Related rates worksheet with solutions

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Another reason is that after the implementation of all these derivatives, we need to be reminded that there are indeed actual applications of derivatives. Sometimes it's easy to forget that there's actually a reason that we spend all this time on derivatives. For these related problems, rates are generally best to just go straight into some problems and see how they work. Example 1 Example 1 Air is pumped into a spherical balloon at a speed of 5 cm3/min. Determine the speed at which the balloon radius increases when the diameter of the ball is 20 cm. Show the solution The first thing we will need to do here is determine what information we have been given and what we want to find. Before we do, let's notice that both the volume of the ball and the radius of the ball will vary depending on the time, and therefore indeed the function of the time, ie \(V\left(t\right)\) and \(r\left(t\right)\). We know that the air is pumped into the balloon at a speed of 5 cm3/min. This is the rate at which the volume increases. Recall that the pace of change is nothing short of derivative, and so we know that \[V'left\(t\right) = 5\] We want to determine the speed at which the radius changes. Again, rates are derivatives, and so it looks like we want to define \[r'\left(t\right) = ?\hspace{0.25in}{\rm{when}}\hspace{0.25in}r\left(t\right) = \frac{d}{2} = 10\,{\rm{cm}}\] Note that we need to convert the diameter to a radius. Now that we have determined what we have been given and what we want to find, we have to link these two quantities to each other. In this case, we can associate the volume and radius with the scope formula. \[V\left(t \right) = \frac{4}{3}\pi {\left; {r\left(t\right)} \right]^3}\] As in the previous section, when we looked at the iby differentiation, we wouldn't normally use \(\left(t\right)\) some of the things in formulas, but since it's time through one of them we will do this to remind ourselves that they really function \(t\). Now we don't really want a connection between volume and radius. What we really want is a relationship between their derivatives. We can do this by differentiating both sides with respect to \(t\). In other words, we will need to make an iutent differentiation according to the above formula. This gives \[V' = 4\pi {r^2}r'] note that at this point we have gone ahead and dropped \(\left(t\right)\) from each term. Now all we have to do is plug in what we know and decide for what we want to find. \[5 = 4\pi \left({{{10}^2}} \right)r'\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.0.0 025in}\hspace{0.25in}r' = \frac{1}{{80\pi }}\,{\rm{cm/min}}\] \[r' = \frac{{dr}}{{dt}}\] The derived units will be numerator units (cm in the previous example) separated by denominator units (min. in the previous example). Let's look at a few more examples. Example 2 Ladder 15 feet rests on the wall. The bottom is originally 10 feet from the wall and pushed against the wall at a speed of \({\textstyle{1 \over 4}}\) ft/sec. How fast does the top of the ladder move up the wall 12 seconds after we start pushing? Show the solution The first thing to do in this case is paint a picture that shows us what is happening. We determined the distance of the bottom of the ladder from the wall to \(x\) and the distance of the top of the ladder from the floor to \(y\). Note also that they change over time, and so we really have to write \(x\left(t\right)\ and \(y\left(t\right)\). As is often the case with related bets/inything-related differentiation problems, we don't write the \(\left(t\right)\) part just to try to remember it in our heads as we get down to the problem.{1}{4} If you're looking for a place to stay, you'll need to make sure that you're not sure what you're looking for. because \(y\) will be increased. As in the first example, we first need a relationship between \(x\) and \(y\). We can get this with the Pythagorean theorem. \[{x^2} + {y^2} = {\left({15} \right)^2} = 225\] All we have to do at this point is differentiate both sides relative to \(t\), remembering that \(x\) and \(y\) are really functions \(t\) and therefore we will need to make an inequal differentiation. This gives an equation that shows the relationship between \[\begin{equation}2xx' + 2yy' = 0\end{equation} \label{eq:eq1}\] Next, let's see which different parts of this equation we know and what we need to find. We know \(x'\) and are asked to identify \(y'\), so it's ok that we don't know what. But we