Negative refraction and self-collimation in the far infrared with aligned carbon nanotube films

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ABSTRACT

This study demonstrates the far-infrared self-collimation and low-loss transmission of aligned carbon nanotube (CNT) films or arrays. The anisotropic dielectric functions of the CNT array is modeled using the effective medium theory considering the degree of alignment. The spectral regions where hyperbolic dispersion is satisfied are in the far-infrared. In the hyperbolic regime, energy propagates inside the CNT film along the optical axis for nearly all incidence angles. The self-collimation effect is also examined for tilted CNT thin films by tracing the Poynting vector trajectories. Low-loss transmission is explored to understand the impact of alignment on the penetration depth and transmission through the film. In conjunction with the surface radiative properties, the self-collimation and transmission characteristics are distinguished between the two hyperbolic bands of the CNT film. The insight obtained from this work may lead to the utilization of CNT arrays in polarization filtering and infrared imaging.

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1. Introduction

A wide range of applications of engineered carbon nanotube (CNT) films have been realized in the last decade. Vertically aligned CNT (VACNT) films have demonstrated high absorption of radiation from the ultraviolet to the far-infrared [1–4]. CNT films have also shown promise as wavelength-selective transparent thin films [5–7]. Studies have also explored transversely oriented CNT arrays as visible light waveguides [8,9]. Using either vertically aligned or obliquely angled CNT arrays for electromagnetic wave manipulation in the infrared or terahertz regime certainly deserves attention as well. The characterization of the unique optical properties of arrayed CNT can contribute toward the development of better sub-diffraction thin film lens, and spectrally and/or polarization-selective thermal sensors [10,11].

Before the turn of the millennium, the scientific community has recognized the potential of nanomaterials for use in confining and manipulating light in unprecedented ways. Pendry and collaborators were early pioneers in designing electromagnetic metamaterials using patterned sub-wavelength standing cylindrical structures [12,13]. Since then, these studies have inspired numerous researchers to realize nanoscale structural analogs, such as silver nanowires [14], doped silicon nanowires [15], and other metallic nanorods [16,17]. The optical anisotropy in such nanowire arrays was identified as a mechanism for negative refraction [18,19]. As the permittivity for electric field along the nanowire axis approaches infinity, a “canalization” or collimation effect can be observed in aligned nanowire arrays [20,21]. This phenomenon is called “self-collimation,” and Kosaka et al. [22] were the first to notice the self-collimation effect in photonic crystals. Depending on the geometry of the dispersion surface, these photonic crystals could achieve beam focusing or simple waveguide interconnects [23–25]. Sun et al. [26] observed negative refraction in a natural material, graphite, when the optical...
axis is perpendicular to the plane of incidence by utilizing the hyperbolic isofrequency contour or dispersion relation in the ultraviolet region. As it turns out, multi-walled aligned CNT arrays have similar optical properties as coordinate-transformed graphite [1,27]. The effective dielectric behavior is a uniaxial medium with hyperbolic dispersion in the mid- and far-infrared.

The present study investigates aligned CNT arrays or films with a focus on their hyperbolic nature, negative refraction, radiative properties, and the low-loss self-collimation in a broad infrared wavelength range. Here, the anisotropic dielectric functions of multi-walled CNT array are modeled based on those of graphite, with a coordinate transformation and effective homogenization. The effects of the packing density (filling ratio) and alignment factor on the spectral radiative properties are also examined. The CNT arrays with various tilting angles are studied using a transfer matrix formulation to assess the effectiveness and limitations of self-collimation in the CNT films.

