Estimating thrust of a water rocket – hussain qasem – kuwait suborbital rockets

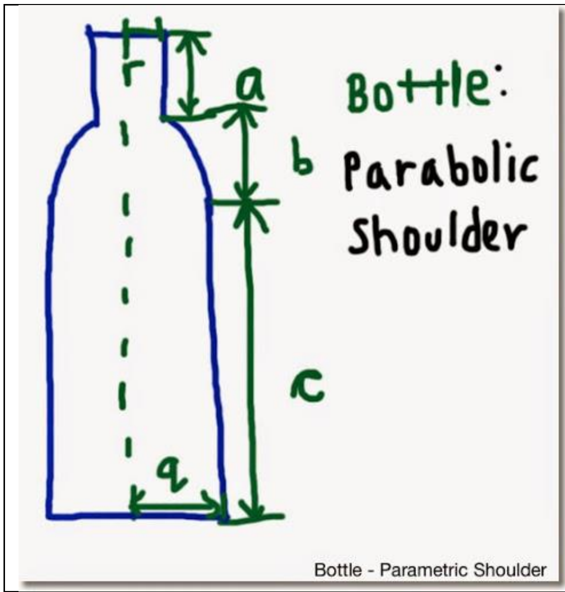
Conservation of mass for deforming control volume

$$\frac{\partial}{\partial t}\int \rho \, dV + \int \rho \, \boldsymbol{W} * \boldsymbol{n} \, dA = 0 \quad (1)$$

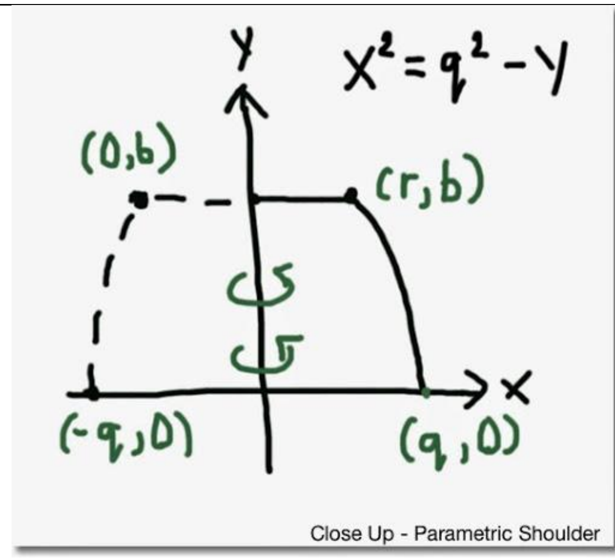
Density is constant

$$\rho \frac{\partial}{\partial t} \int dV + \dot{m}_e = 0 \quad (2)$$

The volume of the water bottle can be expressed as the sum of 3 parts: body, shoulder, and neck. The volume estimation was given on this website:
http://edspi31415.blogspot.com/2014/07/volume-of-bottle-approximate.html



Bottle - Parametric Shoulder



Close Up - Parametric Shoulder

$$V_{neck} = \pi r^2 a \quad (3)$$
$$V_{body} = \pi q^2 c \quad (4)$$
$$V_{shoulder} = \pi \left(b q^2 - \frac{b^3}{3} \right) \quad (5)$$

Thus, the integration in Eq. (2) becomes:

$$\int dV = \pi r^2 a + \pi q^2 c + \pi \left(b q^2 - \frac{b^3}{3} \right) \quad (6)$$

Simplifying we get:

$$\int dV = \frac{\pi}{3}(3ar^2 - b^3 + 3bq^2 + 3cq^2) \quad (7)$$

Plugging Eq (7) into (2)

$$\rho \frac{\partial}{\partial t} \left[\frac{\pi}{3}(3ar^2 - b^3 + 3bq^2 + 3cq^2)\right] + \dot{m}_e = 0 \quad (8)$$

During discharge, a, b, and c are changing with respected to time. Everything else is constant. Equation 8 becomes:

$$\frac{\rho\pi}{3}\left[3r^2 \frac{\partial}{\partial t}(a) - \frac{\partial}{\partial t}(b^3) + 3q^2 \frac{\partial}{\partial t}(b) + 3q^2 \frac{\partial}{\partial t}(c)\right] + \dot{m}_e = 0 \; (9)$$

$$\frac{\partial}{\partial t}(a) = v_a \quad (10)$$

$$\frac{\partial}{\partial t}\ (b) = v_b \quad (11)$$

$$\frac{\partial}{\partial t}\ (c) = v_c \quad (12)$$

In Eq(9), the term $\frac{\partial}{\partial t}\ (b^3)$ is differentiated implicitly. Eq (9) becomes

$$\frac{\rho\pi}{3}[3r^2 v_a - 3b^2 v_b + 3q^2 v_b + 3q^2 v_c] + \dot{m}_e = 0 \quad (13)$$

Rearranging Eq (13)

$$\boxed{\frac{-\rho\pi}{3}[3r^2 v_a + (-3b^2 + 3q^2) v_b + 3q^2 v_c] = \dot{m}_e \quad (14)}$$

Simplifying math further, we can assume that $v_a \gg v_b \gg v_c$, thus equation 14 becomes

$$\dot{m}_e = -\rho\pi r^2 v_a \quad (15)$$

Assuming adiabatic expansion:

$$p = p_o\left(\frac{V_o}{V}\right)^k \quad (16)$$

$$p = \frac{1}{2}\rho v_a^2 \quad (17)$$

Using Eq (16) and (17)

$$p_o\left(\frac{V_o}{V}\right)^k = \frac{1}{2}\rho v_a^2 \rightarrow v_a = \sqrt{\frac{2}{\rho} p_o \left(\frac{V_o}{V}\right)^k} \; (18)$$

Plugging Eq (18) into (15) we get mass flow in terms of current volume of air V

$$\dot{m}_e = -\pi r^2 \sqrt{2\rho p_o \left(\frac{V_o}{V}\right)^k} \quad (19)$$

$$T = \frac{\dot{m}_e^2}{\rho A_e} = \frac{2\pi^2 r^4 \rho p_o \left(\frac{V_o}{V}\right)^k}{\rho \pi r^2} = 2A_e p_o \left(\frac{V_o}{V}\right)^k \quad (20)$$

Taking A = 1, p = 1, Vo = 1, and k = 1.4, T and mdot vs V looks like