still need to identify \(x\) and \(y\). The definition of \(x\) and \(y\) is actually quite simple. We know that first \(x = 10\) and the end is pushed to the wall at a speed of \({\textstyle{1 \over 4}}\)ft/sec and that we are interested in what happened after 12 seconds. We know that \[\begin{align*}{\rm{distance}} = {\rm{rate}} \times {\rm{time}}\\ & = \left({\frac{1}{4}} \right)\left({12} \right) = 3\end{align*}\] So the end of the ladder was pushed 3 feet and so after 12 seconds we should have \(x = 7\). Note that we could calculate this in one step as follows, \[x = 10 - \frac{1}{4}\left({12} \right) = 7\] To find \(y\) (after 12 seconds) all we have to do is reuse Pythagorean theorem with the values \(x\) we just found above. \[y = \sqrt {225 - {x^2}} = \sqrt {225 - 49} = \sqrt {176} \] Now all we have to do is connect to \(\eqref{eq1}\) and decide for \(y'\). \[2\2\left(7 \right)\left(-\frac{1}{4} \right)+2\left(\sqrt{176} \right){y}'=0\Rightarrow {y}'=\frac{{}^{7}/{}_{4}}{\sqrt{176}}=\frac{7}{4\sqrt{176}}=0.1319\,\,\text{ft/sec}\] Note that we got the correct sign for \(y'\). If we had gotten negative value, we would have known we had made a mistake and we could go back and look for it. Before working with another example, we need to make comments about setting up a previous problem. When we marked our sketch, we recognized that hypotenuse is permanent and so simply called it 15 feet. A common mistake students sometimes make here is to also mark hypotenuse as a letter, say \(z\), in this case. Well, it's not really a mistake to label a letter, but it will often cause a problem down the road. If we had denoted hypotenuse \(z\), the Pythagorean theorem and its derivative would have been, \[{x^2} + {y^2} = {z^2}\hspace{0.25in} \to \hspace{0.25in} 2xx' + 2yy' = 2zz'\] Again, there is nothing wrong with this, but it requires that we recognize the values of two more quantities, \(z\) and \(z'\). Since \(z\) is only a hypotenuse that is explicitly \(z=15\). The problem that some students sometimes face is to determine the value of \(z'\). In this case, we must remember that because the staircase, and therefore the hypotenuse has a fixed length, its length cannot change and so \(z' = 0\). Connecting both of these values to the derivative gives us the same equation we got in the example, but required a little more effort to get to. It would be easier to simply mark hypotenuse 15 to begin with and not have to worry about remembering that \(z' = 0\). Fixed quantity marking (the length of the ladder in this example) letter is sometimes easy to forget that it is a fixed amount, and therefore the derivative should be zero. If you don't remember this, the problem becomes impossible to end because you will have two unknown quantities that you will have to deal with. In any problem there has been a quantity fixed and will never be over the course of changing the problem it is always better to simply acknowledge that and mark it with its cost rather than letter. Of course, if we had a sliding ladder that was allowed to change, we would have to mark it with a letter. However, for such a problem we will also need a little more information in the statement about the problem to actually make the problem. Practice issues in this section have several problems in which all three sides of the right triangle change. You should check them out and see if you can work them out. Example 3 Two people are 20 feet apart. One of them begins to walk north at speed so that the angle shown in the diagram below changes at a constant speed of 0.01 councils/min. At what speed does the distance between two people change when \(\theta = 0.5\) radians? Show solution This example is not as complex as it may initially appear. Let's call the distance between them at any given time \(x\) as above. Then we can link all known quantities to one of two trig formulas. \[\cos \theta = \frac{{50}}{x}\hspace{0.25in}\hspace{0.25in}\sec \theta = \frac{x }{{50}}\] We want to find \(x'\), and we could have found \(x\) if we wanted at the point of the issue using the kozin because we also know the angle at that time. However, if we use the second formula, we will not need to know \(x\) as you will see. So let's warm up this formula. \[\sec \theta \tan \theta \,\theta' = \frac{{x'}}{{50}}\] As noted, there is no \(x\) in this formula. We want to identify \(x'\) and we know that \(\theta = 0.5\) and \(\theta' = 0.01\) (do you agree that this is positive?). So, just plug in and decide. \[\left({50} \right)\left({0.01} \right)\sec \left({0.5} \right)\tan \left({0.5} \right) = x'\hspace{0.25in} \0.25in} \0.25in} \. Rightarrow \hspace{0.25in}\hspace{0.25in}x' = 0.311254\,\,{\rm{ft}}/\min \] So far we have seen three related issues. Although each worked differently, the process was essentially the same in each. In each problem, we identified what we were given and what we wanted to find. Next, we recorded a link between all different quantities and used an iquident differentiation to come to a link between different derivatives in the problem. Finally, we connected the known quantities to the equation to find the value we were after. Thus, in general, each problem worked in much the same way. The only real difference between them was to between a known and unknown quantity. This is often the hardest part of the problem. In many problems, the best way to come up with a relationship is to draw a diagram that shows the situation. This often seems like a stupid move, but can make a difference in whether we can find a relationship or not. Let's work out another problem that uses some different ideas and shows some different kinds of things that may appear in related betting issues. Example 4 Water tank in the form of a cone water leak at a constant speed \(2\,{\mbox{ft}}^{3}{\rm{/hour}}\). The tank base radius is 5 feet and the tank's height is 40 feet. At what speed does the depth of water in the tank change when the water depth is 6 feet? At what speed does the radius of the top of the water in the tank change when the water depth is 6 feet? Show all solutions Hide all solutions Show discussions Ok, we should probably start with a quick sketch (probably not scale) of what's happening here. We'll also do a sketch as if we were looking at the tank right in front of it (and so the 3D tank won't be visible) as it will help to see a little bit what's going on. Showing the 3D nature of the tank is a responsibility to just get in the way. So here's a sketch of a tank with some water in it. As you can see, the water in the tank actually forms a smaller cone/triangle (depending on which image we are looking at) from the same central angle as the reservoir itself. The radius of the water cone at any time is provided \(r\) and the height of the cone water at any time is given \(h\). The volume of water in the tank at any time \(t\) is provided, \[V = \frac{1}{3}\pi {r^2}h\] and we were given that \(V' = - 2\). and At what speed does the depth of water in the tank change when the water depth is 6 feet? Show solution For this part we need to identify \(h'\) when \(h =6\) and now we have a problem. The only formula we've got that will associate volume with height also includes a radius, and so if we would differentiate this with respect to \(t\) we would get , \[V' = \frac{2}{3}\pi rr'h + \frac{1}{3}\pi {r^2}h'\] So in this formula we know \(V'\) and \(h\) and want to find \(h'\), but we do not know \(r\) and \(r'\) As we will see, finding \(r\) is not too bad, but we simply do not have enough information at the moment that will allow us to find \(r'\) and \(h'\) at the same time. To fix this, we will need to somehow exclude \(r\) from the formula volume. It's actually easier than it might first look. If we go back to our sketch above and look only at the right half of the tank, we see that we have two similar triangles, and when we say like that, we mean similar exponentially. Recall that the two triangles are called similar, if their angles are identical, it's happening here. When we have two similar triangles, the ratio of either two sides will be equal. For our recruitment, this means that we have, \[\frac{r}{h} = \frac{5}{{14}}\hspace{0.25in}\hspace{0.25in} \Right aarrow \hspace{{{hspace 0.25in}\hspace{0.25in}r = \frac{5}{{14}}h\] If we take this and connect it to our volume formula, which we have, \[V = \frac{1}{3}\pi {r^2}h = \frac{1}{3}\pi {\left({\frac{5}{{14}}h} \right)^2}h = \frac{{25}}{{588}}\pi {h^3}\] Note, however, that this formula is therefore only valid for our cone, so don't be tempted to use it for other cones! If we now differentiate this we have, \[V'\, = \frac{{25}}{{196}}\pi {h^2}h'\] At this point, all we have to do is connect what we know and decide for \(h'\). \[- 2 = \frac{{25}}{{196}}\pi \left({{6^2}} \right)h'\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in }\hspace{0.25in}h' = \frac{{ - 98}}{{225\pi }} = - 0.1386\] height decreases at a speed of 0.1386 ft/h. b At what speed does the radius of the top of the water in the tank change when the water depth is 6 feet? Show solution In this