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Circle chord theorems pdf

The supposed knowledge of the introductory geometry of the plane involving points and lines, parallel lines and transverse, angular amounts of triangles and quadrilateral, and the overall angle of the chase. The experience of a logical argument in geometry, written as a sequence of steps, each justified for a reason. Ruler-and-compass designs. Four standard congruence tests and their application to prove properties and tests for special triangles and quadrilateral. Four standard similarities tests and their application. Trigonometry with triangles. Motivation Most geometry still includes triangles and four-sided, which are formed by intervals on the lines, and now we turn to the geometry of the circles. Lines and circles are the most elementary shapes of geometry - the line is the locus of the point moving in a constant direction, and the circle is the locus of the point, moving at a constant distance from a fixed point, and all our designs are made by drawing lines with a straight edge and circles with compasses. This module introduces tangents, and later tangents become the basis of differentiation in calculus. Circle geometry theorems are not intuitively obvious to the student, in fact most people are quite surprised by the results when they first see them. They clearly need to be proven carefully, and the mind evidence techniques developed in previous modules are clearly displayed in this module. Logic becomes more and more involved - often it takes to divide into cases, and results from different parts of previous geometry modules are often combined within a single piece of evidence. Students traditionally learn more respect and evaluation of mathematics techniques from their study of this creative geometric material. The theoretical importance of circles is reflected in the amazing number and diversity of situations in science, where circles are used to simulate physical phenomena. Circles are the first approximation to the orbits of planets and their satellites, to the movement of electrons in an atom, to the movement of the vehicle around the curve on the road, to the forms of cyclones and galaxies. Spheres and cylinders are the first approximation of the shape of planets and stars, tree trunks, exploding fireballs and water droplets, as well as manufactured objects such as wires, pipes, ball carriers, balloons, pies and wheels. Content Radius and Chords We begin by repeating the definition of the circle and terminology used for circles. All geometry is supposed to be within a fixed plane throughout this module. A circle is a set of all the points in the plane that are at a fixed distance (radius) from a fixed point (in the center). Any interval of attaching a point on a circle to a center is called a radius. By the definition of a circle, any two radii have the same length. Note that the word radius is used to indicate both these intervals and the total length of these intervals. The interval of joining two points on a circle is called a chord. The chord that passes through the center is called diameter. Because the diameter consists of two radii connected to the endpoints, each diameter has a length equal to twice the radius. The word diameter is used to refer to both these intervals and their total length. The line that cuts the circle at two different points is called a sekant. Thus, a chord is an interval through which a circle cuts off the secant, and the diameter is an interval cut off by a secant passing through the center of the circle. The symmetry of the circles have an abundance of symmetries: the circle has all possible symmetries of rotation about its center, in that each rotation of the circle around its center rotates the circle on itself. If the AOB is the diameter of the circle with the center of O, the reflection in the AOB line reflects the circle on itself. Thus, each diameter of the circle is an axis of symmetry. As a result of these symmetries, any P point on the circle can be moved to any other point on the circle. This can be done by spinning through the corner θ the PISE about the center. This can also be done by reflecting in diameter the AOB separating the PO. Thus, each point on the circle is essentially the same as any other point on the circle - no other figure in the plane has this property, except the lines. The congruence and similarity of the circles Any two circles with the same radius are the same, if one circle moves so that its center coincides with the center of the other circle, then the definition indicates that the two circles will be the same. More generally, any two circles are similar - move in one circle so that its center coincides with the center of the other circle, then apply the appropriate extension so that it exactly coincides with the second circle. The circle forms a curve with a certain length called a circle, and it covers a certain area. The similarity of any two circles is the basis of kew's definition, the circumference and diameter ratio of any circle. We saw in the module, Circles, that if the circle has a radius of R, then the circumference of the circle 2'r and the circle area and chords qr2 Radii and chords Let AB be a chord of the circle without passing through its center O. Accord and two equal radius of OA and BO form a triangle of isocels, the basis of which is a chord. The AOB angle is called an angle in the center, a chord to be drawn up. In the Rhombuses, Kites and Trapezia module, we discussed the axis of the symmetry of the isocele triangle. Translating this result into the language of circles: Let AB the chord of the circle with the center of O. The next three lines are the same: the AOB corner encore in the center, the chord is taut. The interval of joining O and ab chord middle. AB perpendicular chord encore. Trigonometry and Chord Designs with radii and chords give many possibilities for using trigonometry. EXERCISE 2 In the chart on the right, the AM interval adjusts the angle of the θ in the center of the O circle of radius 1. Show that: AM and sin θ b OM th cos θ This exercise shows that the sinus can be seen as the length of a half-chord AM in a radius circle 1, and cosine as a perpendicular chord distance from the center. To this day, the blue tables have been composed as table chords or half-chords, and the name sinus is assumed to have come in a complex and confusing way from the Indian word half-chord. Arcs and Sectors Let A and B will be two different points on a circle with the center of O. These two points divide the circle into two opposite arcs. If the AB chord has a diameter, the two arcs are called semicircle. Otherwise, one arc is longer than the other - a longer arc is called the main AB arc, and the shorter arc is called the smaller AB arc. Now join the radius of OA and OB. The AOB's reflex angle is called the angle thought by the AB main arc. The non-flector angle of the AOB is called the angle thought by the minor AB arc, as well as the angle thought by the AB chord. These two radii divide the circle into two sectors, called the main sector of the OAB and the Small Sector respectively. Unsurprisingly, equal chords and equal arcs have equal angles in the center of a fixed circle. The result for chords can be proven by congruent triangles, but congruent triangles cannot be used for arcs because they are not straight lines, so we have to define conversion. The theorem Equal chords of the circle are divided by equal angles in the center. b Equal arcs of the circle are divided by equal angles in the center. The proof on the chart on the right, AOB PO (SSS), so AOB and PO (appropriate angles of congruent triangles) b Turn the circle so that the ARC of PF coincides with the AB or BA arc. Then the angles of PO and AOB are the same, and therefore equal. Segments of the chord AB circle divides the circle into two segments. If AB is in diameter, two congruent segments are called semicircle - the word semicircle is used for both a semicircular arc and a segment enclosed by an arc and diameter. Otherwise, these two segments are called the main segment and the insignificant segment. The word subtend word subtend literally means keeps under, and is often used in geometry to describe the angle. Suppose we have an AB interval or arc and a P point not on AB. Join the AP and BP intervals to form the APB angle. We say that the APB angle is the angle subtended by the interval or the AB arc at point P. Corners in the semicircle Let the AOB be the diameter of the circle with the center of the O, and let the P be any other point on The APB angle, tilted at the AB diameter, is called a semicircle angle. This angle is always right-angle - a fact that surprises most people when they see the result for the first time. The corner theorem in the semicircle is a right angle. Proof Let AB be the diameter of the circle with the center of the O, and let the P be any other point on the circle. Join the PO radius, and let q a and q B. AOP and BOP triangles are iso targets because all radii are equal, so APO and BPO (basic corners of AOP and BOP triangles). So (the angular sum of the APB) is 2 x x 180, so the APB is 90 as needed. This famous theorem is traditionally attributed to the Greek mathematician Thales, the first known Greek mathematician. Exercise 3 Thales theorem can also be proven with the next rectangle test developed in the module, Parallelograms and Rectangles. The four-sided, diagonal of which is equal and split from each other, is a rectangle. Join the PO radius and pronounce it to the diameter of the PO, then join the four-way APB. Explain why the APB is right angles. Building the right angle at the end point of the Thales interval theorem provides a quick way to build the right angle at the end point of the AX interval. 1 Select point O above the AX interval. 2 Draw a circle with the center of the O through the AX crossing again on P. 3 Draw a PO radius and produce it up to the diameter of the PO. 4 Join the AP and then PAZ 90 . The reverse theorem Corner in the semicircle theorem has a simple reverse, which is best expressed as a property of the right-angle triangle: Theorem Circle, the diameter of which is the hypotenusia of the right-angle triangle, passes through all three verticals of the triangle. Proof Let ABC be the right angle on C, and let M be the middle point of AB hypotenuse. We have to prove that MC and MA and MB. Complete the ACBR rectangle. Because the ACBR is a rectangle, its diagonals divide each other and are equal. Thus, M is the middle point of another diagonal CR, and AM is BM and RM. This is a great example of how ideas fit in the geometrically - a significant circles theorem has been proven using the properties of rectangles. Two practical situations to illustrate the reverse theorem Set points in a plane are often referred to as a locus. This term is used, in particular, when the set of dots is a curve, traceable point of motion. For example, a circle can be defined as a point locus that moves so that its distance from some fixed point is constant. The two examples below use the reverse angle in the semicircle theorem to describe the locus. EXERCISE 4 Photographer takes pictures of the decorative facade of the building. He wants the two ends of the front to be pulled at right angles to his camera. Describe a set of all positions where it can stand. 5 Meter-long board resting the flush to the wall, but it slides outwards, with its top sliding down the wall and his feet moving at right angles to the wall. What path can be traced in the middle of the board? The angles in the center and the circumference of the corner theorem in the semicircle can be greatly generalized. On each chart below, AB is the circle arc with the center of the O, and the P is the point on the opposite arc. The AB arc tilts the AOB angle in the center. The arc also tilts the APB angle, called the circle angle, which is applied by the AB arc. In the middle diagram, where the arc is a semicircle, the angle in the center has a straight angle, and on the previous theorem the angle of the circle is right angle - exactly half. We will show that this connection also occurs in two other cases where the arc is a minor arc (left diagram) or the main arc (right chart). The proof uses isocele triangles in the same way to prove the Thales theorem. The corner theorism around the circle circle is half a corner in the center, subordinated to the same arc. Proof Let AB be the arc of the circle with the center of the O, and let the P be any point on the opposite arc. We need to prove AOB 2APB. The evidence is divided into three cases, depending on whether: Case 1: O lies inside ABP Case 2: O lies on APB Case 3: O lies outside the APB. Case 1: Join the PO and produce PO to the WA and OB OP (radiation), so we have two isosceles triangles OAP and OAS. Let OAP and OBP. Then APO (basic corners of OAP isosels) and BPO (basic corners of OBP isoce). Thus, AHO No. 2 (external corner of OAP) and BOS No. 2 (external corner of OBP), AOB 2 2 - 2 (Yap.) . EXERCISE 6 Full evidence in two other cases. EXAMPLE Player stands on the edge of a circular race track. With binoculars, he follows a horse that rides on the track at one revolution per minute. Explain why the player's binoculars rotate at a constant half-revolution rate per minute. Decision As the horse moves from position A to position B, the horse moves the angle of the AOB 2θ about the center of the track O. The circle angle is half the corner in the center, so the APB and θ which means that the player's binoculars rotate at an angle θ the horse moves from A to B. Thus, the player rotates the binoculars at a constant speed, which is half the speed at which the horse rotates around the center. The angles thought of by the same arc On the chart on the right, the two corners of the APB and the A.B.I. are divided by the same (minor) AB arc. Each corner is half the corner of the AOB in the center, thought of by the same arc, so the APB and the ABB. This consequence of the previous theorem is a particularly significant result of the corners in the circles: Theorem Two corners in the circle subordinated to the same arc are equal. Tales' theorem is a special case of this Some alternative terms the last two theorems are often expressed in several different languages, and some explanations are needed to avoid confusion. 