Sequential Choice with Incomplete Preferences*

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Abstract

We study the outcome of a sequential choice procedure based on a potentially incomplete preference relation. A decision maker evaluates alternatives in a list and iteratively updates her choice by comparing the status quo to the next alternative. She favors the status quo whenever the two alternatives are incomparable according to her underlying preference. Developing a revealed preference approach, we characterize all choice functions that can arise from such a procedure, as well as all possible preferences that can rationalize given choices.

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1 Introduction

We consider a decision maker (henceforth DM) who is faced with a finite list of alternatives, and makes choices following a sequential procedure. At any time along the list, the DM compares her current choice to the next alternative, and chooses either one based on an asymmetric binary relation that represents her underlying preference. In the voting literature, this choice procedure is closely related to "amendment voting." However, much of the theoretical analysis for this model has focused on complete preferences, i.e. tournaments (see e.g. Miller [21]).

In many real-world situations, the DM's preference may be *incomplete*. For example, the DM may not be able to compare certain pairs of alternatives due to "just noticeable differences", in the sense of Luce [16]. It is also possible that her preference depends on multiple factors that are difficult to aggregate, as in Dubra et al. [8]. These possibilities motivate us to generalize the classic sequential choice model described above, by allowing the DM to have an incomplete preference. At the same time, we do require the preference to be transitive, so as to capture the aforementioned types of incompleteness that we have in mind.

To define the sequential choice procedure under an incomplete preference, we must specify how the DM chooses between two alternatives that are incomparable. Following the findings in the behavioral economics literature (e.g. Kahneman and Tversky [14]), we assume in our main model that the DM suffers from status quo bias. Thus, she favors her current choice to the next alternative whenever they cannot be compared. While most of our paper concerns individual choice, we mention here that the choice procedure so defined also arises in a social choice setting. Specifically, suppose a group of individuals evaluate a list of alternatives in order, and uses the *unanimous rule* to decide whether or not to choose a new alternative over the status quo. Then, it is as if the group follows our choice procedure with a preference relation that is the intersection of the individual preferences. Note that as long as individual preferences are transitive, the group preference is also transitive but is generally incomplete.

We are primarily interested in the following questions: given observed choices from a collection of lists, can they be rationalized by the sequential choice procedure? And if so, how much can we say about the DM's underlying preference? One difficulty turns out to be that, with limited choice data, often times multiple incomplete preferences lead to the same choice function. Despite this multiplicity, we show that among possible preferences that rationalize given choices, there is a minimum one with respect to the containment of binary relations. This minimum preference can be interpreted as what we can "robustly infer" from observed choices.

We derive the minimum preference using a novel revealed preference method. Say that an alternative y is revealed-preferred to another alternative x, if in a list that begins with x, either y is chosen or moving the position of y can affect the choice. When the DM's true preference is transitive, either of these conditions implies that the revealed preference is contained in the true preference. This establishes the revealed preference as the minimum *possible* underlying preference. The core of our analysis is to show that if a choice function is generated by any preference, then its revealed preference is rich enough to generate the same choices. In fact, our analysis yields a simple characterization of all preferences that are consistent with given choices: such a preference must contain the revealed preference, and must satisfy the additional property that for any two comparable alternatives, swapping their positions at the beginning of any list does not affect the choice. As a corollary, a given choice function is rationalizable if and only if its revealed preference is transitive and satisfies the above behavioral property.

There have already been many models of sequential choice in the literature. To have a more meaningful discussion of the connections, we will defer the literature review to after the model and results, which we now turn to.

2 Model

2.1 Choice Procedure

Let S denote a finite set of N alternatives. A *list* over S is an ordered sequence $\pi = x_1 x_2 \dots x_m$ which enumerates a subset of m $(1 \leq m \leq N)$ alternatives in S without repetition. Throughout, we will write lists in this way, without commas or brackets, to distinguish them from sets. The set of all lists over S is denoted by Π.

Given an asymmetric and transitive binary relation P on S, i.e. a *strict partial order*, we define a choice function c^P on Π in the following recursive manner:

$$c^P(x_1) = x_1;$$

$$c^P(x_1 x_2 \dots x_k) = \begin{cases} x_k, & \text{if } x_k \text{ is } P\text{-preferred to } c^P(x_1 x_2 \dots x_{k-1}); \\ c^P(x_1 x_2 \dots x_{k-1}), & \text{otherwise.} \end{cases} \quad \forall 2 \leq k \leq N. \tag{1}$$

To interpret, imagine a DM who faces a list of alternatives $x_1 x_2 \dots x_m$, which are sequentially presented to her in this order. As she moves along this list, she always keeps one past alternative in mind and compares it to the next alternative in the list. We assume that the DM suffers from *status quo bias*, so that she replaces the current alternative in mind with the new one if and only if the latter is strictly preferred according to P. At the end of the list, the resulting alternative $c^P(x_1 x_2 \dots x_m)$ is the DM's choice from this list, based on the preference P.[1]

A pair of simple examples will help illustrate. In these examples we focus on the choices from lists of length N, i.e. those that enumerate all alternatives in S. First consider $N = 3$ alternatives x, y, z, and suppose the DM's preference P is such that yPx, zPx, but y, z are incomparable. According to the above definition, her choices

[1] We also study a "recency-biased" DM in Appendix B, who chooses the new alternative x_k whenever it is not comparable to $c(x_1 x_2 \dots x_{k-1})$. One reason we focus on status quo bias in our main model is that such a DM is guaranteed to choose a P-maximal alternative from any list (Lemma 1 below). Thus she cannot gain by deviating from the choice procedure (1). This Pareto property does not hold under recency bias.

are $c^P(xyz) = c^P(yxz) = c^P(yzx) = y$ whereas $c^P(xzy) = c^P(zxy) = c^P(zyx) = z$. We see here that with an incomplete preference, different orderings of the same set of alternatives can result in different choices. This example also has the property that either of the two P-maximal alternatives y and z is chosen, depending on which one occurs earlier in the list.

In general, however, the choice depends more subtly on the list. For example, consider $N = 4$ alternatives x, y, z, w and a preference P such that yPx, wPz and all other pairs are incomparable. In this case the DM chooses the alternative w from the list $zxyw$ despite y appearing earlier, and she chooses y from the list $xzwy$ despite w appearing earlier. Such an example captures situations in which the DM evaluates alternatives based on multiple factors, and alternatives that appear earlier in the list "anchor" which factor the DM pays attention to later on.

