.

(11) is an instantiation of Yi's Axiom 9, and (12) features the sort of *one of DP and DP* construction that is necessary for the statement of the Term-Forming Argument. Both are generally regarded as unacceptable.[21]

Of course, it is not news to plural logicians that some uses of $\preccurlyeq$ have awkward translations into English. They often acknowledge that it very odd to follow "is one of" with anything but a plural noun phrase; Oliver and Smiley 2016, in fact, prefer to paraphrase $\preccurlyeq$ as "is one of *or is identical with*."[22] But their attitude seems generally to be that the oddness attaching to sentences like (12) and (11) has to do with syntactic idiosyncrasies of English that can be harmlessly "rounded out" by plural logic.

In my view, however, the violations of the partitive constraint on display in (12) and (11) reflect important principles about grammatical countability. Cardinal partitives can only embed DPs where the nominal is countable. So, for instance, the mass noun *mud* is unacceptable in (13).

(13) # I slipped on one of the mud.

And there is reason to think that (12) and (11) fail for the same reason as (13): like *mud*, *Jon* and *Nebraska and Ohio* do not provide suitable units for counting. For instance, singular referring expressions like *Jon* and coordinated proper names like *Nebraska and Ohio* pattern with mass nouns in the way they interact with fractional partitives (de Hoop 1997, Hoeksema 1996).

(14) Half of the mud was wiped away.

(15) Half of Jon got wet.

(16) Half of Nebraska and Ohio was covered in snow.

The fractional partitive "half" in these cases indicates half of a relevant *quantity* associated with the DP—in this case, volume or surface. So for instance, (16) can be true in a case where half of the combined surface of Nebraska and Ohio is covered in snow, although neither Nebraska nor Ohio is covered in snow. Contrast this with a plural definite description like, "the kids."

[21] See Hoeksema 1996, de Hoop 1997 for discussion of data like (12). I have had some readers suggest that (12) is acceptable and is merely pragmatically awkward because it is equivalent to the simpler "Harris needs to win Nebraska or Ohio." This does not explain the fact that (12) tends to strike speakers as considerably less acceptable than "Harris needs to win one of Nebraska *or* Ohio." Searching the Corpus of Contemporary American English, one finds examples of the disjunctive construction but no corresponding examples for the conjunctive construction.

[22] It is worth pointing out that this gloss yields problems for plural logic too. If $\preccurlyeq$ is translated as "is one of or is identical with," then Axiom 11 seems to have counterexamples. For instance, suppose: Alice and Maya are the couple at table five. Then the couple at table five is either one of or is identical with Alice and Maya. But this does not imply, contrary to Axiom 11, that the couple at table five is one of or identical with Alice or one of or identical with Maya.

(17) Exactly half of the kids got wet.

"Exactly half" in (17) means exactly half cardinality-wise (Pearson 2011, de Vries 2021). "Kids" provides a unit by which to count, and "half the kids" applies to a subcollection of the kids that contains exactly half the number of *kids*. So (17) is false in a situation where there are eight kids who each got soaked to the waist, for instance. Even if the context demands it, plural definite descriptions resist a quantity-based interpretation of fractional partitives. If I have three kids, it will be infelicitious to say:

(18) Half of my kids got wet.

This cannot be interpreted as true in a situation where one kid was entirely dunked and two others waded to the knee. It demands a cardinality-based interpretation, which trivially false because there is no cardinal that is half of three.

This suggests the following generalization: if a noun-phrase is grammatically countable then it only admits of a cardinality-based interpretation when embedded in fractional partitives. Since "Jon" and "Nebraska and Ohio" admit of quantity-based interpretations with fractional partitives, this implies they *do not* express something countable. And if this is correct, we should not write off (10) and (11) as sentences that are semantically in good standing, though unacceptable for superficial reasons that are idiosyncratic to natural language. Rather, they seem to reflect structural facts about the semantics of counting.

4 Semantics for Counting Under a Concept

Thus far I have considered some empirical considerations that suggest that the syntax and semantics of singular/plural morphology in natural language is importantly different from what is found in standard plural logic. Now I will make a positive case that the semantics of natural language plurals is consistent the truth of One-Many claims. There are many extant proposals regarding the semantic that don't rule out the possibility that $[\![\text{one F}]\!] = [\![\text{three Gs}]\!]$. What is common to these proposals is that the units relevant for counting are *relative* to a noun phrase.

4.1 Boolean Background

The domain $\mathbf{B}$ for this semantics consists of a nonempty set that is ordered with respect to $\sqsubseteq$, the *part of* relation characterized by the axioms of Classical Extensional Mereology.[23] We can

[23]This theory has been characterized in a number of not entirely equivalent ways (see Hovda 2009, Varzi 2016). For concreteness, I'll be following Champollion 2017 in assuming the following axioms for for $\sqsubseteq$:

Reflexivity: $\forall x(x \sqsubseteq x)$

Transitivity: $\forall x \forall y \forall z((x \sqsubseteq y \wedge y \sqsubseteq z) \rightarrow x \sqsubseteq z)$

Antisymmetry: $\forall x \forall y((x \sqsubseteq y \wedge y \sqsubseteq x) \rightarrow x = y)$

Unique Unrestricted Sum: $\forall P(P \neq \varnothing \rightarrow \exists z(z = \sqcup P))$

define the notions of *overlap*, *sum*, *generalized sum*, and *binary sum* in terms of $\sqsubseteq$:

(19) **Overlap:** $x \circ y \coloneqq \exists z(z \sqsubseteq x \wedge z \sqsubseteq y)$

(20) **Sum:** $\mathrm{Sum}(x, P) \coloneqq \forall y(y \in P \to y \sqsubseteq x) \wedge \forall z(z \sqsubseteq x \to \exists z'(z' \in P \wedge z \circ z'))$
i.e a sum of a set P is an object that has every member of P as a part and whose parts all all overlap with a member of P

(21) **Generalized Sum:** $\sqcup P \coloneqq \iota x \ \mathrm{Sum}(x, P)$

(22) **Binary Sum:** $x \sqcup y \coloneqq \iota z \ \mathrm{Sum}(z, \{x, y\})$

Finally, I'll use Link 1983's * notation to represent closure under sum.

(23) $^*X \coloneqq \{x \mid \exists Y(Y \subseteq X \wedge x = \sqcup Y)\}$

The axioms of Classical Extensional Mereology guarantee that **B** is closed under $\sqcup$. The fact the metatheory of this semantics involves both mereology and set theory might raise some eyebrows in philosophical circles. But this metatheory shouldn't be invested with metaphysical significance. It's just a tool for illuminating the properties of natural language—the aim of this section is to show that there are empirically supported models of natural language according to which One-Many claims can be true. The sort of metatheory I'm appealing to here is a standard tool in the linguistics literature on plurals (see, e.g. Link 1983, Rothstein 2010, Champollion 2017).[24]

The semantics I develop below is modeled on Fred Landman's "Iceberg Semantics" (2016, 2020). Landman's account is is largely motivated to explain data related to the mass/count noun distinction. This is a motivation that his account shares with other theories (e.g. Rothstein 2010 and Krifka 1995, 1989) that analyze counting constructions in a "concept relative" manner. All of these theories suppose, in contrast to Link 1983, that count nouns and mass nouns are interpreted in a common domain. There are a number of empirical considerations that support this assumption. For instance:

- Within a mass/count language, there are often pairs of mass and count nouns that are apparently about the same things ("coins" vs. "change").
- There are things denoted by mass nouns in some languages that appear to be denoted by count nouns in others (English "hair" v.s. Italian "capelli").
- There are pervasive interactions between mass and count denotations. Mass nouns can combine with portion partitives to yield countable individuals ("six drops of water"). And, in suitable contexts, any count noun can be converted into a mass noun ("After my dog was through with it, there was bathrobe all over the living room.").