1 The angle tilted by the arc is often considered to be standing on the arc. With this terminology, two theorems become: - Angle at the circle circle circle half of a corner on the center standing on the same arc. - Two corners in the circle, standing on the same arc, are equal. In the context of these two theorems, it is best to avoid the phrases standing on the AB chord and subtended by the ABB chord because we need to distinguish between the angles thought of by AB's main arc and the angles that are subsized by ab's shallow arc. 2 In the top chart on the right, the APB angle is called the angle in the (main) segment. Note that it actually stands on a small AB arc, which can be misleading. We've used this terminology before talking about a corner in a semicircle. With this terminology, the last theorem becomes: Two corners in the same segment of the circle are equal. The situation is illustrated in the bottom chart on the right. Expansion - Orthocenter triangle Height triangle perpendicular to any of the three turntables to the opposite side, produced if necessary. These two cases are illustrated in the charts below. There are three heights in the triangle. The next theorem proves that they are simultaneously at a point called the H triangle orthocenter. Surprisingly, circles can be used to prove the consistency of heights. Triangle Height theorem is simultaneous. EXERCISE 7 On the chart right height AP and BH are found in H. Interval CH is made to meet AB, produced if necessary, in R. We must prove that CR ⊥ AB. Explain why q, H, P and C are concyclic, and draw a circle. b Explain why A, B, P and q are concyclic, and draw a circle. c Join the common chord of THE And explain why XX and HF. d Explain why APH and ABH. e Use ABH and ACR to explain why CR ⊥ AB. Cyclical quadrilateral in the module, Congruence, we showed how to draw a circle through the vertices of any triangle. To do this, we showed that the perpendicular two-sectionors of the three sides are simultaneous and that their intersection, called the triangle center, is equal to each peak. No other circle passes through these three verticals. If we tried to take as a center point P, except the center, then P would not lie on one of the perpendicular two-sectors, so it can not be equal to the three vertices. When there are four points, we can always draw a circle through any three of them (assuming they are not collinar), but only on special occasions this circle will pass through the fourth point. The cyclical quadrilateral is a four-sided, whose vertices all lie on the circle. This is type of special four-way that we'll look at. Building a circular cycle of a cyclical four-way, we are given a quadrilateral, which is known to be cyclical but whose center is not shown (perhaps it was rubbed). The center of the quadrilateral is the district center of the triangle, formed by any three of its verticals, so the structure on the right will find its center. Opposite angles of the cyclical four-sided distinctive feature of the cyclical quadrilateral is that its opposite angles are optional. The next proof uses a theoreme that the circumference angle is half a corner in the center, standing on the same arc. Opposite angles of the cyclical quadrilateral are optional. Proof Let ABCD be a cyclical quadrilateral with an O in the center of the circle. Join the OB and OD radii. Let q and g be the corners in the center, as shown in the diagram. Then - 360 (angles in Revolution in O) Also A (corners on the same BCD arc) and C (corners on the same BAD arc), so that A q C - 180 hence and ABC - ADC 180 (corner sum of four-sided ABCD) Here is alternative proof, using the fact that the two corners in the same segment are equal. EXERCISE 8 Join the AC diagonal and BD cyclical quadrilateral ABCD. Let the corners be shown. a Give a reason why DBC and b What other angles are sizes, q and q? c Prove that th th th 180. d So, prove that the opposite corners of the ABCD are optional. The outer corners of the cyclical four-sided outer corner of the cyclical quadrilateral is an addition to the adjacent inner angle, so equal to the opposite corner of the interior. This gives us a consequence of the cyclical four-sided theorem: Theorem The outer corner of the cyclical quadrilateral equals the opposite corner of the interior. Proof In the diagram on the right, B.C. is made for P to form the outer corner of the PCD. This outer angle and A are optional to BCD, so they are equal. EXERCISE 9 Show that the AP CR on the chart on the right. EXERCISE 10 If the cyclical trapezoid is not a rectangle, show that the other two sides are not parallel, but have equal length. Expansion - The test for the cyclical four-sided Quadruple Four-sided Property proved previously that its opposite angles are optional, is also a test for the four-sided to be cyclical. That is true to the contrary. This theorem completes the structure we followed - for each special quadrilateral, we establish its distinctive properties and then set the tests for it. The proof uses evidence of contradictions, and thus a little more complicated than other materials of the year 10. Theorem If the opposite corners of the four-way are additional, then the four-way Proof Let ABCD be four-sided with KK 180. . circle through A, B and D, and suppose by contradiction that the circle does not pass through C. Let DC, produced if necessary, meet the circle again on X, and join the XB. Then A and BXD are optional because ABXD is a cyclical quadrilateral, so the angles of DXB and DCB are equal, so the XB CB is therefore the same line, so that the C and X are the same, that is the circle passes through C. EXERCISE 11 To prove the next alternate shape of the aforementioned theorem: If the outer corner is four-way equal to the opposite corner of the interior, then the four-way loop. EXERCISE 12 On the chart on the right, the two adjacent sharp angles of the trapeze are equal. Prove that the trapeze is cyclical. Expansion - The Sinus Rule and the geometry of the sinus circle rule states that for any ABC triangle, the ratio of either side over the sinus to its opposite angle is constant, each term's ratio of length over pure number, so their total value seems to be length. So, it is reasonable to ask what is the total length? The answer to this question is a surprise - the total length of the diameter of the circumference through the vertices of the triangle. Proof of this result is proof of the sinus rule, which does not depend on the evidence presented in the module, Further Trigonometry. Theorem In any ABC triangle, No. 2R, where R is a circumference radius. Proof Let O be the center of the circle through A, B and C, and let A. enough to prove that it is the diameter of the circumference. There are three cases, as shown below. Case 1: A and O lie on one side of B.C. Case 2: A and O lie on opposite sides of B.C. Case 3: O lies on B.C. In cases 1 and 2, build the diameter of the BOP, and join the PC . EXERCISE 13 Complete the following steps of proof in case 1. I explain why BCP and P y iiComplete proof using BCP. b Follow the following steps of evidence in case 2. I explain why BCP 90 and sin P and sin iiComplete proof using BCP. c Full evidence in case 3. Tangents to Tang's circles to a circle is a line that meets a circle only at one point. The chart below shows that there may be three possibilities, given the line and the circle, that the line can be a sequent, cutting the circle in two points. The line can be tangential, touching the circle only at one point. The line may completely miss the circle. The words sekant and tangent from the Latin secant mean cutting (as in cross section), and tangent means touch (as in tango). The point where the tangent touches the circle is called the point of contact. It is not immediately obvious how to draw a tangent at a certain point on a circle, or even there may be more than one tangent at that point. The next theorem makes the situation clear and uses the Pythagoras theorem in its evidence. Let the Theorem Let T Be On a circle with a center O. Line via T, perpendicular to the OT radius, is tangent to circle. b This line is the only tangent to the circle in T. c Each dot on a tangent, except T itself, lies outside the circle. Proof First, we prove parts A and c. Let there be a line through the T perpendicular radius of OT. Let the P be any other point on, and join the OP interval. Then using the Pythagoras theorem in OTP, OP2 and OT2 and PT2; that's more OT2, so the OP is larger than the OT radius. Hence P lies outside the circle, and not on it. This proves that the line is tangential because it meets a circle only at T. It also proves that every point on, except T, lies outside the circle. It remains to prove part b that there is no other tangent to the circle at T. Let t be tangent on T, and suppose by contradiction that t were not perpendicular to OT. Let the M will be the foot perpendicular to O to t, and build a U point on t, on the other side of the M, so UM and MT. Then with the Pythagoras theorem in OMT and OMU, OT2 - OM2 - MT2 - OM2 , MU2 - OU2, So OU - OT. Thus, U also lies on a circle that contradicts what t on a tangent. Design - Tangents from the outer point Using this radius and tangent theorem, and the angle in the semicircle theorem, we can now build tangents to the circle with the center of O from point P outside the circle. 