2. Energy streamlines in an anisotropic film

Energy streamlines through a homogeneous thin film are formed by tracing the trajectories of Poynting vectors, which depend on the electric and magnetic fields (i.e., $\mathbf{E}$ and $\mathbf{H}$) [28–30]. In the present study, the transfer matrix formulation is applied to a three-layer system shown in Fig. 1, where the top and bottom media (1 and 3) are semi-infinite free space or vacuum with relative permittivity $\varepsilon = 1$ and permeability $\mu = 1$. Medium 2 is made of aligned CNT film tilted in the plane of incidence ($x$–$z$ plane) by an angle $\beta$ with respect to the $z$ axis. The CNT film acts as a uniaxial medium whose optical axis $\hat{c}$ is parallel to the nanotubes. The CNT film is assumed to be nonmagnetic ($\mu = 1$) whose dielectric tensor in the $(x,y,z)$ coordinates is given by [31,32]

$$
\begin{pmatrix}
\varepsilon_{xx} & 0 & \varepsilon_{xz} \\
0 & \varepsilon_{yy} & 0 \\
\varepsilon_{zx} & 0 & \varepsilon_{zz}
\end{pmatrix}
$$

which is constant and independent of $\varepsilon_{zz}$ for a uniaxial slab of thickness $d_2$. Here, $\varepsilon_0$ and $\varepsilon_\infty$ are the dielectric functions of the anisotropic vertically aligned CNT (VACNT) medium for the ordinary wave (electric field $\mathbf{E} \perp \hat{c}$) and extraordinary wave ($\mathbf{E}\parallel \hat{c}$), respectively [4]. The dielectric functions are modeled using the Maxwell–Garnett effective medium theory based on the ordinary ($\varepsilon_{zz}$) and extraordinary ($\varepsilon_{xx}$) dielectric functions of graphite [1,33]. The effective dielectric functions can be expressed as functions of $\varepsilon_{zz}, \varepsilon_{xx}$, the filling ratio $f$, and an alignment factor $\zeta$, viz.

$$
\varepsilon_0 = \varepsilon_0(\varepsilon_{zz}, \varepsilon_{xx}, f, \zeta)
$$

$$
\varepsilon_\infty = \varepsilon_\infty(\varepsilon_{zz}, \varepsilon_{xx}, f, \zeta)
$$

The dielectric functions of graphite for the ordinary wave and extraordinary wave can be found from [34–36]. The filling ratio ($f$) is defined as the volume occupied by a CNT filament per square unit volume [33]. Slight entanglement or random tilting is considered using the quantity $\zeta$, which is not too far from unity (i.e., perfectly aligned case). In practice, the fabricated CNT arrays typically have $\zeta$ values between 0.950 and 0.995 [27]. The effective medium approach using the filling ratio with the misalignment weighting is valid in the infrared, since the electromagnetic wavelength is much greater than the CNT filament diameter and inter-CNT gap spacing [37–39]. Detailed expressions of $\varepsilon_0$ and $\varepsilon_\infty$ can be found from [1].

For a transverse electric (TE) wave incidence, since the electric field is parallel to the $y$ axis, the CNT array behaves as an isotropic medium with a dielectric function $\varepsilon_0$, regardless of the tilting. The interest of this work is for transverse magnetic (TM) waves incident on the CNT film from free space. Since the optical axis is rotated in the plane of incidence, there exist no cross-polarizations. The magnetic field for a TM wave (with an angular frequency $\omega$) is perpendicular to the plane of incidence as given by

$$
H_{yj}(x, z) = (A_j e^{ik_j x} + B_j e^{ik_j x}) e^{j\omega z - j\omega t}, \quad j = 1, 2, 3
$$

Here, $k_j^x$ and $k_j^z$ represent the $z$ component of the wavevector for the forward and backward propagating waves, respectively, and $A_j$ and $B_j$ are the amplitude of the forward and backward waves in the $j$th medium. The $x$ component of the wavevector is the same in all media due to phase matching; that is, $k_j = k_0 \sin \theta_j$, where $\theta_j$ is the angle of incidence and $k_0 = \omega / c_0$ with $c_0$ being the speed of light in vacuum.