A preliminary result is that as long as P is a strict partial order, a DM following the above choice procedure always chooses a P-maximal alternative from each list. This is a desirable property because it guarantees that even if the DM is sophisticated and forward-looking, she cannot profitably deviate from the recursive and myopic choice procedure we have assumed.

Lemma 1. *Suppose the DM follows the choice procedure* (1) *with a strict partial order* P*. Then for any list* π*, no alternative in* π *is* P*-preferred to* $c^P(\pi)$*.*

Proof. Suppose for contradiction that in some list $\pi = x_1 \dots x_m$, an alternative $y \in \pi$ is P-preferred to $c^P(\pi) = x$. Observe that according to the choice procedure, $c^P(x_1 \dots x_k)$ is either equal to or P-preferred to $c^P(x_1 \dots x_{k-1})$, for each $k \leq m$. Thus by transitivity, $x = c^P(x_1 \dots x_m)$ is P-preferred to every $c^P(x_1 \dots x_{k-1})$. Let us choose the subscript k so that $y = x_k$. Then y is P-preferred to x, which is in turn P-preferred to $c^P(x_1 \dots x_{k-1})$. By transitivity, this implies $y = c^P(x_1 \dots x_k)$. But then $x = c^P(x_1 \dots x_m)$ must be P-preferred to $y = c^P(x_1 \dots x_k)$, contradicting the assumption that P is asymmetric. □

2.2 Domain Assumptions

Lemma 1 tells us that by observing the DM's choices from different lists, we can recover partial information about her preference P, assuming that she follows the choice procedure described in (1). Naturally, we may ask the following questions: 1) Given observed choices from different lists, does there exist a preference P that rationalizes these choices? 2) When such a preference P exists, to what extent can we identify it?

To formalize these two exercises of rationalization and identification, we introduce a little more notation. Let $\mathcal{D} \subset \Pi$ be a non-empty collection of lists, which represents the domain of observed choices. The primitive of our study is a choice function $c : \mathcal{D} \to S$ satisfying $c(\pi) \in \pi$ for every $\pi \in \mathcal{D}$. We then define:

Definition 1. *Say that a strict partial order P* rationalizes *the choice function c, if $c = c^P$ on the domain $\mathcal{D}$ of c.*

Given c, we seek to understand the set of preferences P that rationalizes c. When $\mathcal{D} = \Pi$ is the domain of all lists, this question was studied by Guney [11], who showed that P exists if and only if the choice function has a recursive structure, and satisfies the *primacy axiom* which requires a chosen alternative to remain chosen when it is moved to an earlier position in the list. For that domain P must be unique, as the DM's preference can be fully identified from her choices in lists that only contain two alternatives.

In this paper, we consider a more general domain $\mathcal{D} \subset \Pi$. The motivation is that in practice, if the total number N of alternatives is large, it may not be feasible to observe the DM's choices from all lists (whose number, on the order of $N!$, is very large). In addition, even if we could provide the DM with all lists in an experimental setting, it might happen that the DM has limited attention and does not actually go through all the alternatives in each list. In that case what we observe is effectively a choice function defined on a subset of (shorter) lists.

Theoretically, a general domain $\mathcal{D} \subset \Pi$ calls for a different analysis from that of Guney [11]. One particular difficulty is that the same choice function $c : \mathcal{D} \to \Pi$ can sometimes be rationalized by multiple preferences P — for example, if $\mathcal{D}$ consists of all lists with length N, then the constant choice function $c(\pi) = x^*$ is rationalized by any P that prefers x^* to all remaining alternatives (whose ranking can be arbitrary). Our insight is that, there often exists a "minimum" preference P (with respect to containment of binary relations) that rationalizes the choice function — in the preceding example this P makes all alternatives besides x^* incomparable. When such a minimum preference exists, we can think of it as the most amount of information that can be inferred with certainty about the DM's true underlying preference.

It turns out that, as we show in the next section, the minimum preference can be constructed as a revealed preference from the choice function. This revealed preference approach is effective for all domains $\mathcal{D} \subset \Pi$ that satisfy the following assumption:

Assumption 1. *If $\pi \in \mathcal{D}$ and $\tilde{\pi}$ is a permutation of π, then $\tilde{\pi} \in \mathcal{D}$ as well.*

In words, if the DM's choice from a certain ordering of a subset of alternatives is observed, then we in fact observe her choices from all orderings of these alternatives. This is a natural assumption because, after all, we are interested in how an incomplete preference can lead to different choices from the same set of alternatives, depending on their ordering. We maintain Assumption 1 in all our results.

However, our results are strongest under an additional assumption on the domain. To state it, for any list π, we denote by π_{-1} the list that removes the last alternative in π. Then we require

Assumption 2. *If $\pi \in \mathcal{D}$ and z is an alternative not in π, then either $\pi z \in \mathcal{D}$, or $\pi_{-1} z \in \mathcal{D}$.*[2]

The use of this assumption will be explained in the next section. For now let us point out two kinds of domains that satisfy both Assumptions 1 and 2. The first kind is a domain $\mathcal{D}$ that consists of all lists with a prescribed length. The second kind is a domain $\mathcal{D}$ that consists of all lists containing a given subset of alternatives.

[2] Here and later, for any list π and any alternative $z \notin \pi$, the expression πz denotes the list that appends z to the end of π. Similarly for $z\pi$.

More complex domains can be accommodated by taking unions of these simple ones; if $\mathcal{D}_1, \mathcal{D}_2 \subset \Pi$ both satisfy Assumptions 1 and 2, then so does $\mathcal{D}_1 \cup \mathcal{D}_2$.

3 Results

As mentioned, we will take a revealed preference approach. Given any choice function $c : \mathcal{D} \to S$, we define a binary relation P^* such that yP^*x if either of the following two conditions holds:

Condition I: $c(\pi) = y$ for some list $\pi \in \mathcal{D}$ that begins with x;

Condition II: $c(\pi) \neq c(\pi')$ for some list $\pi \in \mathcal{D}$ that begins with x and another list $\pi' \in \mathcal{D}$ obtained from π by moving the alternative y to a different position after x.

Any choice function c determines a *revealed preference* $P^*(c)$ through this pair of conditions, and we will often simply write P^* when there is no confusion. The next lemma shows that when the choice function arises from the choice procedure (1), the revealed preference is part of the true underlying preference.