1 Join the OP and create a mid-M OP. 2 Draw a circle with the center of M through O and P, and let it meet the circle on T and U. The angles of PTO and PUO are the right angles because they are the corners in the semicircle. Thus, the PT and PU lines are tangential because they are perpendicular to OT and OU radii, respectively. Tangents from the outer point have an equal length It is also a simple consequence of the radius theorem and tangent that the two tangent PT and PU have an equal length. Note that here and elsewhere, we use the word tangent in the second sense to mean not the whole line, but simply the interval from the outer point to the point of contact. Tangents to circle from the outer point have an equal length. EXERCISE 14 Prove this result using either congruence or the Pythagoras theorem. Tangents and trigonometry The right angle, formed by radius and tangent, provides additional possibilities for simple trigonometry. The following exercise shows how the names tangent and sekant, and their acronyms tan θ and sec θ, have become used in trigonometry. In the 1st radius circle, the length of the tangent angle θ in the center of the tan θ, and the length of the sequestration from the outer point to the center is θ. EXERCISE 15 Casing PT to the circle of radius 1 touches the circle on T and subtends corner θ in the center of the O. Show that PT and tan θ and PO and sec θ where the function of the trigonometer secant is defined θ = . The four-way relationship with the rounder Follows exercises include a four-way circle in which the circle can be drawn on a tangent for all four parties. These four-way formations form another class of special quadrilateral formations. EXERCISE 16 Parties four-way have tangents to the circle drawn inside it. Show that the sums of opposite sides of the quadrilateral are equal. The common tangent and touching circles of the Line, which has a tangent to two circles, is called a common tangent to circles. When the points of contact are different, there are two cases, as in the charts below. These two circles lie on one side of a straight common tangent and lie on opposite sides of an indirect common tangent. Direct common tangents are common tangents Two circles are said to concern at a common point T, if there are common tangents for both circles at point T. As in the chart below, the circles are externally concerned when they are on opposite sides of the common tangent, and are internally concerned when they are on the same side of the common tangent. Provided that they are different, touching circles have only one common point. Expansion - Center Triangle We are now able to prove a remarkable theoreum on the corner two-sectors of the triangle. These three two-sectors are simultaneous, and their intersection point is called the center of the triangle. Incentre is a center on tangents and twists with all three sides of the triangle, as on the chart on the right. Theorem Corner bisectors of the triangle are parallel, and the resulting center is the center of the twist, that is tangent for all three sides. Proof We have to prove IC bisects BCD Given ABC, build a corner of the two-sector A and B, and let me have their crossing points. Join IC and let th BAI and CAI and z. ABI and CBD. Fall perpendicular IL, CHAT and B.C., CA and AB respectively. Summary - Four Triangle Centers This completes the development of the four most famous centers of the triangle. The results, as well as the associated terminology and notation, are summarized here for reference. Perpendicular bisectories on three sides of the triangle are simultaneously located in the O district center, which is the center of the circle through all three verticals. This has been proven in the module, Congruence. The two-sets of the three corners of the triangle are simultaneous in center I, which is the center of the irrh, which is tangent for all three sides. This has been proven above. The median of the triangle is the interval of attaching the top to the middle of the opposite side. The medians of the triangle are simultaneously on the G centroid, which divides each median in a ratio of 2 : 1. This has been proven in the module, the scale of the drawings and the similarities. The height of the triangle is perpendicular from the top to the opposite side, made if necessary. Triangle heights, too This has been proven previously in this module as an extension in the corners in the center and circle section at the same time. The alternative theorem of the segment Left chart below shows two angles of P and q lying in one segment of the circle - we proved that these two angles are equal. In the next two charts, the angle of the BSU remains the P, as the point moves around the arc closer and closer to A. In the last chart, q coincides with A, and the AU is tangent. From these charts, it seems reasonable that, if limited, the BAU angle will still be P- that's what the alternative segment theorem says. The angle between the chord and the tangent is equal to any corner in the alternative segment. Here the BAU is the angle between the chord and the tangent. The word alternative means another - the AB chord divides the circle into two segments, and the alternative segment is the segment on the left, containing angle P. We have already proved that all angles in this left segment are equal. There are two equally satisfactory evidence of this theorem. One is written out below and the other remains as an exercise. Proof Let AB be a chord circle, and let BU be tangent on B. Let P be a point on the arc that is not in the hands of ABU , and let P and there are three cases, depending on: Case 1: Center O lies on the chord AB. Case 2: Center O lies outside the hands of ABU. Case 3: Center O is in the hands of the ABU. In each case, we have to prove that ABU - Case 1 is easily settled: 90 (a semicircle angle) and ABU 90 (radius and tangent) For two other cases, build a diameter bod, and join the DA. Case 2: ADB (corners on the same AB arc) and BAD (corner in the semicircle), so DBA (corners on the same AB arc) and BAD (corner in the semicircle), so that DBA (adjacent angles in B) Case 3: ADB 180 (cycle THE quadrilateral APBD and BAS (semi-circle angle), so ABD (corner amount ABD) But DBU 90 (radius and tangent) so ABU (adjacent angles on B) Exercise 19 Find alternative evidence in cases 2 and 3 by building radii AO and BO and use corners in the center. The result in the next exercise is amazing. Don't expect parallel lines to appear so easily on a diagram with two touching circles. EXERCISE 20 Prove that AB PL is on the chart on the right, where STU is a common tangent for both laps. AP and BP are straight lines. The similarities and circles of the Final Theorem in this module combine similarities with the