The transfer matrix formulation has been used for calculation of radiative properties of thin-film multilayers, such as metal–dielectric multilayers and left-handed photonic crystals [28–30,40]. The transfer matrix for TM waves is given by

$$
D_j = \begin{pmatrix}
1 & 1 \\
Z_j & -Z_j
\end{pmatrix}, \quad j = 1, 2, 3
$$

where the surface impedance is $Z_j = k_j^z / (\omega \varepsilon_0 \varepsilon_j)$ for an isotropic medium, where $\varepsilon_0$ is the vacuum permittivity and $\varepsilon_j$ is the relative permittivity of medium $j$. Since

![Fig. 1. Illustration of a uniaxial slab of thickness $d_2$ made of CNT arrays with a tilting angle $\beta$, for a plane wave incident from the top vacuum (medium 1). All media are assumed to be nonmagnetic and the uniaxial CNT array is modeled as an effective dielectric tensor whose optical axis $\hat{c}$ is along the nanotube.](image-url)
medium 2 is a uniaxial medium, $Z_2$ is modified as follows [30]:

$$ Z_2 = \frac{1}{\aleph_0} \sqrt{\frac{k_0^2 \varepsilon_{zz} - k_x^2}{\varepsilon_0 \varepsilon_{zz}}} $$

(5)

Between the interfaces, the wave propagation in a layer of thickness $d_2$ is described by a propagation matrix defined as

$$ P_2 = \begin{pmatrix} e^{-ik_2 d_2} & 0 \\ 0 & e^{-ik_2 d_2} \end{pmatrix} $$

(6)

where the z-components of the forward and backward wavevector are given by

$$ k_{z,2}^\pm = \frac{-k_{x,2} \pm \sqrt{\varepsilon_0 \varepsilon_{zz} \left( k_0^2 \varepsilon_{zz} - k_x^2 \right)}}{\varepsilon_{zz}} $$

(7)

It can be seen that $k_{z,2} = -k_{x,2}$ for normal incidence ($\theta_i=0^\circ$) or without tilting ($\beta=0^\circ$). The forward and backward field amplitudes for medium $j$ are related to those in the adjacent medium $(j+1)$ as follows:

$$ \begin{pmatrix} A_j \\ B_j \end{pmatrix} = P_j D_j^{-1} D_{j+1} \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix}, \quad j = 1, 2 $$

(8)

Note that $P_j$ is a unit matrix. For the three-layer system, the forward propagating field amplitude in the incident vacuum medium is set to unity, i.e., $A_1 = 1$. Since there exists no backward propagating wave in the semi-infinite medium 3, the field amplitude is $B_3 = 0$. The remaining field amplitudes can be determined from Eq. (8) and are expressed as follows:

$$ A_2 = \begin{pmatrix} M_{2,11} \\ M_{2,21} \end{pmatrix}, \quad A_3 = \frac{1}{M_{2,11}} \begin{pmatrix} M_{2,11} \\ M_{2,21} \end{pmatrix} $$

$$ B_1 = \begin{pmatrix} M_{1,21} \\ M_{1,31} \end{pmatrix} \quad \text{and} \quad B_2 = \begin{pmatrix} M_{2,21} \\ M_{2,11} \end{pmatrix} $$

(9)

where $M_1 = D_1^{-1} D_2$ and $M_2 = P_1 D_2^{-1} D_3$. Once the magnetic fields are determined, the electric field can be determined based on the Maxwell equation:

$$ -i \omega \varepsilon_0 \mathbf{E} = \nabla \times \mathbf{H} $$

(10)

It is straightforward to calculate the electric field in media 1 and 3 by setting the dielectric tensor to a unity tensor. For medium 2, Eq. (10) results in two coupled equations for which $E_x$ and $E_z$ can be solved in terms of $H_y$ [28,30]. The time-averaged Poynting vector is calculated according to $\langle S \rangle = \frac{1}{2} \text{Real} (\mathbf{E} \times \mathbf{H})$. Subsequently, radiative properties such as the reflectance and transmittance can also be obtained, see [30,31] for details.