[24]The use of mereology is to some extent a convention (see Champollion 2017 p.17-19 for discussion). What really matters is that the ontology of one's model has the structure of a complete Boolean algebra with the bottom element removed; for that for purpose, one could frame one's metatheory in entirely set theoretic terms (or even plural logical terms.)

All of these suggest that grammatical countability should not be analyzed as an objective feature of the denotation of count nouns but as a perspective on that denotation provided by the language. One implementation of this idea is to suppose that count nouns are associated with two-level semantic values that determine an *extension* for the noun as well as a base-set that determines units for counting (Landman 2016, 2020; Sutton and Filip 2016; Sutton 2024). On such accounts, the distinction between mass and count nouns is due not to differences in their extensions but to properties of the base-set. And, as we will see, this leaves open the possibility that two count nouns can count the same extension in different ways.

On the account I propose, all nominals are interpreted in terms of two-level semantic values, represented as an ordered pairs, consisting of an *extension* and an *individuating concept*. I'll call such ordered pairs *tiered denotations*. The term "individuating concept" and its precise formal implementation in my semantics is novel, but it captures an idea familiar from a number of extant semantic theories. I'll first describe what these tiered denotations look like for root nouns, and then show how they are compositionally determined for Number Phrases (4.2), definite descriptions (4.3), and cardinal partitives (4.4).

Any root noun N is associated with a tiered-denotation with the following form, where X is the extension of N, and i is N's individuating concept.[25]

(24) $[\![\mathrm{N}]\!] = < X, i >$

For any N, the extension X is a subset of **B** that is closed under $\sqcup$. I think of an Individuating Concept as a mental representation that provides a rule for dividing an extension into parts. For my purposes, there is no harm in supposing that they are objects like any other, and that the set of Individuating Concepts is subset of **B**—though one could also adopt a typed domain with Individuating Concepts comprising a separate type.[26] What distinguishes Individuating Concepts is their semantic role. The first element of a noun phrase's denotation is its subject matter—what the noun phrase is *about*—whereas the second element is, in Landman's words (2020 p. 123), "perspective" on or "way of viewing" that subject matter. It is a perspective, potentially one among many, that divides that subject matter into units that are relevant to counting constructions. This idea has been given a number of different formal implementations by linguists working on grammatical countability—this is the role played by *base-sets* in Landman 2016, 2020; by *concepts* or *kinds* in Krifka 1989,1995; and as *counting contexts* in

[25]It would be more empirically accurate to suppose that lexical roots are themselves acategorical, and that they combine with the category-forming head n to form nPs, as in the trees above from Harbour 2014 and Martí 2020. This follows consensus in the literature on Distributive Morphology. But since I will not be concerned with any semantic or syntactic processes that occur below the nP level, it is a harmless simplification to treat roots as if they come directly with noun semantic values.

[26]Depending on what else other work wants Individuating Concepts to do, there might be pressure to separate Individuating Concepts from objects. For instance, in Krifka's 1989, 1995 account, *concepts* play a similar role to my Individuating Concepts, but Krifka also uses them as predicative entities that that can be mapped one-one to sets of objects. To avoid Russell's paradox, Krifka supposes that kinds are not in the domain of objects. But needless to say, there are a variety different of choices one can make to avoid the threat of paradox, e.g. rejecting the relevant comprehension principle for kinds. And in my semantics, Individuating Concepts are not utilized to do anything besides determine base-sets.

Rothstein 2010, 2017. In my framework, Individuating Concepts play this role.

An Individuating Concept, relative to an extension, can determine a *base-set.* I model this by means of a function—Base—which takes an ordered pair consisting of a subset of $\mathbf{B}$ and an Individuating Concept (i.e. a noun phrase semantic value) and yields a subset of $\mathbf{B}$. The Base function is constrained by conditions (a)-(d) below.

(25) Base($< X, i >$) $\subseteq \mathbf{B}$ if Base($< X, i >$) is defined then

a. Base($< X, i >$) $\subseteq X$

b. $X \subseteq^*$Base($< X, i >$)

c. $\sqcup X = \sqcup$Base($< X, i >$)

d. For any nonempty $Y \subseteq X$, Base($< Y, i >$) = Base($< X, i >$)$\cap \{x \,|\, \exists y \in Y(x \sqsubseteq y)\}$

I suppose that Base is defined on anything that is the semantic value of a root noun. So any N is associated with a tiered denotation $< X, i >$ such that X is generated by Base($< X, i >$ under $\sqcup$. For example, a tiered denotation for "tree" might look like (26)a., with the associated base given in (26)b.

(26) a. $[\![\text{tree}]\!]) =< \{a, b, c, a \sqcup b, a \sqcup c, a \sqcup c, a \sqcup b \sqcup c\}, \mathbf{tree} >$

b. Base($[\![\text{tree}]\!]$) $= \{a, b, c\}$, where $\forall x \in \{a, b, c\} \forall y \in \{a, b, c\}(\neg x \circ y)$

The base-set associated with a noun phrase determines various properties regarding its interaction with numerals and quantifiers, as well as its status as mass/count. I'll follow Landman (and others) in supposing that what is distinctive of count nouns is that they are associated with a base-set that is *disjoint.*[27] A set G is disjoint if and only if $\forall x \in G\, \forall y \in G(\neg x \circ y)$. The guiding idea is that something only admits of grammatical countability if there are clear criteria for what counts as *double-counting*, and this is violated by collections with overlapping members.[28] So, according to the disjointness analysis, what makes "tree" a count noun is the fact that is denotation has a base which consists of three non-overlapping objects.

By contrast, the denotations of mass nouns have base-sets that are non-disjoint. So for instance, (27) provides a mass interpretation for "mud." Suppose that $\{x_0, ..xi\}$ is the set containing all the pieces of mud that have no distinct pieces of mud as parts.

(27) a. $[\![\textit{mud}]\!]) =< {}^*\{x_0, ..., x_i\}, \mathbf{mud} >$

b. Base($[\![\textit{mud}]\!]$) $= {}^*\{x_0, ..., x_i\}$

The extension of *mud*, ${}^*\{x_0, ..., x_i\}$, is the set containing every quantity of mud. The indi-

[27]For accounts of count nouns based on disjointness, see Landman 2016, 2020; Rothstein 2010, 2017; de Vries and Tsoulas 2024; Sutton and Filip 2016.

[28]There are plausible alternates to disjointness as the defining condition on count noun base-sets. For instance, Krifka 1995 suggests that count nouns are associated with sets that are quantized—sets which have no member that is a proper part of another member—and Sutton 2024 considers a variety of conditions that are weaker than quantization. Nothing in my account hinges on the choice between disjointness, quantization, or the weaker conditions Sutton discusses.

viduating concept **mud** marks every thing in that extension as potentially counting as one with respect to **mud**, i.e. as a member of the base. Any sum of mud quantities can count as one relative to a suitable unit, i.e. "one *portion* of mud." Since there is more than one object in $\{x_0, ..., x_i\}$, $^*\{x_0, ..., x_i\}$ is guaranteed to be disjoint, and, therefore, grammatically uncountable.

I should pause to make a note about the relationship between Individuating Concepts and base-sets. As I've mentioned, in Landman's Iceberg semantics, the second level of a noun phrase denotation simply *is* the base-set—Landman doesn't posit individuating concepts as an intermediary. Individuating concepts are black boxes in the theory that I am sketching; what matters from a compositional perspective is that, relative to an extension, they determine a base. I treat the second element of a tiered denotation as a concept rather than a set of objects for a couple reasons. First, it does justice to the idea, which Landman also endorses, that second element is not a second subject matter but a *perspective* on the subject matter provided by the extension. Namely, it provides perspective by which that extension is divided into things that count as one. Second, ICs can be used to capture patterns that are not visible in the bases themselves. For instance, although for simplicity I've assumed that each root noun is associated with its own IC (e.g. *mud* $\rightarrow$ **mud**, *tree*$\rightarrow$ **tree**), it is not implausible to think that some classes of nouns are associated with a common IC. For instance, perhaps *tree* is associated with a principle of individuation common to all animate nouns, i.e. a principle that takes an extension and carves it into integrated metabolic wholes (**organism**). More generally, associating root nouns directly with base-sets forestalls a legitimate question—why, for some root nouns, are some but not all the objects within the extension singled out as things that can count as one? Individuating concepts, if treated as black boxes, obviously do not answer this question, but they mark the spot where an explanation should go.