geometry of the circle to produce three theorems about intersecting chords, intersecting section and square on tangents. The intersecting chords of the first theorem is dedicated to the chords that intersect within the circle. In this situation, each chord cuts the other in sub-intervals called interceptions. The surprising consequence of such triangles is that in this situation the products of interceptions on each chord are equal. That is, on the chart on the right, AM and MB th PM and MH. When two chords intersect within a circle, the products of interceptions are equal. Proof Let AB and PP be chords intersect on M. Join the AP and BH. In the triangles APM and WPM: PAM and BM (angles on the same PB arc) APM and PM (corners on the same AP arc), so that APM is similar to the BPM (AA): Hence (appropriate sides of similar triangles), so WHAT AM and BM. The last step is particularly interesting. It converts the equality of two length ratios into the equality of two-product lengths. This is a common procedure when dealing with similarities. Sinus rule and similarity Many problems related to similarity can be handled using the sinus rule. The exercise below provides alternative proof of intersecting chord theorems by using the sinusin rule to deal directly with the ratio of the two sides of the triangles. The other two theorems of this section also have alternative evidence using the sinus rule. EXERCISE 21 Use the sinusin rule on the chart in the evidence above to prove that. Secants from the outer point Now assume that the chords do not occur within the circle. If they meet on a circle, the identity above holds trivial, and if they are parallel, there is nothing to say. Otherwise, we can produce chords until they intersect outside the circle, and a similar theorem is applied. Now we are dealing with separations from the outer point. When the secant through the outer point of M meets a circle at two points A and B, the lengths of AM and BM are called sequencing interceptions from the outer point, and, as before, AM and MB and PM. Be careful here, the AB chord is not one of two interceptions. With this definition of interception, the previous theorem can now be listed as follows. Its proof is word for word almost the same, apart from the reasons: Theorem Product interceptions on two seedlings from the outer point are equal. Proof Let the missile defense and MSM be two sicacs from the outer point of M, as shown on the site. Join the AP and BZ. EXERCISE 22 Complete the evidence. EXERCISE 23 If instead we joined the intervals of AP and BP, what appropriate changes should be made to the evidence? Tangent and secant from the outer point Now imagine a secant MP in the previous charts rotating until it becomes tangent at point T on the circle. As the secant rotates, the length of each PM and PM interception approaches the length of the tangent TM to the circle from M, so the PM and M product gets closer to the TM2 tangent square from M. This is proof using limitations. When we draw secant and tangent from M, we saw that the PRODUCT AM and BM equal to the tm2 square The geometric evidence is similar to two evidence, but an alternative segment theorem is required to establish similarities. The theorem of the product of interceptions on the seconta from the outer point is equal to a square tangent from this point. The proof let the missile defense be secant, and TM tangent, from the outer point of M, as shown in the shown. We have to prove that AM and BMTM2. Join AT and BT. The hg by the three common chords Proof of Consistency usually involves logic, which is a little more complex than is required in most evidence. The next theorem is the difficult application of intersecting chord theorems. The EXERCISE 25 Show that the three common chords of AB, PL and ST in three circles on the chart above are simultaneous. Forward Circle geometry links are often used as part of problem solving in trigonometry and calculus. The converse of theorems of the theorems of the theorems of the theorems of the theorems of the theorems shown in this module have dramatic and important reverse conversation theorems that are tests for points to lie on a circle. Evidence of these conversations, and their application, is generally considered inappropriate for 9-10 years, except for the reverse angle in the semicircle theorem that was developed in the module. However, they are so closely related to the material of this module that they were fully developed in the application. Coordinate tangents geometry to circles We identified tangent to circle as a line with one point in common with the round. This definition can be used in coordinate geometry using simultaneous equations. For example, the chart on the right shows the x and y 2 lines and the x2 and y2 circles. Replacing the line equation into the circle equation gives x2 (2 - x)2 - 2 2x2 - 4x, 4 - 2 x2 - 2x 1 - 0 (x) 1)2 and thus the only solution is x No 1. Consequently, the line and circle have only one crossing point T(1, 1), proving that the line is tangent to the circle. Similarly, the dotted line x th th 1 is secant, crossing the circle in two points, and the dotted line x th 3 does not cross the circle at all. Tangents to parabola This simultaneous approach of equations to tangents can be summarized by other curves defined by algebraic equations. For example, the y 2x 1 line is tangent to the parabola chart y x2 at P (1, 1), because solving equations simultaneously gives x2 and 2x 1 x2 - 2x 1 y 1 y 0 From here P (1, 1) is the only crossing point. However, for schedules defined by cabins and higher powers, the definition of tangent needs to be adapted. The first diagram on the right shows that we will think how the tangent at P point can cross the curve again at some other point, the next diagram shows that even with a square graph, our current definition of tangent will mean that each vertical line will be tangent Obviously, we need to change the requirement of a single crossing point and instead develop some idea of the tangent being a straight line that is roughly a curve in the area of contact. These considerations naturally lead to a well-known process of restriction. To build a tangent at point P on a common curve, we build a secant through P and another point on the curve, and then move the point closer and closer to P. This is the traditional beginning of calculus in the school. Apps in motion and speed change When the stone on the string whirls around in a circle and then suddenly let go, it flies off on a tangent to the circle (ignoring the subsequent fall to the ground). We can interpret this situation by saying that the tangent describes the direction in which the stone was traveling at the moment it was released. This leads to the concept of a vector that has both the magnitude and the direction of the speed particle. The study of movement begins with a movement in a straight line, that is, in one dimension. When motion in two dimensions is first seen, circular paths and parabolic pathways are the first paths to be considered because they are simple enough to describe mathematically, and they occur in many practical situations. The rate of change is also described by tangent. This time the tangent has no physical significance, but it is a tangent gradient that gives an instantaneous rate of change. For example, when boiling water is removed from the oven and cooled, the temperature chart looks like a graph on the right. At any time t, the speed at which the water is cooled is given by the tangent gradient at the appropriate point on the curve. Vectors and complex numbers often require placing a set of