3. Results and discussion

3.1. VACNT as a hyperbolic metamaterial

The dielectric functions of vertically aligned carbon nanotubes are shown in Fig. 2 with the volume filling ratio $f=0.05$ and the alignment factor $\zeta=0.98$. From Fig. 2 (a), the real part of the dielectric function in the ordinary direction $\varepsilon_{0}^p$ becomes negative at certain frequencies due to the high conductivity along the CNT axis. The CNT filament is composed of graphite layers wrapped in tubes, where electrons can travel freely along the graphene sheets [36,41]. The shaded regions indicate the wavelength ranges where the hyperbolic dispersion exists. The hyperbolic metamaterial (Type I) is defined such that $\varepsilon_{0}^p > 0$ and $\varepsilon_{0}^s < 0$ since the isofrequency contour becomes a hyperbola in the dispersion relation of an anisotropic medium governed by [42,43]

$$ \frac{k_z^2}{\varepsilon_0^s} = \frac{k_x^2}{\varepsilon_0^p} $$

(11)

Matching between VACNT arrays and vacuum, resulting in low reflectance and high broadband absorption [1–4]. Note that the dielectric function in the extraordinary direction $\varepsilon_{0}^s$ becomes negative at certain frequencies due to the high conductivity along the CNT axis. The CNT filament is composed of graphite layers wrapped in tubes, where electrons can travel freely along the graphene sheets [36,41]. The shaded regions indicate the wavelength ranges where the hyperbolic dispersion exists. The hyperbolic metamaterial (Type I) is defined such that $\varepsilon_{0}^p > 0$ and $\varepsilon_{0}^s < 0$ since the isofrequency contour becomes a hyperbola in the dispersion relation of an anisotropic medium governed by [42,43].
negative in the medium. In general, the direction of wavefront propagation, determined by the ratio of the wavevector components \( k_x \) and \( k_z \), remains positive. It should be noted that the hyperbolic dispersion relation of Eq. (11) describes TM waves for VACNT arrays only. For TE waves, no negative refraction occurs since the electric field is perpendicular to the plane of incidence and the isofrequency contour becomes circular in the \( k \)-space, according to the dispersion relation \( k^2 + k_z^2 = \varepsilon_0 k_0^2 \).

As shown in Fig. 2(a), hyperbolic dispersion is identified for VACNT films in two spectral regions: 14.4 \( \mu \text{m} < \lambda < 63.4 \mu\text{m} \) (Region 1) and 90.1 \( \mu\text{m} < \lambda < 133 \mu\text{m} \) (Region 2). While not shown, it is noted that changing the filling ratio or alignment factor does not significantly affect the wavelength bounds in Region 1, but the upper cutoff wavelength in Region 2 can be pushed farther with smaller filling ratio or higher alignment factor. Fig. 2(b) shows the imaginary parts of the dielectric function in both ordinary and extraordinary directions. The double prime symbol is used to indicate the imaginary part of a complex number. The magnitude of \( \varepsilon \) decreases \( \varepsilon \) do not necessarily reduce the transmission as described by Feng [44,45]. Ultimately, transmission increases with decreasing \( \varepsilon \) but often times also with increasing magnitude of \( \varepsilon \).

Filling ratio and alignment factor can also affect transmission and self-collimation. Sparsely packed CNT arrays (i.e., \( f=0.03 \)) is shown to reduce the imaginary part of the dielectric function in both ordinary and extraordinary directions [1]. Meanwhile, increasing the alignment factor (i.e., \( \zeta = 0.99 \)) does not significantly change \( \varepsilon \) but decreases \( \varepsilon_0 \). In the following calculations, unless specified differently, the filling ratio is taken as 0.05 and the alignment factor is taken as 0.98.