Tiered denotations for complex NPs are determined compositionally in accord with Landman's *Head Principle*: NPs inherit the individuating concepts of their heads. So, for instance, the NP "green tree" will inherit **tree** as its individuating concept, since "tree" is its head. And given that the extension of "green tree" is a subset of the extension of "tree," condition (c) in (25) determines that the counting base for "green tree" is just the green objects among the counting base of "tree."

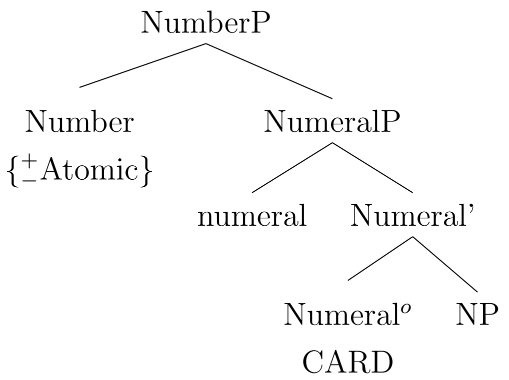
Since my concern in this section is just with the denotations of nominals, I will leave some compositional principles regarding the interpretation of whole sentences underspecified. It will suffice to say that I am thinking of tiered denotations like (26) and (27) as *predicative*: if a matrix predicate has this sort of denotation and it composes with a referring expression, the resulting expressions is true if and only if the referent of the latter is in the extension of the former. I'll refer to predicative tiered denotations as P-type denotations. In the sections below, I introduce semantic values that are functions on P-type semantic values, and I will represent these using λ abstracts binding the variables $P, P', P''....$ So for instance, $\lambda P.\text{Ext}(P)$ is the function that takes a P-type tiered denotation and yields its extension.

4.2 Numerals and Counting

Now I will lay out the semantics for numeral noun constructions. I interpret numerals as denoting numbers.

(28) a. $[\![\text{one}]\!] = 1$
b. $[\![\text{three}]\!] = 3$

I adopt the syntax for numeral-noun constructions given in (29), following Martí 2020 and Scontras 2022. According to this analysis, before NPs interact with numerals or the feature Number, they combine with the cardinality operator CARD to yield a function from numbers to P-type tiered denotations.



(29)

(30) $[\![\text{CARD}]\!] = \lambda P.\lambda n. < \text{Ext}(P) \cap \{x \mid \mu_{\#}(\text{Base}(P))x = n\}, \text{Ind}(P) >$

The semantics given in (30) adapts Martí 2020's semantics for CARD to a framework with tiered denotations.[29] In (30), $\mu_{\#}$ is a cardinality-counting function defined below:

(31) For any set Z, if Z is disjoint then, $\mu_{\#}(Z)$ is the function such that, for any x, $\mu_{\#}(Z)x = |\{y \mid y \sqsubseteq x \wedge y \in Z\}|$. If Z is not disjoint, then $\mu_{\#}$ is undefined.

$\mu_{\#}$ takes a set Z, which is presupposed to be disjoint, and yields a function that takes an object x and returns the cardinality of the set containing all the parts of x that are in Z. (This definition follows Landman 2020 pg. 144).

Finally, we can define semantic values for the grammatical number features that correspond to singular and plural morphology in English: +Atomic and -Atomic. These express functions from tiered denotations to tiered denotations.

(32) a. $[\![+\text{Atomic}]\!] = \lambda P. < \text{Base}(P), \text{Ind(P)} >$
b. $[\![-\text{Atomic}]\!] = \lambda P. < \text{Ext}(P), \text{Ind}(P) >$

The components are in place to derive tiered denotations for numeral constructions like "three trees" and "one copse." And there is no barrier to constructing a model where some object

[29] The idea that a cardinality operator like CARD is a component of numeral-noun constructions is widely accepted. See e.g. Krifka 1989, Hackl 2000, Scontras 2022, Martí 2020, 2024.

satisfies both of these expressions. Suppose "copse" has the tiered denotation given in (33), with the associated base in (34).

(33) $[\![\text{copse}]\!] =< \{a \sqcup b \sqcup c\}, \mathbf{copse} >$

(34) $\text{Base}([\![\text{copse}]\!]) = \{a \sqcup b \sqcup c\}$

Then the semantic values for "three trees" and "one copse" are as follows.

(35) $[\![+\text{Atomic}(\text{one}(\text{CARD}(\text{copse})))]\!] = [\![+\text{Atomic}]\!]([\![\text{one}]\!]([\![\text{CARD}]\!]([\![\text{copse}]\!])))$

a. $= [\![+\text{Atomic}]\!](< \text{Ext}([\![\text{copse}]\!]) \cap \{x \mid \mu_{\#}(\text{Base}([\![\text{copse}]\!]))x = 1\}, \mathbf{copse} >)$

b. $= [\![+\text{Atomic}]\!](< \{a \sqcup b \sqcup c\}, \mathbf{copse} >)$

c. $= < \text{Base}(< \{a \sqcup b \sqcup c\}, \mathbf{copse} >), \mathbf{copse} >$

d. $= < \{a \sqcup b \sqcup c\}, \mathbf{copse} >$

(36) $[\![-\text{Atomic}(\text{three}(\text{CARD}(\text{tree})))]\!] = [\![-\text{Atomic}]\!]([\![\text{three}]\!]([\![\text{CARD}]\!]([\![\text{tree}]\!])))$

a. $= [\![-\text{Atomic}]\!](< \text{Ext}([\![\text{tree}]\!]) \cap \{x \mid \mu_{\#}(\text{Base}([\![\text{tree}]\!]))x = 3\}, \mathbf{tree} >)$

b. $= [\![-\text{Atomic}]\!](< \{a \sqcup b \sqcup c\}, \mathbf{tree} >)$

c. $= < \text{Ext}(< \{a \sqcup b \sqcup c\}, \mathbf{tree} >), \mathbf{tree} >$

d. $= < \{a \sqcup b \sqcup c\}, \mathbf{tree} >$

One object is the extension of both "one copse" and "three trees:" $a \sqcup b \sqcup c$. So this model provides us with a example of how a One-Many claim could be true—$a \sqcup b \sqcup c$ can be correctly described as "one copse" and as "three trees." And given a standard lexical entry for "something," the model will make true the sentence "Something is one copse and three trees."

4.3 Definite Descriptions

Likewise, this semantics can be extended in a natural way to yield the result that "the one copse" and "the three trees" corefer. First, we introduce a lexical entry for "the."

(37) $$[\![\text{the}]\!] = \sigma = \lambda P. \begin{cases} < \sqcup\text{Ext}(P), \text{Ind(P)} > & \text{if } \sqcup \text{Ext}(P) \in \text{Ext}(P) \text{ and } \sqcup \text{Ext}(P) \notin \text{Base}(P) \\ < \sqcup\text{Ext}(P), \mathbf{object} > & \text{if } \sqcup \text{Ext}(P) \in \text{Ext}(P) \text{ and } \sqcup \text{Ext}(P) \in \text{Base}(P) \\ & \text{undefined otherwise} \end{cases}$$

The semantic value of "the" is a modified version of Sharvy's definiteness operator (1980). This function takes a P-type tiered denotation P and yields a tiered denotation whose extension is the supremum of $\text{Ext}(P)$. This introduces a new kind of tiered denotation that I'll call an x-type denotation: a tiered denotation where the extension is an individual rather than a set. Our original characterization of the Base function in (25) only applied to P-type denotations, so it needs to be extended in the following way:

(38) If $x \in X$, and $\text{Base}(< X, i >)$ is defined, then
$\text{Base}(< x, i >) = \{y \mid y \sqsubseteq x\} \cap \text{Base}(< X, i >)$.