complex numbers on a complex plane. The following example requires some knowledge of the vectors and geometry of the circle. EXAMPLE Sketch is a set of complex numbers, so that the ratio is an imaginary number, that is a real multiple i. Solution If z No. 1, then No 0, which is imaginary. If z no.1, it is not determined. So suppose that z is neither 1 nor No.1. The condition on z is that z No 1 is ki fold z 1, where k is real. We know that multiplying by ki rotates the direction of a complex number by 90 degrees or 90 degrees. That is, the vector z No. 1 is perpendicular to the vector z No. 1. We know that z No. 1 is a vector with a tail on 1 and a head on z, and that z No. 1 is a vector with a tail at No.1 and a head on z. Let A, B and P represent points No.1, 1 and z. Then the condition becomes APB is a straight angle. Thus, the z locus is a circle with ab diameter, i.e. a radius of 1 and a center 0, with the exception of No. 1. Many problems like this are not just about theorems, but they're talking, and as developed in the annex to this module. The history and application of Greek geometry was based on the constructions of straight lines and circles, using a straight edge and compasses, which naturally gave circles a central place in their geometry. All theorems developed in the content and application of this module were developed by the Greeks and appear in the elements of Euclid. In the geometry of the coordinates later developed by Descartes in the 17th century, horizontal and vertical length is measured by two aus, and diagonal length is connected to them by the Pythagoras theorem. As a result, the relationship between the circles and the Pythagoras theorem is seen in the circle equation. The square distance point P (x, y) from the source x2 and y2 the pythagorean theorem, which means that the circle equation with the radius and center of origin is x2 and y2 a2. A circle is a simple closed curve with the inside and the outside, a property that it shares with triangles and quadrilateral. In three dimensions, spheres, cubes and tories (donuts) have an inner and outer side, but the torus is clearly connected in a different way than the sphere. The theme called topology, started by Euler and widely developed in the 20th century, begins with such observations. Greek astronomy perfectly used circles and spheres. They knew that the Earth was round, and were able to calculate its circumference with reasonable accuracy. Ptolemy described celestial bodies from the point of view of concentric spheres on which the Moon, planets, sun and stars were embedded. When astronomy was reconstructed in the 16th century by Copernicus, the celestial spheres disappeared, but it still used circles for orbit, with the center moving from Earth to the Sun. At the end of the 17th century, Newton used calculus, his laws of motion, and the universal law of gravity to obtain Kepler's laws. John Dalton reconstructed chemistry in the early 19th century on the basis of atoms that he considered tiny spheres, and in the 20th century models of circular orbits and spherical shells were originally used to describe the movement of electrons around the spherical nucleus. Thus, circles and their geometry have always remained at the center of theories about the microscopic world of atoms and theories about the cosmos and the universe. Geometry continues to play a central role in modern mathematics, but its concepts, including many circles, are becoming more abstract. For example, spheres in a higher dimensional space were noticed in 1965, when John Leach and John Conway made an impressive contribution to modern algebra by studying close packaging of spheres in a 24-dimensional space. On the other hand, the classical Euclidean geometry in the form presented in this module has nevertheless progressed in our time - here are three results obtained in recent centuries. The Euler line In 1765, Euler discovered that: Orthocentre, circumcentre, and the centraloid of any collinear triangle. The centroid divides the interval of joining the circus center and the orthocenter in a ratio of 2:1. The line of accession to these three centers is now called the Euler Line. The proof is simple enough, and presented in the next exercise. EXERCISE 26 On the chart on the right, G and O, respectively, the centroid and the ABC Center. The OG interval is made for X so that OG : GX No. 1 : 2. We will prove that X is the center of the ABC, proving that it lies on each of the three heights of the triangle. Join AX. Let M be mid-BC Join the median AGM and perpendicular OM. Prove that AGX is similar to MGO, with a 2:1 similarity ratio. b Thus, prove that AX OM. c So explain why AX is produced is the height of the ABC. d Full proof is the orthocenter of the triangle. Nine-point circle In the early 19th century Poncelet, Feuerbach and others showed that in any triangle the following nine points are cyclical: the middle of each side of the triangle, the leg of each height, the middle of the interval, attaching each top of the triangle to the orthocenter. That is, these nine points lie on the circle. He also showed that the center of this nine-point is on the Euler line and is the middle point of the center's accession to the orthocenter. Again, the proof is easy enough to present here as a structured exercise, although proving that the center of a nine-point circle is the average point of oh is pretty uncomfortable. Exercise 27 Chart on the right shows the ABC with the heights of AU, BV and CW meeting at the H. Points E, F and G are respective intermediate points AH, BH and CH. Points P, q and R are intermediate points of the parties B.C., CA and AB. Prove that RS and FG are parallel to B.C. and half its length. b Prove that the RF and the zG are parallel AH and half its length. c C Thus, prove that the FGHD is a rectangle. d Prove also that FP-E is a rectangle. e Let N will be the middle F point, and use the properties of the rectangle to prove that RN and GN and therefore that R, G, F, q, P and E are concyclic, lying on a circle with center N . f Considering the FWF, prove that N is equivocing from q and V, and therefore that the circle also passes through U, V and W. g Chart on the right shows the height of the AU, the middle of P B.C., the middle of the S UP, and the district center O, the H orthocenter, and the nine-point center N. i Explain why SN is a perpendicular bicector UP. ii Explain why SN If necessary, bisects OH. iii Explain why N lies on OH, and thus its middle point. The transformation of the extension associated with the Euler line and the nine-volume circle here is quite dramatic proof, using one extension that establishes the existence of the Euler line and the position on it of the G, Center O, Center H and Nine-Point Center N. Notation is the same as in exercises 25 and 26. In this design, all that is used about the nine-point circle is that it is a circle through P, q and R. The fact that the remaining six items lie on it will be proven afterwards. The medians trisect each other, so AG : GP and BG : GZ and CG : GR No 2 : 1. Consider an extension with a G center and an extension factor. This extension fixes G, and cards A to P, B to q and C to R, so the ABC circumference is displayed to the P'R circle. Thus, the O center is displayed at nine points in Center N, and G thus divides NO in a ratio of 1 : 2. ABC's H Orthocenter is displayed in the PDR Orthocenter, and the PDR Orthocenter is about the center of the O ABC, because the perpendicular bisectories of the ABC parties are the heights of the PRA, so G divides HO in a ratio of 2 : 1. Thus, the four points H, N, G and O are