The wavevector refraction angle \( (\theta_k) \) and the Poynting vector refraction angle \( (\theta_S) \) are defined respectively as follows [15,23]:

\[
\tan(\theta_k) = \frac{\text{Re}(k_z)}{\text{Re}(k_z)}_{z=0^+} \tag{12}
\]

\[
\tan(\theta_S) = \frac{S_z}{S_z}_{z=0^+} \tag{13}
\]

When the optical axis of the uniaxial anisotropic medium is in the \( z \) direction (i.e., \( \beta = 0^\circ \)), the Poynting vector components are \( S_x = \text{Re}(k_x/\varepsilon) \) and \( S_z = \text{Re}(k_z/\varepsilon_0) \). In this case, when the magnitude of \( \varepsilon \) is very large, the Poynting vector is close to normal or parallel to the CNTs. The refraction angles versus the angle of incidence are shown in Fig. 3 at three wavelengths chosen from three different regions. At \( \lambda = 1 \mu\text{m} \) with elliptic dispersion, both refraction angles are positive for all incidence angles. For \( \lambda = 20 \mu\text{m} \) in Region 1 and 100 \( \mu\text{m} \) in Region 2 hyperbolic bands, \( \theta_S \) is positive, but the Poynting vector or energy refraction angle \( \theta_S \) becomes negative. The negative refraction reflects the characteristics of a hyperbolic material. Furthermore, the magnitude of \( \theta_S \) is very small, suggesting that the energy refraction tends to align with the optical axis or along the carbon nanotubes.

The radiation penetration depth in a CNT array can be expressed as [1,27]

\[
\delta = \frac{1}{2 \text{Im}(k_z)} \tag{14}
\]

where \( k_z = k_0 \sqrt{\varepsilon_0 (1 - \varepsilon E^{-1} \sin^2 \theta)} \) for TM waves. Note that at normal incidence or for TE waves, the penetration depth only depends on the ordinary dielectric function, such that \( \delta = \lambda / \text{Im}(4\pi \sqrt{\varepsilon_0}) \). The penetration depth versus incidence angle at the two wavelengths selected from different hyperbolic bands is shown in Fig. 4. The penetration depth is greater at \( \lambda = 100 \mu\text{m} \) than at \( \lambda = 20 \mu\text{m} \) for all incidence angles. Furthermore, the penetration depth at \( \lambda = 100 \mu\text{m} \) depends little on the incidence angle. This is because the magnitude of \( \varepsilon \) is so large that \( \varepsilon E^{-1} \sin^2 \theta < 1 \) for any \( \theta \).
Fig. 4 also suggests that improving alignment can increase the penetration by several times. Reducing the filling ratio alone from $f=0.10$ to $0.03$ also improves penetration depth (not shown), but it is not as effective as changing the alignment factor from $\zeta=0.95$ to $0.99$. Although the penetration depth in Region 2 ($\lambda=100\ \mu\text{m}$) may be desired, the following section reveals some setbacks at the far-infrared wavelengths, namely higher reflection at the interface between air and the CNT array [1].

3.2. Tilted CNT arrays

In the remaining discussion, the CNT array is tilted at various tilting angles ($\beta$) and compared to the vertically aligned case ($\beta=0^\circ$). The reflectance is calculated at $\lambda=20\ \mu\text{m}$ and $100\ \mu\text{m}$ using the Fresnel reflection coefficient, $R=|r_{12}|^2$, at the interface [30,46]. The results are plotted in Fig. 5 as functions of incidence angle for both polarizations. Since the CNT array is treated as a homogeneous medium with a smooth interface, surface scattering and volume scattering are not considered here. As mentioned earlier, the reflectance for TE waves is independent of the tilting angle. For the VACNT array, the incidence angle for minimum reflectance is described by the Brewster angle for TM waves which increases with the effective refractive index [1]. It is interesting to note that at oblique incidence, the reflectance is minimized at incidence angles close to the tilting angle, especially for large $\beta$, at both wavelengths. In tilted CNT arrays, reflectance is higher at normal incidence for both wavelengths. At $\lambda=20\ \mu\text{m}$, the reflectance is reduced by several order magnitudes when the incidence angle coincides with the tilting angle ($\theta_i=\beta$). Due to loss, the reflectance at $\lambda=100\ \mu\text{m}$ is generally higher than that at $\lambda=20\ \mu\text{m}$. Despite the relatively greater penetration depth at $\lambda=100\ \mu\text{m}$, as discussed in the preceding section, surface reflection loss may be more significant at longer wavelengths.