σ is only defined on a semantic value P if the supremum of Ext(P) is a member of Ext(P). The second element of the denotations it yields depends on whether the supremum of Ext(P) is a member of Base(P). If not, then σ yields Ind(P); if so, then σ yields the *sui generis* IC **object** as the individuating concept of the resulting tiered denotation.

The introduction of **object** is novel to my account, and later on I will use it to explain the data from 3.2. We saw in 3.2 that definite singular noun phrases pattern with mass nouns in partitive constructions, which suggests that expressions like "John" or "the dog" are akin to "the mud" in the sense that they aren't associated with units for counting.[30] There are various ways of dividing the referent of "the dog" into parts but none determines semantically distinguished units. We can capture that in the present framework by supposing that **object** is an IC that puts no constraints on how an extension is to be divided up:

(39) For any object x that is not a set, Base($< x, \textbf{object} >$)$= \{y \mid y \sqsubseteq x\}$

This characterization of how **object** interacts with the Base function implies that, for any individual x, the base-set that **object** determines for x is the set of all x's parts. So if x has any proper parts, Base($< x, \textbf{object} >$) will be non-disjoint, like the base-sets for mass nouns. (I will return to a discussion of **object** in Section 5. It plays no role in showing, below, that "the three trees" and "the one copse" corefer.)

Given (37), we can compute the semantic values for "the three trees" and "the one copse," along with the associated base-sets.

(40)
a. $[\![\text{the three trees}]\!] = [\![\text{the}]\!]([\![-\text{Atomic(three(CARD(tree)))}]\!])$
b. $[\![\text{the}]\!]([\![-\text{Atomic(three(CARD(tree)))}]\!]) = \sigma(< \{a \sqcup b \sqcup c\}, \textbf{tree} >)$
c. $= < a \sqcup b \sqcup c, \textbf{tree} >$
d. $\text{Base}([\![\text{the three trees}]\!] = \{y \mid y \sqsubseteq a \sqcup b \sqcup c\} \cap \text{Base}([\![\text{tree}]\!]) = \{a, b, c\}$

(41)
a. $[\![\text{the one copse}]\!] = [\![\text{the}]\!]([\![+\text{Atomic(one(CARD(copse)))}]\!])$
b. $[\![\text{the}]\!]([\![+\text{Atomic(one(CARD(copse)))}]\!]) = \sigma(< \{a \sqcup b \sqcup c\}, \textbf{copse} >)$
c. $= < a \sqcup b \sqcup c, \textbf{object} >$
d. $\text{Base}([\![\text{the one copse}]\!]) = \{y \mid y \sqsubseteq a \sqcup b \sqcup c\}$

I understand the first element of the ordered pairs in (40)c. and (41)c. as the *referent* of the respective expressions; and so I regard "the one copse" and "the three trees" as coreferential in this model. The two expressions do not have the same semantic value—they are associated with two different individuating concepts, and, as (40)d. and (41)d. show, different base-sets. As we will see shortly, this means that the two expressions do not always have the same semantic-compositional import. But this doesn't undermine the suggestion that the two expressions *corefer*. The proposal on offer analyses NPs and DPs as having a two-level semantic value, and, as I suggested earlier, only the first level corresponds to the intuitive notion of reference

[30] I locate this in the determiner because singular morphology itself doesn't render a noun phrase uncountable. "The one dog" is acceptable, while "one of the dog" is not.

or "what an expression is about." This is born out by the fact that, it seems, we can truly say things like:

(42) The copse I referred to is those three trees.

So, just as in a Fregean two-level semantics that distinguishes sense and reference, we should allow that two expressions can be *coreferential* without having entirely the same semantic value.

This sketch shows, then, that there is a reasonable semantics for natural languages like English according to which singular and plural expressions can corefer and One-Many claims can be true. Besides my idiosyncratic treatment of individuating concepts, all the parts of this semantics are drawn from existing semantic theories with an independent empirical motivation. The possibility of true instances of One-Many claims is a straightforward consequence of the fact that numeral nouns constructions are sensitive to the base-set associated the noun-phrase they embed, in addition to its extension. In the terms of my framework, we can say that they are *IC sensitive.*

4.4 Cardinal Partitives

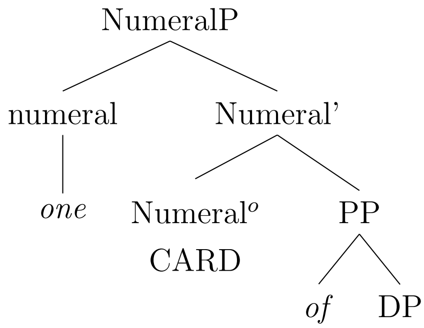
Since counting constructions in general are IC sensitive, it follows that *cardinal partitives*, the construction at the heart of the arguments of Section 2, should receive an IC sensitive analysis. "One of the tree" applies to something that is among the trees and counts as one. But counting as one is only intelligible, in the present framework, if we are provided with a suitable IC. This IC sensitivity gives rise to substitution failures inside cardinal partitives. Suppose we have a model with six trees and two copses, such that "the trees" and the "the copses" corefer. Clearly, (43) and (44) will not have the same truth conditions.

(43) One of the trees caught fire.

(44) One of the copses caught fire.

This difference falls out naturally from our semantic theory once we adopt provide a semantic value for "of." For the sake of simplicity, I'll assume that the syntax of "one of DP" is simply that given in (45).[31]

[31] It is a matter of ongoing debate whether the syntax of partitive constructions should be analyzed as containing one or two NPs. On the two NP analysis, "three of the dogs" actually has the structure "three *e* of the dogs," where *e* is a null NP, the content of which reconstructed from the DP argument to the partitive. Roughly, on this view, "three of the dogs" really has a structure like "three dogs of the dogs." For an overview of these two accounts, see Cardinaletti and Giusti 2017 p. 27-29. If the two NP account is correct, the semantics I give below needs to be complicated but not substantially revised.



(45)

(46) $[\![\text{of}]\!] = \lambda x. < \{x | x \sqsubseteq \text{Ext}(x) \wedge x \in {}^*\text{Base}(x)\}, \text{Ind}(x) >$

"Of" takes an x-type tiered-denotation, x, and returns a P-type tiered-denotation with an extension consisting of the set of all parts of Ext(x) that are in *Base(x) and Ind(x) as an individuating concept (c.f. Ionin et al. 2006, Barker 1998).[32] So, "of the trees" should have an extension that includes all the parts of the referent of "the trees" that are trees or pluralities of trees. Now we can give a model like the one below to represent a situation where we have six trees and two copses, and the copses just are the six trees.

(47) a. $[\![\text{tree}]\!]) =< {}^*\{a, b, c, d, e, f\}, \textbf{tree} >$
b. $\text{Base}([\![\text{tree}]\!]) = \{a, b, c, d, e, f\}$

(48) a. $[\![\text{copse}]\!] =< \{a \sqcup b \sqcup c, d \sqcup e \sqcup f, \sqcup\{a, b, c, d, e, f\}\}, \textbf{copse} >$
b. $\text{Base}([\![\text{copse}]\!]) = \{a \sqcup b \sqcup c, d \sqcup e \sqcup f\}$

(49) $[\![\text{the trees}]\!] =< \sqcup\{a, b, c, d, e, f\}, \textbf{tree} >$

(50) $[\![\text{the copses}]\!] =< \sqcup\{a, b, c, d, e, f\}, \textbf{copse} >$

According to (49) and (50), "the trees" and "the copse" have the same referent in this model: $\sqcup\{a, b, c, d, e, f\}$. But, since the two terms are associated with different ICs, they make different semantic contributions in cardinal partitives, as indicated in (51)-(54).