Lemma 2. *If $c = c^P$ for some strict partial order P, then $P^*(c) \subset P$.*

Proof. It suffices to show that if y is not P-preferred to x, then y is not revealed-preferred to x. From transitivity, we see that Condition I can never be satisfied. In fact, when faced with a list that begins with x, the DM never chooses y at any time along the list. Thus the position of y does not affect the final choice, and Condition II will not be satisfied either. $\square$

To see why Condition II is important, consider 4 alternatives x, y, z, w and a preference P such that $wPyPx, zPx$ and all other pairs are incomparable. A DM following our choice procedure chooses w from the list $xyzw$ but z from the list $xzyw$. Only by Condition II (and not Condition I) can we infer that the DM prefers y to x, which is essential to explain the above choices.

Let $\overline{P^*}$ be the transitive closure of the binary relation P^*. Our next result shows that $\overline{P^*}$ is the most we can infer about the true preference from observed choices.

Specifically, if the choice function arises from the choice procedure (1) for some strict partial order P, then $\overline{P^*}$ is a strict partial order that is contained in P, and $\overline{P^*}$ also rationalizes the choice function. In this sense, $\overline{P^*}$ is the *minimum rationalizing preference.*[3]

Theorem 1. *Fix a choice function $c : \mathcal{D} \to S$, where the domain $\mathcal{D}$ satisfies Assumption 1. Define P^* be the revealed preference of c, and $\overline{P^*}$ be its transitive closure. Then the following conditions are equivalent:*

(i) $c = c^P$ *for some strict partial order P;*

(ii) $\overline{P^*}$ *is a strict partial order, and $c(xyA) = c(yxA)$ for any pair of alternatives $y\overline{P^*}x$ and any list A such that $xyA \in \mathcal{D}$;*

(iii) $\overline{P^*}$ *is a strict partial order, and $c = c^{\overline{P^*}}$.*

Proof. It is immediate that (iii) implies (i). That (i) implies (ii) is also relatively straightforward. Indeed, by Lemma 2, if $c = c^P$ then $P^* \subset P$. Because P is transitive, the transitive closure $\overline{P^*}$ of P^* is also contained in P, which implies that $\overline{P^*}$ is a strict partial order. Moreover, $c(xyA) = c(yxA)$ holds for any $y\overline{P^*}x$ because the choice procedure (1) has the property that $c^P(xyA) = c^P(yxA)$ whenever yPx.

The implication (ii) $\implies$ (iii) is more difficult. Fix any list $\pi \in \mathcal{D}$. Let x_1 denote the first alternative in π, and for $k \geq 1$ let x_{k+1} denote the first alternative after x_k in this list that is $\overline{P^*}$-preferred to x_k. In this way we can write $\pi = x_1A_1x_2A_2\ldots x_nA_n$, so that $c^{\overline{P^*}}(\pi) = x_n$. Because any alternative y in A_1 is not $\overline{P^*}$-preferred (thus not P^*-preferred) to x_1, Assumption 1 and Condition II imply that we can move y to the end of the list without changing the DM's choice or going out of the domain $\mathcal{D}$. Applying this observation to every alternative in A_1, we obtain

$$c(x_1A_1x_2A_2\ldots x_nA_n) = c(x_1x_2A_2\ldots x_nA_nA_1). \tag{2}$$

[3]The existence of a minimum rationalizing preference relies on Assumption 1. For a counterexample, consider 7 alternatives x, y, z, y_1, z_1, u, v, and suppose we only observe the choices from two lists: $c(xyzy_1z_1vu) = u$ and $c(xzyz_1y_1uv) = v$. One preference that rationalizes these choices consists of $uPyPx$ and $vPzPx$, with all other pairs of alternatives incomparable. Similarly, the preference Q with uQy_1Qx and vQz_1Qx also leads to these choices. Nevertheless, the intersection of P and Q only has u and v preferred to x, which is not sufficient to explain the observations.

But $x_2\overline{P^*}x_1$, so by condition (ii) in the theorem we have

$$c(x_1x_2A_2\ldots x_nA_nA_1) = c(x_2x_1A_2\ldots x_nA_nA_1) = c(x_2A_2\ldots x_nA_nx_1A_1), \qquad (3)$$

where the second equality follows because $\neg x_1P^*x_2$ and Condition II. From (2) and (3) we obtain $c(x_1A_1x_2A_2\ldots x_nA_n) = c(x_2A_2\ldots x_nA_nx_1A_1)$. Repeating this argument, we can eventually derive

$$c(x_1A_1x_2A_2\ldots x_nA_n) = c(x_nA_nx_1A_1\ldots x_{n-1}A_{n-1}). \qquad (4)$$

Because x_n is $\overline{P^*}$-preferred to each x_k and no alternative in A_k is $\overline{P^*}$-preferred to x_k, transitivity and asymmetry of $\overline{P^*}$ imply that no alternative in the list $x_nA_nx_1A_1\ldots x_{n-1}A_{n-1}$ is $\overline{P^*}$-preferred to x_n. Thus no alternative in this list is P^*-preferred to x_n, which implies by Condition I that the choice $c(x_nA_nx_1A_1\ldots x_{n-1}A_{n-1})$ can only be x_n. By (4), $c(\pi) = x_n = c^{\overline{P^*}}(\pi)$ as we desire to show. □

Theorem 1 shows what we can robustly infer about the DM's preference from observed choices. In fact, we have a complete characterization of possible preferences that rationalize a given choice function:

Lemma 3. *Fix a choice function $c : \mathcal{D} \to S$, where the domain $\mathcal{D}$ satisfies Assumption 1. Define P^* be the revealed preference of c, and let P be any strict partial order. Then $c = c^P$ if and only if $P^* \subset P$ and $c(xyA) = c(yxA)$ whenever yPx.*

Proof. The "only if" direction is simple, whereas the "if" direction resembles the implication (ii) $\implies$ (iii) in Theorem 1. Specifically, we can write any list $\pi \in \mathcal{D}$ as $x_1A_1x_2A_2\ldots x_nA_n$ where x_{k+1} is the first alternative after x_k in this list that is P-preferred to x_k. The rest of the proof is essentially the same as before, with the preference P taking the place of $\overline{P^*}$. □

The condition "$c(xyA) = c(yxA)$ whenever yPx" can be written as $P \subset \hat{P}$, where $\hat{P}$ is a binary relation derived from the choice function c, such that $y\hat{P}x$ if $c(xyA) =$

$c(yxA)$ for all A. Thus, any strict partial order P that rationalizes c is "bounded between" P^* and $\hat{P}$, as described in the following result.