The penetration depth is plotted in terms of the wavelength as shown in Fig. 6(a) and (b) with $\beta=30^\circ$ and $60^\circ$, respectively. The penetration depth for tilted CNT

![Fig. 6](image-url)
arrays is calculated using Eq. (14) by setting $k_z = k^2_{zz}$ according to Eq. (7). The spectral behavior of the penetration depth is associated with the trend of the extinction coefficient due to the free carriers and far-infrared interband transition in graphite [1]. The penetration depth is the highest at all wavelengths when $\theta_i = \beta$. This trend supports the low-loss transmission characteristic, but now along the coordinate-transformed optical axis. In addition, the magnitudes of the penetration depth differ between the two tilting angles. In more heavily tilted CNT, the average penetration depth is reduced almost by half. This is explained by the longer length of heavily tilted CNT filaments at a given depth from the surface. The minimal reflectance and maximal penetration depth when the incidence angle is coincident with the tilting angle demonstrate high transmission in CNT arrays along the optical axis. In the following, the all-angle self-collimation is demonstrated for CNT arrays with or without tilting.

Fig. 7 shows the isofrequency contour plots in the $k_x$–$k_z$ space at $\lambda = 20 \mu$m for various tilting angles. Note that the wavevectors are divided by $k_0$. Taking Fig. 7(a) for VACNT as an example, the unit circle is the isofrequency curve for wavevector $k_i$ of the incident wave divided by $k_0$. The incident Poynting vector ($\mathbf{S}_i$) is parallel to the wavevector and perpendicular to the isofrequency curve. For the hyperbolic dispersion described by Eq. (11), the solutions of the real part of $k_z$ are the upper and lower hyperbolic branches. Due to loss, the solution also results in a positive imaginary part of $k_z$, as indicated by the dotted curve. The resultant Poynting vector ($\mathbf{S}_{CNT}$) is perpendicular to the isofrequency curve for $\text{Re}(k_z/k_0)$ [15,43]. The wavevector in the CNT, $k_{CNT}$, begins from the origin and ends at the hyperbolic dispersion curve with the condition that $k_z = k_0 \sin \theta_i$. Furthermore, the requirement for the $z$-component of $\mathbf{S}_{CNT}$ to be positive necessitates that the upper branch of the isofrequency curve corresponds to waves incident from vacuum to the CNT. As indicated in Fig. 7(a), the Poynting vector points slightly leftward, suggesting negative energy refraction. Since the isofrequency contour is rather flat, the Poynting vector is nearly parallel to the $z$-axis or the nanotubes. This is in agreement with the energy refraction angles seen in Fig. 3.

The remaining isofrequency contours shown in Fig. 7(b, c, and d) demonstrate the refraction and self-collimation.

**Fig. 7.** Normalized hyperbolic isofrequency contours at $\lambda=20 \mu$m for (a) VACNT, and tilted CNT arrays at (b) $\beta=30^\circ$, (c) $\beta=60^\circ$, and (d) $\beta=70^\circ$. 
for tilted CNT arrays at $\beta = 30^\circ$, 60$^\circ$, and 70$^\circ$, respectively. Since the optical axis forms an angle $\beta$ with respect to the surface normal, the dispersion relation of Eq. (11) is modified to

$$
\frac{(k_z \cos \beta + k_x \sin \beta)^2}{\varepsilon_0} + \frac{(k_z \sin \beta - k_x \cos \beta)^2}{\varepsilon_E} = k_0^2 \tag{15}
$$