collinear, with HN : NG : GO No 3 : 1 : 2. It also shows that the nine-point circle has a half-circle radius. Theorem of the Morley Trisector In 1899, the American mathematician Frank Morley discovered an amazing equilateral triangle that formed inside each triangle. When each corner of the triangle is trisected, the intersection points of the trisectors of adjacent verticals form an equilateral triangle. The result has a lot of evidence of similar triangles, and we refer to the reader, in particular, the proof of John Conway and the version of Bollobas. The result can also be proven by complex angular trigonometry formulas, and thus available enough for students in senior calculus courses. See 27s_triangle app - Converse to Circle Theorems This evidence is best written as evidence of contradictions. The original theorem is used to prove each theorem of the reverse conversation. We saw this approach when the Pythagoras theorem was used to prove the opposite of the Pythagoras theorem. The reverse side of the corner in the central theorem is the key theorem of this module: The circle circle circle angle is half the corner in the center, pulled by the same arc. The easiest reverse of these: Let the OAB be isosceles with OA and OB, and let P be a point on the same side of the AB interval as O. Suppose that AB subtends the angle at the P half-sized angle of the AOB. Then the circle with the center of O through A and B passes through P. Proof Let APB and AOB Build a circle through A and B. Suppose, by contradiction, that the circle does not pass through the P, and let the circle cross the AP, produced if necessary, on the X. Join BX. Then AXB and AOB (center angles and circumference on the same AB arc) BP (appropriate angles are equal); so the BX and PX lines are the same, and therefore the points X and P are the same. Thus P lies on a circle, contrary to the assumption that the circle does not pass through the P. Testing whether the four data points are concyclic One of the main axioms of geometry is that the line can be drawn through any two different points A and B. When there are three different points A, B and C, we can ask whether the three points are collarar, or the triangle form. Similarly, any three non-coulinar points A, B and C are concyclical. When there are four points A, B, C and D, there are no three collinars, we can ask whether these four points are concyclic, that is, they lie on the circle. The rest of the reverse theorems all provide tests on whether the four data points are concyclic. In each case, we draw a unique circle through three of them and prove that the fourth point lies on this circle. Let's assume that the fourth paragraph does not lie on the circle and does not create contradictions. Reverse from corners on the same arc theorem If the interval subtends equal angles at two points on the same side of the interval, then the two points and the end points of the interval are concyclic. Proof Let the interval AB subtend equal angles at points P and q on the same side of B.C. Build a circle through A, B and P and assume, by contradiction, that the circle does not pass through Kew. Let the AK produced if necessary, meet the circle again on X. Then the AXB (corners on the same AB arc), so that the BX BH (appropriate angles are equal) So bh and BX are the same line, so q coincides with the X, which is a contradiction. Test for cyclical four-sided We have proven previously as an extension of content, two tests for cyclical four-sided: If the opposite corners of the cyclical four-sided are optional, then the four-way cyclical. If the outer corner of the quadrilateral is equal to the opposite corner of the interior, the four-sided cyclical. The proof of the contradiction of the first test is almost identical to the evidence of the previous theorem of the reverse conversation. The second test is a simple consequence of the first test. Crossing the chord test We have proven that the product intercepts intersecting chords are equal. The reverse is giving another test for four points to be concyclic. The evidence continues in exactly the same lines. Let AB and PL theorems will be intervals intersecting on M, with AM and BM and PM. Then the four endpoints A, B, P and q are concyclic. EXERCISE 28 Complete the proof of this theorem. We leave it to the reader to articulate and prove (true) conversations with the remaining two theorems about seedlings from the outer point, and tangent and seekant from the outer point. Similar methods can also prove the opposite theorem, that if the quadrilateral has a round, the sums of its opposite parties are equal. ANSWER TO EXERCISE EXERCISE 1 Each translation of any distance along the map on itself. Let O be any point on. Then the reflection in the line m through O perpendicular, and the rotation of 180 o O as the map on itself (and identical in their action on the points). The reflection of the plane in the line fixes the line because it fixes each point on. b Translation of the plane by moving P to the cards on itself, and the card P to q. Reflection in the perpendicular bicep of the P' maps interval on itself, and exchanges points P and q. (rotation 180 about mid P' does the same.) EXERCISE 2 Use simple trigonometry in AMO. EXERCISE 3 All ranges are equal, so OA and OP th Oz, so the quadrilateral APB is a rectangle because its diagonals are equal and corrode each other. Thus, the APB is right-hand. EXERCISE 4 At all times, in front of the building of the hypotenuse right-angle triangle, the third top of which is the photographer. Therefore, the circle in diameter, the facade of the building, always passes through the photographer, and his possible positions are points on the semicircle in front of the building. EXERCISE 5 At all times the triangle formed by the floor, wall and floor, has the right angle with a hypotenusia length of meters. On the reverse theorem above, the middle dot of the board is therefore always meters from the corner. Thus, the middle point traces the quadrant of the circle with the center on the corner and the radius of meters. EXERCISE 6 Case 2: It will be enough to prove the result when O lies on the AP. Let the OBP and Z. then BPO (basic angles of THEP isocele), so that AOB Nos. 2 and 2 APB (external angle obp). Case 3: Join the PO and produce PO to W. Let OAP and OBP (basic angles isosceles OBP), then APO (base angle issosceles AOP) OPB (basic angle OBP) Hence, APB (outer corner OAP) and BOS 2 (external corner OBP) Hence IOB 2 2 - 2 (I) - 2 - APB EXERCISE 7 Interval HC subtends right angles on P and q, so that a circle with a diameter of HC passes through them. b Interval AB tilts the right angles to P and q, so a circle with ab diameter passes through them. c They stand on the same arc of the headquarters of the first circle. d They stand on one arc of the second round. E In the triangles of AB and ACR, ABH and RCA in parts c and d, and A is common, so arc and AB 90 . EXERCISE 8 DAC and DBC angles stand on the same DC arc, so DBC and BDC (corners on the same B.C. arc), ABD (corners on the same AD arc), ADB (corners on the same AB arc). 