Theorem 2. *Fix a choice function $c : \mathcal{D} \to S$, where the domain $\mathcal{D}$ satisfies Assumption 1. Define P^* and $\hat{P}$ according to the above discussions. Then $c = c^P$ for a strict partial order P if and only if $P^* \subset P \subset \hat{P}$.*

To better understand this theorem, let us recall an earlier example where $\mathcal{D}$ consists of all permutations of S, and the choice function c is constant: $c(\pi) = x^*$ for every list π with length N. Then x^* is revealed-preferred to all other alternatives, which are incomparable to each other under P^*. On the other hand, $\hat{P}$ by definition is the complete symmetric binary relation such that $y\hat{P}x$ holds for all x, y. Thus, just as we expect, Theorem 2 tells us that the choice function c can be rationalized by *any* strict partial order P that contains P^*, i.e. any P that prefers x^* to all other alternatives. Note that these possible underlying preferences have a minimum element P^*, but do not have a maximum element.[4]

Either Theorem 1 or Theorem 2 answers the question of when there exists *some* preference that rationalizes the choice function c. A necessary and sufficient condition is that the revealed preference P^* is acyclic, and its transitive closure $\overline{P^*}$ satisfies the condition $c(xyA) = c(yxA)$ whenever $y\overline{P^*}x$. However, due to having to take the transitive closure, the last condition "$c(xyA) = c(yxA)$ whenever $y\overline{P^*}x$" is not very easy to verify.

To address this caveat, below we assume that the domain $\mathcal{D}$ also satisfies Assumption 2. In this case it turns out that the revealed preference P^* is itself a strict partial order (without having to take the transitive closure). This leads to the following strengthening of Theorem 1:

[4]More generally, we can show that a maximum rationalizing preference exists only when there is a unique rationalizing preference, $\overline{P^*}$. Indeed, if Q is a maximum rationalizing preference that strictly contains $\overline{P^*}$, then there exists x, y such that yQx but $\neg y\overline{P^*}x$. Thus $y\hat{P}x$, which implies $x\hat{P}y$ by the construction of $\hat{P}$. Since $\overline{P^*}$ is a strict partial order and $\neg y\overline{P^*}x$, the binary relation $\overline{P^*} \cup (x, y)$ is seen to be acyclic. Taking its transitive closure then leads to a preference P bounded between $\overline{P^*}$ and $\hat{P}$. By Theorem 2, this P also rationalizes the choice function. But by construction xPy, and Q is maximum, so we must also have xQy. This contradicts the asymmetry of Q as we started out assuming yQx.

Theorem 3. *Fix a choice function* $c : \mathcal{D} \to S$*, where the domain* $\mathcal{D}$ *satisfies both Assumptions 1 and 2. Define* P^* *be the revealed preference of* c*. Then the following conditions are equivalent:*

(i) $c = c^P$ *for some strict partial order* P*;*

(ii) P^* *is a strict partial order, and* $c(xyA) = c(yxA)$ *for any pair of alternatives* yP^*x *and any list* A *such that* $xyA \in \mathcal{D}$*;*

(iii) P^* *is a strict partial order, and* $c = c^{P^*}$*.*

Relative to Theorem 1, here we just need to show that if $\mathcal{D}$ satisfies both Assumptions 1 and 2, then the revealed preference P^* derived from $c = c^P$ is transitive. The proof of this is technical and relegated to the appendix. Let us however explain why Assumption 2 is important for this conclusion. Indeed, suppose we have three alternatives x, y, z satisfying yP^*x and zP^*y. This means, by definition, that there exists a list $\pi \in \mathcal{D}$ which begins with x, and in which either y is chosen or y's position affects the choice. Similarly, there exists another list $\pi' \in \mathcal{D}$ that "witnesses" the revealed preference zP^*y. But without Assumption 2 or a similar assumption, the domain $\mathcal{D}$ need not contain *any* list in which the alternatives x and z simultaneously appear. If that happens then z will not be revealed-preferred to x, violating the desired transitivity of P^*. Assumption 2 ensures that the domain is rich enough for the revealed preference to be transitive.

Given the equivalence of (i) and (ii) in Theorem 3, we obtain a set of necessary and sufficient conditions on choice behavior for it to be consistent with the choice procedure (1). Specifically, by the definition of P^*, we can rewrite the condition "$c(xyA) = c(yxA)$ whenever yP^*x" as the following: *If there exists a list starting with* x *such that* y *is chosen or* y*'s position affects the choice, then in every list that begins with* xy*, swapping these first two alternatives does not affect the choice.* Similarly, the transitivity and asymmetry of P^* can also be explicitly stated as conditions on the choice function, without referring to the revealed preference P^*.

Admittedly, such behavioral axioms as described above are still not the simplest. But as far as we are aware, more natural axioms often turn out to be necessary but not sufficient. For example, one may conjecture that $c = c^P$ for some strict partial order P if and only if the choice function satisfies *primacy* and *independence of later alternatives*. Primacy means that if $c(\pi)$ is moved to an earlier position in the list π, then it continues to be chosen. Independence of later alternatives means that if the alternatives after $c(\pi)$ are permuted, then $c(\pi)$ continues to be chosen.[5]

While these properties are satisfied by $c = c^P$, they are not enough to guarantee $c = c^P$ even in the special case where the domain $\mathcal{D}$ consists of all permutations of S.[6] More general domains $\mathcal{D}$ complicate things further, since the above two axioms do not constrain the relation between choices from longer lists and those from shorter lists. It is an interesting but perhaps challenging question for future work to obtain a more behavioral characterization of which choice functions are rationalizable.

In Appendix A, we show that Theorem 3 extends to cases where the true preference P is restricted to be an *interval-order* or *semi-order*.[7] To be specific, we prove that $c = c^P$ for some interval-order/semi-order if and only if the revealed preference P^* is an interval-order/semi-order and $c = c^{P^*}$. This extension further demonstrates the usefulness of the revealed preference approach.

4 Related Literature

Following the paper of Arrow [3], there has been a large literature relating a choice *correspondence* to the underlying preference relation. Notably, Jamison and Lau [13] and

[5] We thank an anonymous referee for suggesting these axioms.