(51) a. $[\![\text{of the trees}]\!] = [\![\text{of}]\!](< \sqcup\{a, b, c, d, e, f\}, \textbf{tree} >)$
b. $= \lambda x. < \{x | x \sqsubset \text{Ext}(x) \wedge x \in {}^*\text{Base}(x)\}, \text{Ind}(x) > (< \sqcup\{a, b, c, d, e, f\}, \textbf{tree} >)$
c. $=< \{x | x \sqsubset \sqcup\{a, b, c, d, e, f\} \wedge x \in {}^*\{a, b, c, d, e, f\}\}, \textbf{tree} >$
d. $=< {}^*\{a, b, c, d, e, f\}, \textbf{tree} >$

(52) a. $[\![\text{of the copses}]\!] = [\![of]\!](< \sqcup\{a, b, c, d, e, f\}, \textbf{copse} >)$
b. $= \lambda x. < \{x | x \sqsubset \text{Ext}(x) \wedge x \in {}^*\text{Base}(x)\}, \text{Ind}(x) > (< \sqcup\{a, b, c, d, e, f\}, \textbf{copse} >)$
c. $=< \{x | x \sqsubset \sqcup\{a, b, c, d, e, f\} \wedge x \in {}^*\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\}\}, \textbf{copse} >$
d. $=< {}^*\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\}, \textbf{copse} >$

[32] This entry for "of" differs from Ionin et al. 2006 and Barker 1998 by (a) introducing ICs but also (b) by including the extra condition $x \in {}^*\text{Base}(x)$. This difference washes out, however, since these authors assume that the part-of relation involved, for count nouns, is individual-part rather than material-part (Link 1983). On their accounts as well as mine "of the trees" has a denotation that consists of trees and pluralities of trees. Barker proposes that "of" only returns *proper* parts of its argument, but Ionin et al. 2006 argue convincingly against this.

(53) a. $\llbracket \text{one of the trees} \rrbracket = \llbracket \text{one} \rrbracket(\llbracket \text{CARD} \rrbracket(\llbracket \text{of the trees} \rrbracket))$

b. $= \llbracket \text{one} \rrbracket \lambda n. < \{\text{Ext}(\llbracket \text{of the trees} \rrbracket) \cap \{x \mid \mu_{\#}(\text{Base}(\llbracket \text{of the trees} \rrbracket))x = n\},$
$\text{Ind}(\llbracket \text{of the trees} \rrbracket) >$

c. $= \llbracket \text{one} \rrbracket \lambda n. < \{ {}^*\{a, b, c, d, e, f\} \cap \{x \mid \mu_{\#}(\{a, b, c, d, e, f\})x = n\}\}, \textbf{tree} >$

d. $= < \{ {}^*\{a, b, c, d, e, f\} \cap \{x \mid \mu_{\#}(\{a, b, c, d, e, f\})x = 1\}\}, \textbf{tree} >$

e. $= < \{a, b, c, d, e, f\}, \textbf{tree} >$

(54) a. $\llbracket \text{one of the copses} \rrbracket = \llbracket \text{one} \rrbracket(\llbracket \text{CARD} \rrbracket(\llbracket \text{of the copses} \rrbracket))$

b. $= \llbracket \text{one} \rrbracket \lambda n. < \{\text{Ext}(\llbracket \text{of the copses} \rrbracket) \cap \{x \mid \mu_{\#}\text{Base}(\llbracket \text{of the copses} \rrbracket)x = n\},$
$\text{Ind}(\llbracket \text{of the copses} \rrbracket) >$

c. $\llbracket \text{one} \rrbracket \lambda n. < \{ {}^*\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\} \cap \{x \mid \mu_{\#}(\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\})x = n\}\}, \textbf{copse} >$

d. $=< \{ {}^*\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\} \cap \{x \mid \mu_{\#}(\{a \sqcup b \sqcup c, d \sqcup e \sqcup f\})x = 1\}\}, \textbf{copse} >$

e. $=< \{a \sqcup b \sqcup c, d \sqcup e \sqcup f\}, \textbf{copse} >$

So although "the copses" and "the trees" corefer, what counts as "one of the trees" does not count as "one of the copses."[33]

5 Responding to the Arguments

The analysis of cardinal partitives in the last sections provides a diagnosis for where the arguments of Section 2 go wrong. Both of those arguments involve applications of Leibniz's Law to expressions that are arguments to $\preccurlyeq$. This amounts to the assumption that if two expressions are coreferential, than they can be substituted *salva veritate* as arguments to $\preccurlyeq$. But, as we have just seen, this does not hold of the supposed English counterpart of $\preccurlyeq$, "is one of." Natural language paraphrases of this application of Leibniz's Law are not valid:

(57) a. The copse is these four trees.

b. The oak is one of these four trees.

c. $\nrightarrow$? The oak is one of the copse.

(58) a. The two copses are these ten trees.

[33] A more complete semantics would need to specify how other expressions interact compositionally with Individuating Concepts—in particular, expressions like distributive quantifiers and reciprocals. Consider for instance (55) and (56).

(55) a. The trees were each home to an owl.

b. #The copse was each home to an owl.

(56) a. The trees intertwined with each other.

b. #The copse intertwined with each other.

On the assumption that "the trees" and "the copse" are co referential, the fact that the a. elements of these pairs are well-formed whereas the b. elements are not calls for an IC sensitive semantic value for "each" and "each other." And this is not surprising—the function of "each" is to distribute down to things that count as one, and, in the current framework, counting as one is always an IC relative matter. I leave a detailed account of reciprocals to future work.

b. I chopped down one of the trees.

c. $\nrightarrow$ I chopped down one of the copses.

In natural language, cardinal partitive constructions are a non-extensional context since they are sensitive to the IC associated with the DP they embed. And, therefore, it isn't valid to apply Leibniz's Law within them. The plural logical treatment of $\preccurlyeq$ abstracts away from two important features of *one of DP*: first, that it is semantically complex and contains a counting construction, and, second, that counting constructions have to interact semantically with ICs.

It might be objected that, by analyzing cardinal partitives as non-extensional contexts, we introduce a complexity to our semantics that is unjustifiably *ad hoc*. This would be fair if the only point of the analysis was to clear the way for the truth of One-Many claims. But that isn't so. The account of Section 4 is a natural extension of extant empirically motivated treatments of the mass-count distinction. And it offers an explanation of some further empirical facts that, from the perspective of plural logic, look anomalous.

First, the semantics offered here allows us to account for the disanalogies between "is one of" and $\preccurlyeq$ discussed in Section 3.2: namely, (i) that singular DPs cannot be felicitously used in construction *one of DP*, (ii) that singular DPs pattern with mass DPs rather than plural DPs in fractional partitive constructions. Consider the tiered denotation of a singular definite description like "the dog:"

(59) a. $[\![\text{dog}]\!] =< \{j\}, \mathbf{dog} >$

b. $\text{Base}([\![\text{dog}]\!]) = \{j\}$

(60) a. $[\![\text{the dog}]\!] = [\![\text{the}]\!]([\![\text{+Atomic}]\!]([\![\text{dog}]\!]))$

b. $= < j, \mathbf{object} >$

object is the special IC that was introduced for singular definite descriptions in (37). Relative to an individual, **object** determines a base-set consisting of the all the parts of that individual. This is meant to the reflect the apparent fact that, when something is referred to in a singular way, it is not associated with a rule for decomposing it into privileged units. So, $\text{Base}([\![\text{the dog}]\!])$ $= \{x \mid x \sqsubseteq j\}$. And if j is supposed to be a dog, then, in any naturalistic model, $\{x \mid x \sqsubseteq j\}$ will not be disjoint. The set will include many things that are proper parts of j—j's nose, j's tail, etc. Consider, then, how we might compute a semantic value for "one of the dog."