2 2 - 2 2 360 (corner sum four-way ABCD) The rest is clear. EXERCISE 9 Join the common chord of BH, and produce ABC for X, and let A. Then BCR (opposite outer angle of the cyclical quadrilateral) and RCX (opposite outer angle of the cyclical quadrilateral), so that AP CR (appropriate angles are equal) EXERCISE 10 In this proof we build two triangles of isoce. Let ABCD be a cyclical trapezoid with AB DC. Because the ABCD is not a rectangle and its angles add up to 360 degrees, one of its angles is sharp. Let A and be sharp, and produce BC. Then DCP (opposite outer angle of the cyclical quadrilateral) so B - (appropriate angles, AB) DC). Thus, B is also acute, so that AD and B.C. is made to meet at point M, and CDM (appropriate angles, AB DC). Thus, DMC and AMB are both isosceles, with DM and CM and AM th BM, so AD and B.C. EXERCISE 11 Neighboring inner corner is an addition to the outer corner, and therefore equals the opposite corner of the interior. Therefore, the quadrilateral is a cyclical theorem. EXERCISE 12 D - 180 (joint interior angles, AB) DC), so D and B 180 hence the four-sided ABCD is cyclical (opposite angles are optional). EXERCISE 13 a In Case 1, BCP - 90 (corner in a semicircle) and P (corners on the same arc BC) hence sin (simple trigonometry in BCP), so 2R. b In the case of 2, BCP -90 (semicircle angle) and P - 180 (opposite angles of the cyclical four-sided BACP), so sin P - sin (180 - y) - sin, hence sin (simple trigonometry in BCP) , yu 90 (corner in the semicircle), so sin No 1. The circumference diameter is a, so No. 2R. EXERCISE 14 Let the tangents from the outer P point touch the circle on T and U. Join the OT and OU radii and the OP interval. Since OP is common, the radii are equal, and the radii are perpendicular to tangential, OPT OPU (RHS) and PT and PU (appropriate sides of congruent triangles). Exercise 15 Triangle OPT has the right angle in T. The rest is a simple trigonometry. EXERCISE 16 Tangents from the outer point are equal, so we can denote the lengths in the picture, as shown. Then both amounts of opposite sides of the quadrilateral are a b q q d. EXERCISE 17 a i 0 ii 1 iii2 iv3 v4. b The first chart below illustrates the situation with two indirect common tangents. Indirect tangents at and BU intersect in M. Then AM and TM and UM, because tangents from the outer point are equal, and the addition of length gives AT th BU. C With straight common tangents, there are two cases to consider. Suppose, first, that the direct tangents of AT and BU intersect in M when manufacturing, as in the second chart above. On the other hand, AM and TM and UM, and subtraction gives AT and BU. Now let's assume that the direct common tangents of AT and BU are parallel, as on Chart. The radii of AO and BO are perpendicular to parallel tangents, tangents, they lie on the same line and form a diameter, and the radii of TK and UH also form a diameter. Thus, ABUT is a rectangle, so its opposite sides AT and BU are equal. EXERCISE 18 Using the AAS test. b They match the sides of congruent triangles. C Use the RHS test. d Hence the LCI and MCI, so the IC is also the angle of the bisection, thus proving the consistency of the angular two-sector. In addition, a circle with a center I and a radius of LI and MI NI on a tangent for all three parties on the radius and tangent theorem. EXERCISE 19 In case 2, AOB No. 2 (angles in the center and circumference on the same AB arc), so OBA 90 (corner amount of OAB isocels) But OBU 90 (radius and tangent) so ABU (adjacent angles on B). In the case of 3, the reflex AOB 2 (angles in the center and circumference on the same AB arc) is so non-flector AOB - 360 (angles in the revolution) OBA 90 (corner of OAB isoceles) OBU 90 (radius and tangent) ABU (adjacent angles in B). EXERCISE 20 Let THE A. then BTU (alternative segment theorem), so that STP and (vertically opposite angles) so that the PP (alternative angles are equal). EXERCISE 21 We have already proven that A and that P and B. The rule of sinuses in the XY triangle can also be written in form using this form of the sinus rule in two triangles, . EXERCISE 22 Proof: In the triangles APM and BPM : PAM and BSM (external angle of the cyclical quadrilateral ABP) AMP and BMH (common angle), so that APM is similar to the BPM (AA). So. (appropriate sides of similar triangles), so am and BM th PM and M. EXERCISE 23 Proof: In the triangles of APM and PBM : A and P (corners on the same BK arc) M is common for both triangles, so that the APM is similar to PBM hence (appropriate sides of similar triangles), so AM and BM: In atM and TBM triangles: TAM and BTM (alternative segment theorem) AMT and TMB (general) So the ATM is similar to TBM (AA) from here and (appropriate sides of similar triangles) AM and BMTM2 EXERCISE 25 Let the chords AB and PL meet on M. Join the SM and produce it to meet the circles on X and Y. We have to prove that the points X and Y match. Using the intersecting chord theorism in each circle in turn, SM and MX - AM (from Circle 1) - PM (from Circle 2) - SM - MY (from Circle 3). EXERCISE 26 a First, AG : GM No. 2 : 1, because the centroid divides each median in a ratio of 2 : 1. Second, XG : GO No 2 : 1 on building Point X. Third, AGX and MGO because they are vertically opposite angles. b So the AGX is similar to the MGO on the SAS similarity test. Thus, the corresponding angles of XAG and OMG are equal, and since these are alternative angles, AX OM. c So the AX meets B.C. at right angles, so that the X lies at the height of A. AX produced by the height of the ABC. d We showed X is at the same height, and the same argument shows X is on all three Thus X is the H orthocenter of the triangle. Triangle. 27 a b These follow because the interval, joining the middle of the two sides of the triangle, parallel to the third side and half of its length. C Thus, the FGHD is a parallelogram and rectangle, because AU ⊥ BC d This follows from a similar argument to parts a c. e It follows because the diagonals of each rectangle are equal and divided into each other. Note that the FH is the diagonal of both rectangles. f Since N is the middle point of the hypotenuse right-angle triangle F'V, it follows from the reverse angle of the theorem, proven earlier in this module that N is equally disturbing from F, q and V. Similarly, N is equally useful from E, P and U, and from G, R and W. Thus, the circle runs through all nine points. g i Perpendicular UP biscector is the locus of all points equal to you and P, and this includes the nine-point center N. ii Full UPOT rectangle. Then na HTU, so NS bisects OT using opposite sides of rectangles, and just splitting oh using the similarity theorem on triangles. So NS meets OH on Y. Iii For similar arguments to parts I and ii, perpendicular BISectors UP, V and WR pass through N, and all OH bicect. Thus, the three perpendicular bisectionors are simultaneous, and the middle point oh and point N coincide at the intersection of these three lines. EXERCISE 28 Build a circle through A, B and P, and suppose, by contradiction, that the circle does not pass through the PL, let the PP, produced if necessary, meet the circle again on X . Then AM and PM hmm (intersecting chords), so XM and M and X both lie on the same side of M. So X coincides with the q, which is a contradiction. The 2009-2011 School Mathematical Education Improvement Project (TIMES) was funded by the Australian Department of Education, Employment and Employment. The views expressed here are in line with those of the author and do not necessarily reflect those of the Australian Government's Department of Education, Employment and Employment. © of the University of Melbourne on behalf of the International Centre of Excellence in Mathematics (ICE-EM), the Education Department of the Australian Institute of Mathematical Sciences (AMSI), 2010 (except when stated otherwise). This work is licensed under the Creative Commons Attribution-Non-Profit-NoDerivs 3.0 Unported License. circle chord theorems pdf. circle chord theorems worksheet. circle theorems tangent and chord. circle chord angle theorems. circle theorems length chord

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