[6] An example involves $N = 4$ alternatives x, y, z, w, where the choice function $c(\pi)$ is defined as follows. If π begins with y, z, w then $c(\pi)$ is this first alternative. If π begins with xy, then $c(\pi) = y$. Finally if π begins with xz or xw, then $c(\pi) = w$. Such a choice function does satisfy primacy and independence of later alternatives. However it is not rationalizable because z is revealed-preferred to x as $c(xzyw) \neq c(xyzw)$, but the "swapping condition" is violated as $c(xzyw) \neq c(zxyw)$.

[7] An interval-order P is a strict partial order that satisfies $PIP \subset P$, where I is the binary relation of incomparables associated with P, and PIP denotes the concatenation of these binary relations. Any such preference admits an "interval representation": yPx if and only if $U(y) > U(x) + b(x)$, for some functions $U(\cdot)$ and $b(\cdot) > 0$. If in addition $PPI \subset P$, then one can take $b(x)$ to be the constant 1 and obtain a semi-order. See Beja and Gilboa [4] and Fishburn [9], Section 2.4.

Fishburn [10] establish necessary and sufficient conditions for a choice correspondence $c : 2^S \backslash \{\emptyset\} \to S$ to be given by $c(T) = \{P\text{-maximal alternatives in } T\}$, where P is an interval-order or a semi-order. In our model, if we fix T and consider all lists that permute T, then the set of alternatives that are chosen from these lists is precisely the set of P-maximal alternatives in T. However, our analysis focuses on the different choices across these lists with the same alternatives, and uses this information to recover the underlying preference.

In a similar spirit, and closer to our model, the literature has studied choice correspondences of the form $c(T, x)$, where T is a subset of alternatives, and x is a status quo/reference point. Examples of such papers include Masatlioglu and Ok [19], Apesteguia and Ballester [1] and Dean et al. [7]. However, in our model the status quo is sequentially updated along a list, and is not observed in general (unless the domain $\mathcal{D}$ satisfies the additional assumption that $\pi \in \mathcal{D}$ implies $\pi_{-1} \in \mathcal{D}$). This distinction makes our exercise quite different from the aforementioned papers.

Empirically, the order effect on choice has been well documented. Miller and Krosnick [20] and Krosnick et al. [15] find statistically significant and sometimes large effects of being listed first on the choice shares of major party alternatives in the U.S. state and federal elections. The "first-position advantage" these papers highlight is a special case of the status quo bias captured by our model. On the other hand, Bruine de Bruin [5] reports on panel decisions in contests such as the World Figure Skating Competition and the Eurovision Song Contest. He finds that the last few participants in the contest have an advantage, corresponding to the model with recency bias in Appendix B.

Rubinstein and Salant [23] introduce an early model of choice from lists.[8] They consider a DM whose underlying preference is a weak order, and who uses the list to resolve indifferences by choosing the first (or the last) most-preferred alternative. Choosing the earliest maximal alternative is equivalent to satisficing in the sense of Simon [26], which specifies a set A and chooses the earliest alternative that belongs

[8] The Rubinstein and Salant [24] model of choice with frames is more general. We study the ordering of alternatives as a specific type of frame.

to A. This is a special case of our model with status quo bias, when the DM's true preference is such that yPx if and only if $y \in A$ and $x \notin A$.

Salant [25] presents a more general model of iterative choice from lists, not restricting attention to the status quo-biased choice procedure that we focus on. He proves that framing effects generally exist, and he characterizes choice rules that exhibit optimal tradeoff between maximizing utility and minimizing computational complexity. A further generalization is obtained by considering a list as a special case of a decision tree, for example see Mukherjee [22].

As discussed, the work of Guney [11] is closest to the current paper. She considers the same choice procedure as we do, but she works with the larger domain of all lists over S. In the main text we have explained how a smaller domain may arise in practice, and how limited choice data makes it more difficult to identify the preference. Because of this, our analysis based on the revealed preference differs significantly from that of Guney [11].

There are many other related models of sequential choice. The agenda-rationalizable choice of Apesteguia and Ballester [2], the tournament choice of Horan [12] and the list-rationalizable choice of Yildiz [28] are models in which the DM also performs pairwise comparisons along a list. However, these papers assume a *fixed but unknown* order in which the alternatives are evaluated, and they attempt to endogenously derive this order. Their choice domain is unordered subsets of alternatives, while we consider observations from ordered lists.[9] Furthermore, these papers consider underlying preference relations that are tournaments, which are complete but not necessarily transitive binary relations. We have however focused on incomplete but transitive preferences. Finally, Caplin and Dean [6] and Masatlioglu and Nakajima [18] consider choice by search, a different kind of dynamic choice procedure. They seek to derive the search rule, while we take the ordering of alternatives as the natural search rule.

[9]The sequentially rationalizable choice model of Manzini and Mariotti [17] has a similar structure to these papers. The distinction is that their DM sequentially evaluates "rationales", i.e. multiple underlying preferences, to eliminate certain alternatives. Likewise, Xu and Zhou [27] study choice functions that arise from the subgame-perfect Nash equilibria of a game tree, where different nodes may have different complete preferences over the alternatives.

Appendix

A Proof of Theorem 3

Since Theorem 1 has been proved in the main text, here we just need to show that when the domain $\mathcal{D}$ satisfies both Assumptions 1 and 2, then $c = c^P$ for a strict partial order P implies that P^* is also a strict partial order. We will further show that if P is an interval-order or semi-order, then so is P^*. This leads to generalizations of Theorem 3 that have been mentioned in the main text.

A.1 The Case of Strict Partial Order

Suppose $c = c^P$ for a strict partial order P. By Lemma 2 we know that $P^* \subset P$ and thus P^* is asymmetric. To show P^* is a strict partial order, it remains to show it is transitive. Thus take three distinct alternatives x, y, z such that yP^*x and zP^*y. We want to show that zP^*x also holds. The following lemma proves this conclusion under the weaker assumption that yP^*x, zPx and $\neg yPz$ (note that yP^*x and zP^*y imply zPx by Lemma 2 and the transitivity of P):

Lemma 4. *Let $c = c^P$ be defined on a domain $\mathcal{D}$ that satisfies Assumptions 1 and 2, with P being a strict partial order and P^* being the revealed preference of c. Then for any three alternatives x, y, z, the relations $yP^*x, zPx, \neg yPz$ together imply zP^*x.*

Proof. Throughout we suppose $\neg zP^*x$ and try to deduce a contradiction. Since yP^*x, by definition of the revealed preference either Condition I or Condition II holds. First consider Condition I, which tells us that y is chosen in a list that begins with x. Since moving y to an earlier position in this list does not change the fact that it is chosen, we have a list $xyA \in \mathcal{D}$ such that $c(xyA) = y$. We then distinguish 3 cases:

(a) A is the empty list. In this case $xy \in \mathcal{D}$, which implies by Assumption 2 that $xz \in \mathcal{D}$ or $xyz \in \mathcal{D}$. If $xz \in \mathcal{D}$, then $c(xz) = c^P(xz) = z$ because zPx, which leads to zP^*x. If instead $xyz \in \mathcal{D}$, then $c(xyz) = z$ or y. The former directly leads to zP^*x by Condition I. But even in the latter case, $c(xyz) = y$ and $c(xzy) = c^P(xzy) = z$ (by $zPx, \neg yPz$) again lead to zP^*x by Condition II. In any case we have a contradiction!