(61) a. $[\![\text{of the dog}]\!] = [\![\text{of}]\!] < j, \mathbf{dog} >$

b. $= < \{x \mid x \sqsubseteq j\}, \mathbf{object} >$

(62) a. $[\![\text{one of the dog}]\!] = [\![\text{one}]\!]([\![\text{CARD}]\!]([\![\text{the dog}]\!]))$

b. $= [\![\text{one}]\!](\lambda P.\lambda n. < \text{Ext}(P) \cap \{x \mid \mu_{\#}(\text{Base}(P))x = n\}, \text{Ind}(P) >)([\![\text{of the dog}]\!])$

c. $=[\![\text{one}]\!]\lambda n. < \text{Ext}([\![\text{of the dog}]\!]) \cap \{x \mid \mu_{\#}(\text{Base}([\![\text{of the dog}]\!]))x = n\}, \text{Ind}([\![\text{of the dog}]\!]) >$

At (62)c. the computation crashes, because $\text{Base}([\![\text{of the dog}]\!])$ doesn't satisfy the presupposition of $\mu_{\#}$. $\mu_{\#}$ requires its first argument to be disjoint, but $\{x \mid x \sqsubseteq j\}$ is not. So "one of the dogs" is predicted to be infelicitous.[34] With some additional assumptions, this explanation can be extended to the other anomalous cases we considered in Section 3.2—proper names ("one of Jon") and conjoined definite singular DPs ("one of Nebraska and Montana"). For instance, if, as seems natural, singular proper names are directly associated with **object** as an IC, then the same explanation applies as above for "one of Jon."[35]

The **object** IC can also be used to explain the contrast in (63) and (64).

(63) a. The trees were burnt.
b. All of the trees were burnt.

(64) a. The dog was burnt.
b. All of the dog was burnt.

Any situation in which (63)a. is true is also one in which (63)b. is true (c.f. Križ 2015), and neither requires that every part of each tree was burnt. But (64)a. can be true in a situation in which only the dog's leg has been burnt, whereas (64)b. requires that every (relevant) part of the dog's body be burnt. This is a straightforward consequence of our analysis: in (63), "all" quantifies over $[\![\text{of the trees}]\!]$, a set containing pluralities of tree-atoms, whereas in (64), "all" quantifiers over $[\![\text{of the dog}]\!]$, which contains not just the single dog-atom but all of the dog's parts.

To account for data regarding fractional partitives we need to introduce denotation for a fractional expression. (66) provides a semantic value for "half" (c.f. Ionin, Matushansky, and Ruys 2006). $[\![\text{half}]\!]$ is a function that takes a P-type denotation P and yields a P-type denotation that has as an extension the set containing any object that is a member of a two membered set which (i) partitions $\sqcup\text{Ext}(P)$ and (ii) is such that all its members are equal with respect to some contextually determined measure function μ_C. I use Φ to represent the relevant partition relation, defined in (65) (c.f. Schwartzschild 1996, Gillon 1984).

(65) $\forall x \forall Y(\Phi(Y, x) \leftrightarrow (\sqcup Y = x \wedge \forall y \in Y\, \forall z \in Y(\neg y \circ z)))$

[34]If there are DPs that genuinely refer to mereological atoms, this explanation is consistent with those being felicitous in "one of DP" constructions. If such cases exist, they are extremely marginal, and I'm doubtful that our intuitions about them should be given much weight.

[35]To account for conjoined definite singular DPs we could give a denotation for *and* according to which a DP conjunction inherits the IC of both of its conjunctions if they are the same. I won't attempt to give a compositional interpretation for "and" here for two reasons. First, there is an ongoing debate about the semantic type of "and" between so-called Boolean and non-Boolean approaches (see Champollion 2016, Schmitt 2020) that I would prefer not to take a stand on. Second, as Rothstein 2017 points out, nominal conjunctions are constrained in some non-obvious ways in their interactions with numerals. The conjunction of two arbitrary countable NPs or DPs will not always yield an expression that is felicitously countable. For instance, "Get me three dogs and hats" is not easily interpreted as a command to fetch three things that are either dogs or hats. But "There are twenty men and women waiting outside" easily receives a twenty people interpretation, i.e. twenty things that are men or women. A compositional semantics for nominal conjunctions should address this difference, which seems to very likely to involve pragmatic factors. I leave this to future work. I agree with Rothstein (2017 p.107) that these issues tend to confirm the hypothesis that counting is a concept-relative matter, rather than raise trouble for it.

(66) $[\![\text{half}]\!] = \lambda P. < \{x \mid \exists Y(\Phi(Y, \sqcup\text{Ext}(P)) \wedge |Y| = 2 \wedge x \in Y$
$\wedge \forall y \forall z((y \in Y \wedge z \in Y) \rightarrow \mu_C(y) = \mu_C(z)))\}, \text{Ind(P)} >$

It is typical to assume that context plays a role in determining which measure function is relevant to the interpretation of fractional and quantifiers—in one context "half of the mud" might mean half by volume, whereas in another it might mean half by mass. On the hypothesis that when interpreting "half", given some input P, the cardinality-based measure function $\mu_{\#}(\text{Base}(P))$ is always selected as the contextually most salient measure function if it is available, then we will predict the sort of pattern we observed in Section 3.2. Namely, plural definite descriptions always are associated with a cardinality-based interpretation of fractional partitives, whereas mass nouns and singular definite descriptions are not.

(67) $[\![\text{half of the trees}]\!] =< \{x \mid \exists Y(\Phi(Y, \text{Ext}[\![\text{of the trees}]\!] \wedge |Y| = 2 \wedge x \in Y$
$\wedge \forall y \forall z((y \in Y \wedge z \in Y) \rightarrow \mu_{\#}(\text{Base}([\![\text{of the trees}]\!]))y = \mu_{\#}(\text{Base}([\![\text{of the trees}]\!]))z))\}, \textbf{tree} >$

(68) $[\![\text{half of the dog}]\!] =< \{x \mid \exists Y(\Phi(Y, \text{Ext}[\![\text{of the dog}]\!] \wedge |Y| = 2 \wedge x \in Y$
$\wedge \forall y \forall z((y \in Y \wedge z \in Y) \rightarrow \mu_C y = \mu_C z)))\}, \textbf{object} >$

In (67), $\mu_{\#}\text{Base}([\![\text{of the trees}]\!])$ is defined. Since it is always the contextually most salient measure function if defined, $\mu_{\#}\text{Base}([\![\text{of the trees}]\!]) = \mu_C$. So "half of the trees" receives a cardinality-based interpretation, with units given by the base of $[\![\text{tree}]\!]$. But, since $\mu_{\#}\text{Base}([\![\text{of the dog}]\!])$ is undefined, the contextually determined measure function in (68) has to be something else—e.g. a volume or surface area based measure function.

Even if we bracket the truth of One-Many claims, we arguably can observe the non-extensional character of cardinal partitives in other natural language examples. For instance, it appears that that some DPs which violate the partitive constraint corefer with DPs that do not violate the partitive constraint.

(69) a. ? One of Femi and Franny left a footprint in my kitchen.
b. Femi and Franny$_i$ used my kitchen. One of them$_i$ left a footprint.

(70) a. ? Two of my family ordered soup at the lunch counter.
b. My family$_i$ sat at the lunch counter, and two of them$_i$ ordered soup.

(69-a) and (70-a) are infelicitous, and this infelicity has been analyzed as a violation of the partitive constraint: the DPs in (69-a) and (70-a) do not provide suitable arguments for *of.* But if we replace those DPs with plural anaphoric pronouns, as in (69-b) and (70-b), the infelicity goes away. If we suppose that the anaphoric pronouns are coreferential with their antecedents, then partitive constructions must be sensitive to something *further* than the referent of the DP they embeds. Treating partitive constructions as IC sensitive suggests a possible explanation.[36]

[36] Another example of the non-extensional character of cardinal partitives comes from free relatives. In the following example "the books" and "what I bought" plausibly corefer.