case we ask \(r'\) and there is an easy way to do this part and a difficult (well, more difficult than easy way anyway ....) way to do it. The tricky way is to replicate the work partially (a) higher only this time, \[\frac{h}{r} = \frac{{14}}{5}\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}h = \frac{{14}}{5}r\], to get the volume from the point of view of \(V\) and \(r\), and then continue as before. It's not scary difficult, but it's more of a job that we need so. Recall from the first part we have \[r = \frac{5}{{14}}h\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}r' = \frac{5}{{14}}h'\] So, as we can see, if we take a link that refers to \(r\) and \(h\) that we used in the first part and differentiate it we get a link between \(r'\) and \(h'\). At the moment, everything what we need to do here is use the result from the first part to get, \[r' = \frac{5}{{14}}\left({\frac{{ - 98}}{{225\pi }}} \right) = - \frac{7}{{45\pi }} = - 0.04951\] Note that we were only able to do this easier because we asked \(r'\)exactly at the same time as we asked \(h'\) in the first part. If we hadn't used the same time, then we would have no choice but to do it in a difficult way. In the second part of the previous problem, we saw an important idea in solving related rates. In order to find the requested course all we need is an equation that relates to the speed we are looking for to the speed we already know. Sometimes there are several equations that we can and sometimes one will be easier than the other. In addition, this problem has shown us we will often have an equation that contains more variables about which we have information, and so in these cases we will need to eliminate one (or more) variables. In this problem, we eliminated an additional variable using the idea of similar triangles. It won't always be how we do it, but many of these problems use similar triangles, so make sure you can use the idea. Let's work out a few more problems. Example 5 Corito water length of 8 meters, and its ends - in the form of triangles of isossels, the width of which is 5 meters, and height - 2 meters. If the water is pumped at a constant speed \(6 \mbox{m}^{3}\mbox{/sec}\). At what speed does the height of the water change when the water is 120 cm high? At what rate does the width of the water change when the water is 120cm high? Show solution Note that the isossel triangle is only a triangle in which the two sides have the same length. In our case, the sides of the tank have the same length. Add in some sizes for water sketch on top. Now in this problem we know that \(V' = 6 \mbox{m}^{3}\mbox{/sec}\) and we want to define \(h'\) when \(h = 1.2\,{\rm{m}}\). Note that because \(V'') in terms of counters, we also need to convert \(h\) to counters. So we will need an equation that will bind these two quantities and the volume of the tank will do so. The volume of this kind of tank is easy to calculate. The volume of the end area is many times greater than the depth. For our case, there is water in the tank, \[\begin{align*}V &= \left({{\mbox{End area}}} \right)\left({{\rm{depth}}} \right)\\ & =\left({{\textstyle{1 \over 2}}{\rm{base}} \\ times {\rm{height}}} \right)\left({{\rm{}}} \right)\\ & = {\textstyle{1 \over 2}}hw\left(8 \right)\\ & = 4hw\end{align*}\] As in the previous example, we have an additional number here, \(w\), that also changes over time, and so we have to eliminate it from the problem. To do this, we will again use the idea of such triangles. If we look at the end of the tank, we'll see that we have two similar triangles again. One for the tank itself and one is formed by water in the tank. Again, remember that with such triangles, the aspect ratio should be equal. In our case, we will use, \[\frac{w}{5} = \frac{h}{2}\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}w = \frac{5}{2}h\] Connecting this volume gives a volume formula (and only for this reservoir), which included only water height. \[V= 4hw = 4h\left({\frac{5}{2}h} \right) = 10{h^2}\] Now we can differentiate this to get, \[V' = 20hh'\] Finally, all we have to do is connect and decide for \(h'\). \[6 = 20\left({1.2} \right)h'\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}h' = 0.25\,\,{\rm{m/sec}}\] So, water height rises at a speed of 0.25 m/sec. Inches answering the second part of this question is not all that hard. We will need \(w'\) to respond to this part and we have the following equation from a similar triangle that are bound width to height and we can quickly differentiate it to get a