(b) A is a list that contains z. In this case, since we assume $\neg zP^*x$, in the list xyA we can move z to immediately after x without changing the fact that y is chosen. Let $\tilde{A}$ be the list obtained from A by removing z (while maintaining the ordering of other alternatives in A). Thus $c(xzy\tilde{A}) = y$. But this is impossible because $c(xzy\tilde{A}) = c^P(xzy\tilde{A}) = c^P(zy\tilde{A}) \neq y$ since zPx and $\neg yPz$.

(c) A is a non-empty list that does not contain z. In this case Assumption 2 implies that either $xyAz \in \mathcal{D}$ or $xyA_{-1}z \in \mathcal{D}$. If $xyAz \in \mathcal{D}$, then $c(xyAz) = c^P(xyAz) = c^P(yz)$ because $c^P(xyA) = c(xyA) = y$. Thus, $c(xyAz) = z$ or y. The former contradicts $\neg zP^*x$ and Condition I, while the latter implies $c^P(xzyA) = c(xzyA) = y$ by Condition II, which then contradicts zPx and $\neg yPz$. If instead $xyA_{-1}z \in \mathcal{D}$, then a similar analysis yields $c(xyA_{-1}z) = z$ or y and leads to a contradiction.

So far we have ruled out the possibility that y is chosen in a list that begins with x. In what follows we consider the remaining possibility that in some list that begins with x, moving the position of y can affect the choice. This can be written as

$$c(xAywB) \neq c(xAwyB).$$

Clearly $w \neq z$, for otherwise zP^*x by Condition II. We next argue that B is non-empty. Indeed, if B is empty, then by writing $v = c^P(xA)$ we obtain $c^P(vyw) = c(xAyw) \neq c(xAwy) = c^P(vwy)$. It is easy to see that $c^P(vyw) \neq c^P(vwy)$ can happen only if $c^P(vyw) = y$ and $c^P(vwy) = w$. But then $c(xAyw) = c^P(vyw) = y$, showing that y is chosen in a list that begins with x. This has already been ruled out.

Thus $B \neq \emptyset$. We next show it is without loss to assume z is the last alternative in B. If $z \in A \cup B$, then because $\neg zP^*x$, we can move z to the end in both lists $xAywB$ and $xAwyB$ without affecting the choices from these lists. The choices from the resulting two lists are still different. If instead $z \notin A \cup B$, then by Assumption 2 we have either $xAywBz \in \mathcal{D}$, or $xAywB_{-1}z \in \mathcal{D}$. Suppose the former happens, then because $c(xAywBz) \neq z$, we must have $c(xAywBz) = c(xAywB)$. Similarly $c(xAwyBz) = c(xAwyB)$. Thus the two choices $c(xAywBz)$ and $c(xAwyBz)$ are different. Suppose instead $xAywB_{-1}z \in \mathcal{D}$, then similarly $c(xAywB_{-1}z) = c(xAywB_{-1})$ is different from $c(xAwyB_{-1}z) = c(xAwyB_{-1})$.[10]

Hence, in any case we can deduce the following situation for some lists $\tilde{A}, \tilde{B}$:

$$c(x\tilde{A}yw\tilde{B}z) \neq c(x\tilde{A}wy\tilde{B}z).$$

By $\neg zP^*x$ and Condition II, we can move z to immediately after x and obtain

$$c(xz\tilde{A}yw\tilde{B}) \neq c(xz\tilde{A}wy\tilde{B}).$$

Since $c = c^P$ and zPx, the above simplifies to $c^P(z\tilde{A}yw\tilde{B}) \neq c^P(z\tilde{A}wy\tilde{B})$. But this is impossible because $\neg yPz$ implies that the position of y should not affect the choice in any list that begins with z. This contradiction completes the proof. □

A.2 The Case of Interval-order

Suppose $c = c^P$ where P is an interval-order, we want to show that P^* is also an interval-order. We already know P^* is a strict partial order. Let I^* be the relation of incomparable pairs associated with P^*, defined by yI^*x if $\neg yP^*x$ and $\neg xP^*y$. It remains to check $P^*I^*P^* \subset P^*$.

Thus let us take four alternatives x, y, z, w with wP^*z, zI^*y, yP^*x. In particular, $w \neq z, y \neq x$ and $w \neq x$ (otherwise yP^*xP^*z, contradicting zI^*y). If $w = y$ or $z = x$, the result wP^*x is immediate. If $z = y$, the same result follows from the transitivity of P^*. Henceforth we can assume x, y, z, w are distinct.

Since P is an interval-order, it admits an interval representation $[U(x), U(x) + b(x)]$. From wP^*z we have wPz and $U(w) > U(z) + b(z)$. If $U(y) \geq U(w)$, then

[10]Note that $c(xAywB) \neq c(xAwyB)$ implies $c(xAywB_{-1}) \neq c(xAwyB_{-1})$ due to our recursive choice procedure.

$U(y) > U(z) + b(z)$ which yields yPz. We thus have wP^*z, yPz and $\neg wPy$ (because $U(y) \geq U(w)$). By Lemma 4, these three relations together imply yP^*z, contradicting the assumption that zI^*y.

Thus $U(y) < U(w)$ must hold, implying that $\neg yPw$. We also have wPx because $U(w) > U(y) > U(x) + b(x)$, where the last inequality uses yP^*x and thus yPx. So yP^*x, wPx and $\neg yPw$, which together imply wP^*x by Lemma 4. This proves $P^*I^*P^* \subset P^*$ so that P^* is an interval-order.