(71) a. I bought some books. One of the books was French.

5.1 Comparison

There is a voluminous literature of responses to the arguments from Sections 2, typically developed as part of a defense of CAI. Rough analogues of the analysis I've given of where those arguments go wrong and the semantics I have developed for counting constructions have both been endorsed, in different forms, by other authors. For example, other authors have proposed:

- that Yi's Axioms 10 should be denied (Payton 2019, Loss 2019)
- that that "is one of" introduces an opaque context (Cotnoir 2008, Bohn 2014)
- that counting constructions should be analyzed in a concept-relative or perspective-relative fashion (Baxter 1988; Cotnoir 2008; Wallace 2011b, 2023 pg.56-60; Bohn 2014; Nicolas and Payton 2025)
- that plural reference and singular reference can offer distinct modes-of-presentation of the same subject matter (Cotnoir 2008; Wallace 2023 pg. 57; Bohn 2014.)

However, my account differs significantly from the works above in methodology and motivation. I have been concerned to defend to the truth of One-Many claims *in natural languages* and my semantics is intended as an empirical hypothesis. The defenses of CAI that I've referred to above tend to be proposals made *within* the framework of plural logic for emending the its standard semantics to allow for the truth of One-Many claims. This orientation means that these projects generally adopt the background assumptions about the nature of the singular/plural distinction that, I argued in Section 3, are empirically misguided. To illustrate this, I'll compare my account to Cotnoir 2008 and to the recent proposals made by Jonathan D. Payton (2019, 2021, 2022), which I take to be representative of this tendency.

Payton 2019 proposes an emendation to the standard semantics of plural logic according to which there is no prohibition on singular terms/variables referring to the same thing(s) as plural terms(s) variables. The core of Payton's response to the arguments of Section 2 is to reject Yi's Axiom 10: $\forall x \forall y(x \preccurlyeq y \rightarrow x = y)$. As he points out this effectively stipulates that no properly plural term can corefer with any singular term.[37] But Payton suggests there is nothing incoherent about the logic that results from dropping Axiom 10. Without this axiom, the Term-Forming Argument and the Pure-$\preccurlyeq$ Argument are no longer valid.

However, although Payton does block the arguments of Section 2, he preserves the aspects of plural logic that, I suggested, make it objectionable as a regimentation of natural language

b. I bought some books. #One of what I bought was French.

(Cotnoir 2008 discusses data from free relatives to make a different point. He suggests that "what I bought" is "semantically plural but syntactically singular.")

[37]Again, on the standard understanding of plural logic, a *properly* plural term refers to some things such that, for distinct x and y, x is one of them and y is one of them, and Axiom 10 guarantees that no singular term refers to such things.

grammatical number. First, Payton accepts that singular/plural marking corresponds to a difference between two sorts of semantically primitive terms. His emendation is to propose that these categories are not mutually exclusive: terms can be both singular and plural. In his account, he preserves the formally distinguished classes of terms/variables of standard plural logic (i.e. a, x vs. aa, xx), but gives an analysis of *singular* and *plural* as semantic rather than syntactic categories.[38] According to that analysis:

(72) *Definition of Singular and Plural Terms* (Payton 2019)[39]

a. A term α is singular on an assignment σ iff $\ulcorner\exists x((\text{ ``}\alpha\text{'' denotes } x \text{ on } \sigma) \wedge \forall xx(\text{``}\alpha\text{'' denotes } xx \text{ on } \sigma \rightarrow xx \preccurlyeq x))\urcorner$ is true

b. A term α is plural on an assignment σ iff $\ulcorner\exists x\exists y(\text{ ``}\alpha\text{'' denotes } x \oplus y \text{ on } \sigma \wedge x \neq y)\urcorner$ is true.

Since a term can meet both these conditions, it is possible for a term to be both semantically singular and semantically plural. This approach departs from the natural language phenomenon in a new way. In natural languages, singular/plural morphology in nominals corresponds to the alternative values of the features in Number. In a language like English, for instance, I've followed Harbour 2011, 2014 and Martí 2020, in supposing that the singular/plural distinction corresponds to the values +Atomic/−Atomic. But crucially: the values +Atomic and −Atomic *are* mutually exclusive. This crucial because we use the fact that these features are incompatible to explain the distributional differences between singular and plural nominals in English. For instance, *two dogs* is acceptable while **two dog* is not because $[\![$+Atomic(two(CARD(dog)$]\!]$ is guaranteed to have an empty extension, unlike $[\![$−Atomic(two(CARD(dog)$]\!]$. Payton's understanding of the singular/plural distinction—according to which a referring expression can be *both* singular and plural—is not suitable for describing the semantic import of singular/plural marking for a language like English.

Furthermore, dropping Axioms 10 still leaves us with an unfaithful regimentation of "is one of." Payton 2019 maintains that $\preccurlyeq$ is reflexive, and consequently that $\forall x\exists y(y \preccurlyeq x)$ is a logical truth. But this is intended as a regimentation of "Everything is such that something is one of it" which is unacceptable in English. And nothing is added to capture the IC sensitive character

[38]As Trueman and Thunder 2025 point out, since every syntactic environment is neutral between the formally distinguished of terms/variables, Payton's language is only "superficially" two-sorted. This is not entirely innocent because Payton's presentation of his view depends on associating variables like x and xx with different semantic roles. In his telling, $\exists xFx$ expresses that an individual is F, whereas $\exists xxFxx$ does not, and the latter does not imply the former. It seems to me that Payton's 2019 view is more perspicuously expressed by the formalism in Trueman and Thunder 2025, where the terms are one-sorted and there is a primitive predicate I that expresses *being an individual.*

[39]In this definition Payton intends "denotes" to be read collectively—so to say that α denotes $x \oplus y$ relative to σ does not imply that α denotes x or y relative to σ. Payton also provides an alternative definition of plural terms that works when "denotes" is read distributively. Though this is tangential to my main points, I would also note that it is odd that, according to Patyon's definitions, the distinction between singular and plural terms in the object language rests on the *truth* of object language sentences containing terms referring to linguistic expressions and predicates expressing *denotation relative to an assignment.* Presumably, a language need not have these sorts of sophisticated metalinguistic resources in order to have a distinction between singular and plural expressions.

of the "is one of" construction. Although Payton's semantics allows him to provide a true regimentation of identity statements like "The copses are those trees," there is nothing in the logic that blocks the regimentation of "a is one of the trees" from implying the regimentation of "a is one of the copses."[40]

Other authors defending CAI do attempt to explain the substitution failures inside counting constructions and cardinal partitives. Cotnoir 2008, for instance, introduces *covers* to the semantics of plural logic and suggests that $\preccurlyeq$ should be treated as cover-sensitive. A cover of a set A is a set P of nonempty sets $p_1, ..., p_n$ such that $\bigcup p_1, ..., p_n = A$, i.e. a partion whose cells may overlap.[41] Linguists like Gillon 1987, 1990 and Schwartzschild 1996 have introduced covers to explain so-called non-atomic distributive readings. For example, consider (73) uttered in the context of a shop containing 40 pairs of shoes:

(73) The shoes cost fifty dollars. (Lasersohn 1998)

(73) is naturally interpreted as predicating *costs fifty dollars* not to the total collection of shoes or to each of the shoes individually, but rather to each *matched pair* of shoes. It is a distributive reading, but the units of distribution are not individual shoes. To explain this, Gillon and Schwartzschild suppose that the verb phrase is predicated of the shoes *with respect to* some way of dividing up those shoes provided by the context—a cover. So, roughly, context provides a cover c of $[\![$the shoes$]\!]$ that divides the 80 shoes into matching pairs, and (73) is interpreted as: every member of c costs fifty dollars.