In Fig. 7(b), the Re($k_z/k_0$) line is slanted along the tilting angle, resulting in all Poynting vectors pointing toward approximately $\theta_i = 30^\circ$. For $\beta = 60^\circ$, as shown in Fig. 7(c), the self-collimation mechanism remains the same as the Poynting vectors point toward the tilting angle. However, toward large negative $k_x$, the Re($k_z/k_0$) curve is slightly bent. The situation is more prominent in the case for $\beta = 70^\circ$ as shown in Fig. 7(d), where the dispersion curve undulates toward negative $k_x$. Self-collimation breaks down when radiation is incident from the opposite side (i.e., $\theta_i < 0^\circ$) for CNT arrays with large tilting angles. The vertical dotted lines in Fig. 7(b–d) are for $k_z/k_0 = \sin \beta$ or $\theta_i = \beta$. It can be seen that Im($k_z$) is near its minimum or loss is minimized at $\theta_i = \beta$ as mentioned previously.

For isofrequency contours at $\lambda = 100 \mu m$ (not shown), Im($k_z/k_0$) lines are greater in magnitude. Interestingly, for heavily tilted CNT ($\beta = 70^\circ$) at $\lambda = 100 \mu m$, the undulation of the Re($k_z/k_0$) line at negative $k_x$ is not as evident. This signifies that self-collimation is more inclusive at $\lambda = 100 \mu m$, but the waves propagating in CNT may suffer from more surface and internal losses. Even though the absolute penetration depth is slightly greater, when normalized to the wavelength, $\delta/\lambda$ generally decreases toward longer wavelengths.

The refraction angles for tilting CNT angles are shown in Fig. 8 for $\lambda = 20 \mu m$ along with that for VACNT for comparison. The wavevector refraction angle has the same sign as the incidence angle, as shown in Fig. 8(a). The energy refraction angles shown in Fig. 8(b) can help better interpret the observations made above for Fig. 7. It should be noted that the energy refraction angle is determined from Eq. (13), where the Poynting vector components for tilted CNTs are given by

$$
S_x = \text{Re} \left( \frac{k_x \varepsilon_{xx} + k_z \varepsilon_{xz}}{\varepsilon_0 \varepsilon_E} \right) \tag{16a}
$$

$$
S_z = \text{Re} \left( \frac{k_z \varepsilon_{xx} + k_x \varepsilon_{zz}}{\varepsilon_0 \varepsilon_E} \right) \tag{16b}
$$

Clearly, $\theta_3$ largely follows the CNT tilting angle for any incidence when $\beta < 60^\circ$. With heavily tilted CNTs, especially when $\beta = 70^\circ$ and for negative $\theta_i$, $\theta_3$ deviates significantly from the tilting angle. In the case with $\lambda = 100 \mu m$, $\theta_3$ is almost constant for all incidence angles (although not shown here), indicating near-perfect self-collimation. In Fig. 8(b), the data points indicate the Poynting vector refraction angle calculated using transfer matrix formulation for a thin film of $d_2 = 10 \mu m$. The agreement suggests that surface refraction dominates the Poynting vector direction in the CNT film, although interference effects can modify it slightly. Further discussion on the radiative properties and self-collimation throughout the vertical or tilted CNT thin films is provided in the following section.

### 3.3. CNT thin films

Energy streamlines are obtained for a thin CNT film with $d_2 = 10 \mu m$ for various incidence angles and with different tilting angles, as shown in Fig. 9. The energy streamlines are traces of the Poynting vector trajectories for plane waves with $\lambda = 20 \mu m$ incident from vacuum onto the film. All the streamlines are assumed to originate at $x = 0$ and $z = -0.5d_2$. Due to the low reflection at the interfaces between vacuum and CNT film, the streamlines are nearly straight in each region. As shown in Fig. 9(a) for VACNT, the streamlines are symmetric with respect to $x = 0$. The upper curves are for $\theta_i > 0^\circ$ and the lower curves are for $\theta_i < 0^\circ$. As seen in Fig. 9(a), the energy streamlines show slight negative bending, but largely travel along the optical axis of the CNT array. In the exiting medium, $z > d_2$, the streamlines or rays continue to spread away from the “source.” Essentially, from $z = 0$ to $z = d_2$, the beam is...
collimated and the shape of the incoming rays from the incident interface to the outgoing interface is conserved. In Fig. 9(b) and (c), with CNT tilting angles of $\beta = 30^\circ$ and $\beta = 60^\circ$, respectively, the self-collimation characteristic through the thin film is also conserved. The rays inside the CNT always travel along its optical axis. However, at $\beta = 70^\circ$, as shown in Fig. 9(d), self-collimation breaks down at negative incident angles, as evident from the bottom three streamlines. For the energy streamlines at $\lambda = 100 \, \mu m$ (not shown), near-perfect self-collimation for tilting angles up to $70^\circ$ is still observed.