A.3 The Case of Semi-order

Suppose $c = c^P$ where P is a semi-order, we want to show that P^* is also a semi-order. We already know P^* is an interval-order. To show P^* is in fact a semi-order, we need to check $P^*P^*I^* \subset P^*$. Thus assume wP^*z, zP^*y, yI^*x, which implies $w \neq z, w \neq y, w \neq x, z \neq y$. If $z = x$ or $y = x$, the result wP^*x is immediate. Henceforth we assume x, y, z, w are distinct.

Let $[U(x), U(x) + 1]$ be a representation for the semi-order P. If $U(x) \geq U(z)$, then from zPy we obtain $U(x) \geq U(z) > U(y) + 1$, and thus xPy. We have the relations $zP^*y, xPy, \neg zPx$, which together imply xP^*y by Lemma 4. This contradicts the assumption that yI^*x.

Thus $U(z) > U(x)$ must hold. Below we show that wP^*z and $U(z) > U(x)$ imply wP^*x. The proof below mostly resembles the earlier proof for Lemma 4, with some modifications at the end. First note that wP^*z implies wPz and $U(w) > U(z) + 1$. Thus $U(w) > U(x) + 1$, and wPx holds. We also have $\neg xPz$ because $U(z) > U(x)$.

Since wP^*z, either Condition I or Condition II holds. We consider these two cases in turn:

(a) Suppose w is chosen in a list that beings with z. Then there exists a list $zwA \in \mathcal{D}$ such that $c(zwA) = w$. There are now 3 sub-cases. First suppose A is the empty list, then $zw \in \mathcal{D}$ and $wz \in \mathcal{D}$ by Assumption 1. By Assumption 2, we further have $wx \in \mathcal{D}$ or $wzx \in \mathcal{D}$. On one hand, $wx \in \mathcal{D}$ would imply $xw \in \mathcal{D}$ and

$c(xw) = c^P(xw) = w$, so that wP^*x. On the other hand, $wzx \in \mathcal{D}$ would imply $xzw \in \mathcal{D}$ and $c(xzw) = c^P(xzw) = w$ (because wPz and wPx), and wP^*x also holds.

Next suppose A is a list that contains x. Let $\tilde{A}$ be the list obtained from A by removing x. Then since $\neg xPz$, we can move x without affecting the choice. This gives $c(zwx\tilde{A}) = c(zwA) = w$. Now note that wPz, wPx, so $c^P(zwx) = c^P(xzw) = w$ which imply

$$c(xzw\tilde{A}) = c^P(xzw\tilde{A}) = c^P(w\tilde{A}) = c^P(zwx\tilde{A}) = c(zwx\tilde{A}) = w.$$

Thus $c(xzw\tilde{A}) = w$, leading to the desired result wP^*x.

Finally suppose A is a non-empty list that does not contain x. Then by Assumption 2, either $zwAx \in \mathcal{D}$ or $zwA_{-1}x \in \mathcal{D}$. Either way we have some $\hat{A}$ that contains x, such that $zw\hat{A} \in \mathcal{D}$. Moreover, since $\neg xPz$, the fact that x is appended to the end does not affect the choice. So $c(zw\hat{A}) = w$, which returns to the previous situation that we analyzed.

(b) Suppose the position of w affects the choice in some list that begins with z. We can write this as

$$c(zAwvB) \neq c(zAvwB)$$

for some alternative v and lists A, B. Thus vP^*z which implies vPz and in particular $v \neq x$ (recall $U(z) > U(x)$ so $\neg xPz$).

If B is empty, then by writing $u = c^P(zA)$ we deduce from the above that

$$c^P(uwv) = c^P(zAwv) = c(zAwv) \neq c(zAvw) = c^P(zAvw) = c^P(uvw).$$

But $c^P(uwv) \neq c^P(uvw)$ can only happen if $c^P(uwv) = w$ and $c^P(uvw) = v$. So $c(zAwv) = c^P(uwv) = w$, which reduces to the case above where w is chosen in a list that begins with z.

Thus assume B is non-empty. We next argue it is without loss to have x be the last alternative in B. Indeed, if $x \in A \cup B$, then since $\neg xPz$, we can move x to the end of the lists $zAwvB$ and $zAvwB$ without affecting the choices. The resulting lists $z\tilde{A}wv\tilde{B}x$ and $z\tilde{A}vw\tilde{B}x$ still lead to different choices. If instead $x \notin A \cup B$, then by Assumption 2 we have either $zAwvBx \in \mathcal{D}$ or $zAwvB_{-1}x \in \mathcal{D}$. In both cases we deduce from $\neg xPz$ the situation that $c(z\tilde{A}wv\tilde{B}x) = c(zAwvB)$ is different from $c(z\tilde{A}vw\tilde{B}x) = c(zAvwB)$.

Hence we have found $\tilde{A}, \tilde{B}$ such that $c(z\tilde{A}wv\tilde{B}x) \neq c(z\tilde{A}vw\tilde{B}x)$. From $\neg xPz$ we can move x to immediately after z and deduce

$$c(zx\tilde{A}wv\tilde{B}) \neq c(zx\tilde{A}vw\tilde{B}).$$

We can further assume every alternative in $\tilde{A}$ is P-preferred to z, since all other alternatives can be moved into $\tilde{B}$ without affecting the choices above. Now observe that $c^P(zx\tilde{A}wv\tilde{B}) = c^P(xz\tilde{A}wv\tilde{B})$, because the first alternative in $\tilde{A}w$ is P-preferred to z and thus also P-preferred to x (thanks to the semi-order representation and $U(z) > U(x)$). By a similar argument using vPz, we also have $c^P(zx\tilde{A}vw\tilde{B}) = c^P(xz\tilde{A}vw\tilde{B})$. Putting it all together,

$$c(xz\tilde{A}wv\tilde{B}) = c(zx\tilde{A}wv\tilde{B}) \neq c(zx\tilde{A}vw\tilde{B}) = c(xz\tilde{A}vw\tilde{B}).$$

This shows wP^*x by Condition II, and completes the proof.

B The Model with Recency Bias

In this appendix we consider the variant of our model with recency bias: when the DM cannot compare two alternatives according to P, she favors the one that just appeared. Her choices under such a procedure are thus defined as follows:

$$c_P(x_1) = x_1;$$

$$c_P(x_1x_2\dots x_k) = \begin{cases} c_P(x_1x_2\dots x_{k-1}), & \text{if } c_P(x_1x_2\dots x_{k-1}) \text{ is } P\text{-preferred to } x_k; \\ x_k & \text{otherwise.} \end{cases} \quad \forall 2 \leq k \leq N. \tag{5}$$

Note that we write $c_P(\pi)$ with the subscript P, to distinguish these choices from our main model.