link between \(w'\) and \(h'\). \[w = \frac{5}{2}h\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0 .25in}w' = \frac{5}{2}h'\] what needs to be done is connect this to this equation and we will have an answer. \[w' = \frac{5}{2}\left({0.25} \right) = 0.625\,\,{\rm{m/sec}}\] Therefore, the width increases at a speed of 0.625 m/sec. Example 6 Light is on top of a 12ft tall pole and a 5ft 6in tall man walks away from the pole at a speed of 2ft/sec. At what speed does the tip of the shadow move away from the pole when a person is 25 feet from the pole? At what speed does the tip of the shadow move away from a person when a person is 25 feet from the pole? Show all solutions Hide all solutions Show discussion Let's start by entering all the corresponding quantities in the thumbnail on top. Here \(x\) is the distance of the tip of the shadow from the pole, \({x_p}\) is the distance of the face from the pole, and \({x_s}\) is the length of the shadow. Also note that we have turned individuals over 5.5 feet high as all other measurements are in their legs. The tip of the shadow is determined by the rays of light that just beamed past the person, and so we can see how they form a set of similar triangles. This will be useful down the road. And at what speed does the tip of the shadow move away from the pole when the person is 25 feet from the pole? Show solution In this case, we want to define \(x'\) when \({x_p} = 25\), considering that \({x'_p} = 2\). The equation we need here is: \[x = {x_p} + {x_s}\], but we will need to exclude \({x_s}\) from the equation to get an answer. To do this, we can again take advantage of the fact that two triangles are like getting, \[\frac{{5.5}}{{12}} = \frac{{{x_s}}}{x}\hspace{0.25in}\hspace{0.25in}{\rm{Note: }}\frac {{5.5}}{{12}} = \frac{{\textstyle{{11} \over 2}}}{{12}} = \frac{{11}}{{24}}\] \[{x_s} = \frac{{11}}{{24}}x\] Then we can connect this to the equation above and decide for \(x\) as follows. \[x = {x_p} + {x_s} = {x_p} + \frac{{11}}{{24}}x\hspace{0.25in} \Rightarrow \hspace{0.25in} x = \frac{{24}}{{13}}{x_p}\] Now everything, what we need to do is differentiate this, connect and decide for \(x'\). \[x' = \frac{{24}}{{13}}{x'_p}\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in }x' = \frac{{24}}{{13}}\left(2 \right) = 3.6923{\rm{ ft/sec}}\] The tip of the shadow then moves away from the pole at a speed of 3.6923 ft/sec. Note also that we should never have used the fact that \({x_p} = 25\) for this issue. will occur in rare cases. b At what speed does the tip of the shadow move away from the person when the man is 25 feet from the pole? Show solution This part is actually quite simple if we have an answer from (a) in hand that we do, of course. In this case, we know that \({x_s}\) represents the length of the shadow or the distance of the shadow tip from the person, so it looks like we want to define \({x'_s}\) when \({x_p} = 25\). Again, we can use \(x = {x_p} + {x_s}\), but unlike the first part we now know that \({x'_p} = 2\) and \(x' = 3.6923{\rm{ ft/sec}}\), so all we have to do is differentiate the equation and connect to all known quantities. \[\begin{align*}x' &= {{x'}_p} + {{x'}_s}\\ 3.6923 &= 2 + {{x'}_s}\hspace{0.] 5in}{{x'}_s} = 1.6923{\rm{ ft/sec}}\end{align*}\] The tip of the shadow then moves away from the person at a speed of 1.6923 ft/sec. Example 7 Spot light is on the ground 20 feet from the wall, and a 6-foot tall man walks toward the wall at a speed of 2.5 feet/sec. How quickly does the height of the shadow change when a person is 8 feet from the wall? Is the shadow increasing or decreasing in height at this time? Show solution Below is a copy of the sketch in the statement of the problem with all the corresponding quantities added in. The upper part of the shadow will be determined by the light rays running over the human head, and so we will get another set of similar triangles again. In this case, we want to define \(y'\) when the person is 8 feet from the wall or \(x = 12{\rm{ ft}}\). Also, if a person moves toward a wall at 2.5 feet/sec, then a person should move away from the spotlight at 2.5 feet/sec, and so we also know that \(x' = 2.5\). In all previous problems that have used similar triangles, we have used similar triangles to eliminate one of the variables from the equation we worked with. In this case, however, we can get an equation that refers to \(x\) and \(y\) directly from two similar triangles. In this case, the equation we will work with is, \[\frac{y}{6} = \frac{{20}}{x}\hspace{0.25in}\hspace{0.25in}\hspace{0 \Rightarrow \\ hspace{0.25in}\hspace{0.25in}y = \frac{{120}}{x}\] what we need to do is differentiate and connect values to the solution to get \(y'\). \[y' = - \frac{{120}}{{x^2}}x'\hspace{0.25in}\hspace{0.25in} \Rightarrow \hspace{0.25in}\hspace{0.25in}y ' = - \frac{{120}}{{{{{12}^2}}}\left({2.5} \right) = - 2.0833{\rm{ ft/sec}}\] Shadow height decreases at 2.0833 ft/sec. Ok, we've worked quite a few problems now that involved similar triangles in one way or another, so make sure you can do such problems. Now it's time to make a problem which, while similar to some of the problems we've done up to this point, is also quite different that it can cause until you've seen how. Example 8 Two Two on bicycles separated by 350 meters. Man A starts driving north at a speed of 5 m/sec and after 7 minutes Man B starts driving south at a speed of 3 m/sec. At what speed does the distance separating the two people change 25 minutes after Man A starts driving? Show solutions there are many to digest here with this problem. Let's start by sketching out a situation that shows each person's whereabouts someday after both people start driving. Now we are after \(z'\), and we know that \(x' = 5\) and \(y' = 3\). We want to know \(z'\) after Man A drove for 25 minutes and Person B traveled to \(25 - 7 = 18\) minutes. After turning these times into seconds (because our fares are all in m/sec) it means that at the time we are interested in each of the cyclists driving, \[x = 5\left({25 \times 60} \right) = 7500{\rm{ m}}\hspace{0.25in}\hspace{0.25in}y = 3\left({18 \times 60} \right) = 3240{\rm{ m}}\] Next, Pythagorean theorem tells us that, \[\begin{equation}{z^2} = {\left({x + y} \right)^2} + {350^2}\label{eq:eq2}\end{equation}\] Therefore, 25 minutes after Man A starts riding two bike riders, there is \[z = sqrt {{{\left({x+ 2y} \right)}^2} + {{350}^2}} = \sqrt {{{\left({7500 + 3240} \right)}^2} + {{350}^2}} = 10745.7015\,\,{\rm{m}}\] apart. To determine the speed at which the two riders move apart all we have to do is differentiate between \(\eqref{eq:eq2}\) and connect all the quantities we know to find \(z'\). \[\begin{align*}2zz' & = 2\left({x+ y} \right)\left({x' + y'} \right)\\ 2\left({10745.7015} \right)z' & = 2\left({7500 + 3240} \right)\left({5 + 3} \right)\\z' & = 7.9958{\rm{ m/sec}}\end{align*}\] So the two riders diverge at a speed of 7.9958 m/sec. Every problem we've worked on up to this point boils down to the need for a geometric formula, and we should probably run a quick problem that isn't geometric in nature. Example 9 Assume that we have two resistors related in parallel to the supports \({R_{\,1}}\) and \({R_{\,2}}\) measured in omahs (\(\Omega \)). Total resistance, \(R\), is then granted by \[\frac{1}{R} = \frac{1}{{{{R_{{{\,1}}}} + \frac{1}{{{{{R_{\,2}}}}\] Suppose \({R_ {\,1}}\) increases at a speed of 0.4 \(\Omega \)/min and \({R_{\,2}}\) decreases at a speed of 0.7\(\Omega \)/min. At what speed \(R\) changes when \({R_{\,1}} = 80\,\Omega\) and \({R_{\,2}} = 105\,\Omega \)? Show the solution well, unlike previous problems, doesn't really have much to do here. First, note that we are looking for \(R'\) and what we know \({R'_{\,1}} = 0.4\) and \({R'_{\,2}} = - 0.7\). Be careful with the signs here. Also, since we will eventually need it, let's define \(R\) at a time when we are interested. \[\frac{1}{R} = \frac{1}{{80}} + \frac{1}{{105}} = \Rightarrow \hspace{0.25in}\hspace{0.25in}R = = = 45.4054\,\Omega \] Next, we need to differentiate the equation specified in the problem operator. \[\begin{align*} – \frac{1}{{{R^2}}}R' & = - \frac{1}{{{\left({{R_{\,1}}} \right)}^2}}{R '}_{\,1}} - \frac{1}{{{{\left({{R_{\,2}}} \вправо)}^2}}}{R'}_{\,2}}\\ R' &= {R ^2}\left({\frac{1}{{{{\left({{R_{\,1}}} \вправо))^2}}{R'}_{\,1}} + \frac{1}{{{\\ліворуч({{R_{\,2}}} \right)}^2}}{R'}_{\,2}}} \right)\end{align*}\] Нарешті, все, що нам потрібно зробити, це підключити до цього і зробити деякі швидкі обчислення. \[R' = {\left({45.4054} \right)^2}\left({\frac{1}{{{{80}^2}}}\left({0.4} \right) + \frac{1}{{{{{105} ^2}}}\left({ - 0.7} \right)} \right) = - 0.002045\] So it looks like \(R\) decreases at a speed of 0.002045\(\Omega \)/min. We have seen quite a few related fare problems in this section that cover a wide range of possible problems. There are still many other different kinds of related betting problems out there in the world, but the ones we've worked with here should give you a pretty good idea of how to at least start most of the problems you're responsible with. In.