Fig. 10 shows the transmittance ($T$) through a 10-\(\mu\)m-thick CNT array as a function of the incidence angle. As shown in Fig. 10(a) for $\lambda = 20 \, \mu m$, the transmission is symmetric and reaches a peak at normal incidence for TE waves regardless of tilting or for TM wave when $\beta = 0^\circ$ (VACNT). For tilted CNT thin films, the transmission is favored when the incidence angle is around the tilting angle. For example, in the CNT thin film with $\beta = 60^\circ$, the transmittance at negative incidence angles is nearly zero.

For $\lambda = 100 \, \mu m$, the transmittance is generally higher and insensitive to incidence angle for $-60^\circ \leq \theta_i \leq 60^\circ$ and $0^\circ \leq \beta \leq 30^\circ$, as shown in Fig. 10(b). For VACNT, the transmittance plateaus in a broad incidence angle range $|\theta_i| < 60^\circ$. Moreover, in tilted CNT thin films, the transmittance remains high even with negative incidence angles until $\beta$ exceeds $60^\circ$. The different transmission trends between the two wavelengths have implications on design of radiation manipulators in the far infrared. In Region 1 ($\lambda = 20 \, \mu m$), the tilted CNT array becomes a spatial filter or polarizer especially at larger tilting angles. In the far-infrared hyperbolic band, the CNT film serves as a lossless coupler, with possible application in fiber-optic-like thermal radiation and terahertz wave detection.

4. Conclusions

Two broad hyperbolic dispersion bands in the far-infrared region are predicted based on the effective dielectric tensor of VACNT. With the hyperbolic region, the energy

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**Fig. 9.** Energy streamlines for various incidence angles at $\lambda = 20 \, \mu m$ for a CNT thin film of $d_2 = 10 \, \mu m$ with different tilting angles: (a) $\beta = 0^\circ$ (VACNT), (b) $\beta = 30^\circ$, (c) $\beta = 60^\circ$, and (d) $\beta = 70^\circ$. Curves above $\theta_i = 0$ are for positive $\theta_i$ and below $\theta_i = 0$ are for negative $\theta_i$. 

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The tilting angle, especially at surface reflectance is reduced and the penetration depth is further promote this mechanism. For tilted CNT arrays, hyperbolic region. Improving the alignment factor can in infrared at $\lambda_{collimation}$ in CNT arrays are demonstrated in the far infrared at $\lambda = 20 \mu m$ and $100 \mu m$, selected from each hyperbolic region. Improving the alignment factor can further promote this mechanism. For tilted CNT arrays, surface reflectance is reduced and the penetration depth is increased when the incidence angle is nearly coincident with the tilting angle, especially at $\lambda = 20 \mu m$. The refraction and self-collimation of light through the tilted CNT array is strong for most tilting angles. For heavily tilted CNT, however, self-collimation may break down when radiation is incident at negative angles. The transmittance of the CNT thin film is shown to distinguish the two far-infrared wavelength regions for their selectivity in angular filtering. This study suggests the possibility of using VACNT and tilted CNT arrays as subwavelength devices in the far infrared such as beam collimators, couplers, and spatial filters or polarizers.

**Fig. 10.** Transmittance through a CNT thin film of thickness $d_2=10 \mu m$ with various tilting angles at (a) $\lambda=20 \mu m$ and (b) $\lambda=100 \mu m$.

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