Cotnoir proposes that "is one of" is cover-sensitive and that this creates an opaque context for substitution of identicals. Suppose, again, that we have some trees and some copses, and that the copses just are the trees. Cotnoir suggests that we regard "This tree is one of the trees" as true and "This tree is one of the copses" as false because we interpret the former with respect to a cover i that divides down the level of individual trees, whereas we interpret the latter with respect to a distinct cover j that only divides down to copses. But, this account leaves us with a number of explanatory holes. For one, it implies that "The tree is one of the copses" is *true* relative to a cover j. But this sentence appears to be simply false, relative to whatever context. So we are owed some explanation why certain covers are available for certain sentence types but blocked for others.[42] Secondly, the proposal runs the risk of being

[40] c.f. Payton 2021. In the semantics provided in this paper for definite descriptions, in a case where the bricks in the wall are the atoms in the wall and a is a brick in the wall, Payton's regiments "a is one of the atoms in the wall" as true.

[41] This is to define covers in set theoretic terms, which is the sort of metatheory Cotnoir 2008 adopts. In mereological terms, a cover of an object x is a set P such that $\bigsqcup P = x$. For a novel characterization of covers in a plural logical context, see Payton 2021, 2025 and Nicolas and Payton 2025.

[42] Nicolas and Payton 2025 go some way towards offering such a principled answer to this question. They define conditions under which a cover counts as *fitting* with respect to a noun phrase, and propose that sentences must always be interpreted with respect to covers that are fitting with respect to the noun-phrases they contain. Payton and Nicolas's semantics allows for true One-Many claims and looks like a fruitful alternative to the sort of semantics I sketched in Section 4. I don't discuss it in further depth here because it is technically quite complex, and, as formulated, it has the unwanted consequence that there is no cover i such that "Anna and Beth are a pair of people" is true with respect to i and i is a fitting cover for "pair of people." So I am doubtful

ad hoc, because, although covers do have an empirical motivation, it isn't obvious that there is an independent empirical basis for thinking that *cardinal partitives* are cover-sensitive. The semantics I develop in Section 4 addresses both of these short-comings. In that account, base-sets roughly do the work that Cotnoir wants covers to do, but base-sets are associated with noun-phrases as part of the grammar and they have an independent basis in theories of the mass/count distinction.

To summarize: although each of the components of my defense of One-Many claims have been defended by other contributors to the debate on CAI, I hope to have supplied a new basis for believing those components. Previous authors have tended to adopt plural logic as a regimenting language and to suggest more-or-less piecemeal adjustments to its standard semantics to address objections to CAI. I've tried to show that many of the things that defenders of CAI have wanted to say—that singular and plural referring expressions can corefer, that counting is concept-relative, that "is one of" is not adequately modeled by $\preccurlyeq$—are plausible empirical hypotheses that can be given a well-motivated compositional semantics and fit into a unified picture of grammatical number.

6 Import for Logic and Metaphysics

I have argued that there is a plausible semantics for natural languages with a singular/plural distinction according to which singular referring expressions can corefer with proper plural referring expressions and there can be true One-Many claims. And, therefore, whatever motivation there is in standard plural logic for is for restricting such coreference and judging those claims to be logically false, it does not come from our best empirical accounts of grammatical number. In natural language, that something is referred to in a singular/plural way need not imply anything of deep metaphysical significance about what is referred to.

This point is, in my view, interesting in its own right as a claim about natural language, but one might wonder: what is the upshot for the logician or metaphysician? The main lesson is that plural logic is not a faithful regimentation of natural language arguments involving grammatical number marking. On the standard scheme of regimentation, there are valid arguments in plural logic that are translated into invalid or ungrammatical arguments in natural language, and plural logical falsehoods that may have true natural language translations. Plural logicians often present their work precisely as an attempt to regiment the logical behavior of natural language arguments involving plural marking (Yi 2005; Florio and Linnebo 2021 p. 29; Oliver and Smiley 2016 Chapter 1). So, in my view, they fail on this score. And as a corollary: the fact that One-Many claims are logically false when naively translated into Standard Plural Logic is not a decisive refutation of those claims—this scheme of translation distorts the logic of natural language plurals.

that fitting covers, as defined in Nicolas and Payton 2025, offer an empirically adequate way of constraining available covers.

This lesson has broader consequences for the way plural logic is motivated and framed. Plural logicians sometimes suggest that formal languages that regiment natural language plural discourse using, e.g., singular terms for sets or sums are guilty of "changing the subject," whereas the true subject is preserved by the resources of standard plural logic (Oliver and Smiley 2016 Chapter 3).[43] This strikes me as mistaken on two counts. First, again, the plural variables and terms of standard plural logic do not appear to be syntactically or semantically faithful to anything in natural language. And second, the suggestion that something referred to plurally can *only* be referred to plurally looks parochial in the full natural language context. *Prima facie*, we do not change the subject when we refer to some coins, using plural marking, as *the coins* v.s. when we refer to them, without plural marking, as *the change*; nor does the Mandarin speaker change the subject when they translate *the coins* into an expression that lacks number morphology entirely.

I should stress that my argument leaves many questions open regarding its import for plural logic. In particular: does it provide reasons for thinking that standard plural logic does not succeed *as a logic*? I have said a number of times that the distinction between plural and singular terms that one finds in standard plural logic is an *invention*, not something found in natural language. But to say it is an invention is not to say that it is a mistake. More generally, does the argument suggest that standard plural logic is not the right logic to use when we are doing metaphysics and thinking about ontology?

I do not propose to answer those questions here, but my argument does provide some guidance for approaching those them. For example, some proponents of plural logic have presented it an alternative to other formal systems like set theory and second-order logic with respect to, e.g., projects in the foundations of mathematics. In this context, it is sometimes suggested that plural logic is to be preferred, since, unlike set theory and second-order logic, plural logic merely regiments expressive resources that are already present in natural language. But this, I've argued, is not so. Plural logic is not obviously any superior to set theory or second-order logic as regimentation of natural language, so this suggestion cuts no dialectical ice. It's is also common for proponents of plural logic to offer general arguments that plural reference is *ineleminable* in favor of singular reference (e.g. paradox of plurality arguments).[44] But these arguments invariably turn precisely on the sort of naive importation of plural logical principles to natural language constructions involving "is one of" that I've argued are fallacious.

Before closing, I want to respond to one further objection that might seem to undercut the metaphysical significance of my argument. In the semantics that I sketched in Section 3, the distinction between "the copse" and "the tree" was characterized as a mode-of-presentation difference—"the copse" presents the trees as one under the counting concept **copse**. That might suggest that One-Many claims have no place in serious ontology. When when we make true

[43] Yi's (2005) discussion of Link 1983 as an attempt to provide a "reductive" account of plurals in a singular idiom suggests a similar perspective.

[44] For presentations of these arguments, see Boolos 1984, Schein 1993 p. 23, Florio and Linnebo 2021 p.38, Oliver and Smiley 2016 p.40

One-Many statements in natural language, the thought might go, we are simply talking about what is *really* many things in a singular way. The trees are a feature of the mind-independent reality, and "the copse" is just a way we have of talking about those trees, associated with a distinct counting concept. On this understanding, Composition as Identity in combination with Classical Mereology might look, at best, like a trivial addition to an already complete ontology. I think this complaint has a dubious presupposition: that if there is a plural way of viewing some subject matter ("the trees") as well as a singular way ("the copse"), the plural way always reflects the more fundamental perspective. This might well be the case with "mere aggregrates" like heaps of stones—where each individual stone is more fundamental than the heap. But it is less obviously true of other more interesting examples. If there are cases where the whole is prior to any of its individual parts—maybe, the whole of space-time v.s. particular spacetime events (c.f. Schaffer 2010)—then it would seem that the singular perspective on the whole would be metaphysically more perspicuous that then the plural perspective.

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