If P could be any binary relation, then this model is equivalent to our main model under a transformation of the underlying preference. Specifically, if a choice function c is generated by (1) with true preference P, then it is also generated by (5) with true preference Q, where yQx if and only if $\neg xPy$. However, this equivalence breaks down once we restrict attention to transitive preferences.

We are again interested in the question of whether a given choice function $c : \mathcal{D} \to S$ can be rationalized by c_P for some strict partial order P. The answer depends on a different revealed preference P_* defined by the following condition:

Condition III: yP_*x if in some list $\pi \in \mathcal{D}$, y precedes x and $c(\pi) = y$.

Clearly, if $c = c_P$, then the revealed preference P_* is contained in P because $c(\pi) = y$ implies yPx for every later alternative x, according to the recency-biased choice procedure (5). In fact, the choice function c_P can be characterized as follows:

Lemma 5. *Suppose P is a strict partial order. Then for any list π, $c_P(\pi)$ is the earliest alternative in π that is P-preferred to every later alternative in π.*

Proof. Let us write $\pi = x_1A_1x_2A_2\dots x_nA_n$, where x_{k+1} denotes the first alternative after x_k such that $\neg x_kPx_{k+1}$. Then $c_P(\pi) = x_n$ according to (5). Note that in the list π, by construction x_n is indeed P-preferred to every later alternative in A_n. Moreover, for each $k \leq n-1$ we have $\neg x_k P x_{k+1}$ and $x_k P y$ for every $y \in A_k$. Thus by transitivity $\neg y P x_{k+1}$ also holds. This shows that every alternative earlier than x_n is *not* P-preferred to some later alternative. Thus x_n is indeed the earliest alternative in π that is P-preferred to every later alternative, proving the result. □

As will be clear, this simple characterization is the primary reason why our analysis for the recency-biased model is substantially simpler than for our main model.

Similar to Theorem 1, our main result is that the transitive closure $\overline{P_*}$ of the revealed preference is the minimum preference rationalizing the given choice function c.

Theorem 4. *Consider the recency-biased model. Given any choice function c and its revealed-preference P_*, the following conditions are equivalent:*

(i) $c = c_P$ *for some strict partial order* P*;*

(ii) $\overline{P_*}$ *is a strict partial order, and for any list* $\pi \in \mathcal{D}$ *and any alternative* $y \in \pi$ *preceding* $c(\pi)$*, there exists* $x \in \pi$ *later than* y *such that* $\neg y \overline{P_*} x$*;*

(iii) $\overline{P_*}$ *is a strict partial order, and* $c = c_{\overline{P_*}}$*.*

Proof. Clearly (iii) implies (i). To show (i) implies (ii), suppose $c = c_P$ for a strict partial order P. Then as discussed above the revealed preference P_* is contained in P, which implies that the transitive closure $\overline{P_*}$ is a strict partial order. In addition, if in some list $\pi \in \mathcal{D}$ an alternative y precedes $c(\pi)$, then by Lemma 5 y is not P-preferred to some later alternative x. Since $\overline{P_*} \subset P$, we also have $\neg y \overline{P_*} x$ as stated in (ii).

It remains to show (ii) implies (iii). Take any list $\pi \in \mathcal{D}$ and let y be the earliest alternative in π that is $\overline{P_*}$-preferred to every later alternative. Then by Lemma 5, $c_{\overline{P_*}}(\pi) = y$. To show $c(\pi) = y$, first note that by Condition III, $c(\pi)$ is P_*-preferred (and thus $\overline{P_*}$-preferred) to every later alternative. But we have chosen y to be the earliest such alternative in π, so $c(\pi)$ cannot precede y. At the same time, $c(\pi)$ cannot be later than y because that would imply by the second part of (ii) that y is not $\overline{P_*}$-

preferred to some later alternative, again contradicting the choice of y. Hence $c(\pi) = y$, proving (iii) and the theorem. $\square$

It is worth pointing out that, relative to Theorem 1, the above result for the recency-biased model does not require any assumption on the domain $\mathcal{D}$. If we do assume the domain satisfies Assumptions 1 and 2, then similar to Theorem 3 we can obtain a stronger result that gets rid of the transitive closure:

Theorem 5. *Consider the recency-biased model. Given any choice function $c : \mathcal{D} \to S$ where $\mathcal{D}$ satisfies Assumptions 1 and 2, the following conditions are equivalent:*

(i) *$c = c_P$ for some strict partial order P;*

(ii) *P_* is a strict partial order, and for any list $\pi \in \mathcal{D}$ and any alternative $y \in \pi$ preceding $c(\pi)$, there exists $x \in \pi$ later than y such that $\neg y P_* x$;*

(iii) *P_* is a strict partial order, and $c = c_{P_*}$.*

Proof. It suffices to show that if $c = c_P$ for some strict partial order P, then the revealed preference P_* is transitive. Thus assume zP_*y and yP_*x, which in particular imply $zPyPx$. Now, from zP_*y we know that there exists a list $\pi \in \mathcal{D}$ in which z is chosen, and z precedes y. We consider two cases, depending on whether $x \in \pi$ or not. If $x \notin \pi$, then by Assumption 2 we have either $\pi x \in \mathcal{D}$ or $\pi_{-1}x \in \mathcal{D}$. In either of these lists, because zPx, z remains the earliest alternative that is P-preferred to every later alternative.[11] So by Lemma 5, z is chosen in either list, which implies zP_*x as desired. A Theory of Reference-Dependent Behavior Suppose instead that $x \in \pi$. If x is later than z, then we already have zP_*x. If x precedes z, then using Assumption 1 we can obtain a new list in $\mathcal{D}$ by moving x to the end of π. In this new list, z remains P-preferred to every later alternative. In addition, any alternative u preceding z remains not P-preferred to some later alternative. Hence by Lemma 5 again, z is chosen in this new list and zP_*x also holds. $\square$

[11]To show z is the earliest, note that every alternative u preceding z is not P-preferred to some later alternative v in π. If v is even later than z, then because zPv, u is also not P-preferred to z. Thus we can without loss assume v either precedes z, or is z itself. This implies that v belongs to both of the lists πx and $\pi_{-1}x$ (in the latter case recall that z precedes y in π, so π_{-1} contains z).

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