



# Bank efficiency and failure prediction: a nonparametric and dynamic model based on data envelopment analysis

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## Abstract

For decades, the prediction of bank failure has been a popular topic in credit risk and banking studies. Statistical and machine learning methods have been working well in predicting the probability of bankruptcy for different time horizons prior to the failure. In recent years, bank efficiency has attracted much interest from academic circles, where low productivity or efficiency in banks has been regarded as a potential reason for failure. It is generally believed that low efficiency implies low-quality management of the organisation, which may lead to bad performance in the competitive financial markets. Previous papers linking efficiency measures calculated by Data Envelopment Analysis (DEA) to bank failure prediction have been limited to cross sectional analyses. A dynamic analysis with the updated samples is therefore recommended for bankruptcy prediction. This paper proposes a nonparametric method, Malmquist DEA with Worst Practice Frontier, to dynamically assess the bankruptcy risk of banks over multiple periods. A total sample of 4426 US banks over a period of 15 years (2002–2016), covering the subprime financial crisis, is used to empirically test the model. A static model is used as the benchmark, and we introduce more extensions for comparisons of predictive performance. Results of the comparisons and robustness tests show that Malmquist DEA is a useful tool not only for estimating productivity growth but also to give early warnings of the potential collapse of banks. The extended DEA models with various reference sets and orientations also show strong predictive power.

**Keywords** Bank failure · Bank efficiency · Data Envelopment Analysis · Bankruptcy prediction · Dynamic model

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# 1 Introduction

Financial institutions such as banks are key players in a country's financial system. Reducing and controlling the systemic risk which can possibly spread from problematic banks to other solvent banks is one of the main duties of regulators (Schulte & Winkler, 2019). Regulations on banks, such as the BASEL Accord, are used across the globe to ensure banking operations are functional and stable. However, in certain downturns, such as a financial crisis, banks that find themselves under increasing pressure may fail for various reasons. In the last financial crisis in 2008, the entire financial sectors, including the banking industry, suffered great losses and many banks became insolvent during the recession which followed (De Haas & Van Horen, 2013). This greatly impacted the global economy and the stability of many countries' financial systems.

In much the same way that the predicting of corporate bankruptcy has been well discussed in business and credit risk literature (see reviews in Véganzones and Severin (2020)), the prediction of bank failure has also been well documented in banking and finance studies. Various statistical and intelligent algorithms have been proposed to classify failed banks and non-failed banks, for example, Discriminant Analysis (Meyer & Pifer, 1970), Logistic Regression (Martin, 1977), the Cox Proportional Hazard model (Lane et al., 1986), Neural Networks (Tam, 1991), Trait Recognition models (Kolari et al., 2002), Probit model (Canbas et al., 2005), Support Vector Machines (Boyacioglu et al., 2009), with these models being comprehensively reviewed by Altman and Saunders (1997), Ravi Kumar and Ravi (2007), and Demyanyk and Hasan (2010). Many of the models are already successfully being used as Early Warning Systems (EWS) to give signals of possible bank failure and thus prevent potential crises for the market.

A bank's efficiency may be judged by different factors such as cost, profit, or management, and there is much literature available which discusses the relationship between efficiency and bankruptcy. As Berger and Humphrey (1997) commented, management quality should be positively related to efficiency and banks should logically show signs of low efficiency rates prior to failure. Barr et al. (1993) found that the efficiency of failed banks is significantly lower than that of healthy banks in the years prior to their eventual failure. Barr et al. (1994) later became the first to use data envelopment analysis (DEA) as a non-parametric method to predict bank failure. Costs and profits are two important aspects that banks need to take into consideration when making efficiency analyses. Cost efficiency is used to measure costs under the condition of the same output. The idea behind profit efficiency is to measure revenue streams. Assaf et al. (2019) investigated the effect of efficiency on the survival and profitability of banks in normal periods (i.e., non-recession periods) and found that cost efficiency plays a vital role, while profit efficiency has only a limited effect.

In the wider scope of corporate bankruptcy prediction, DEA is a popular method and has been tested in many different contexts, examples being Premachandra et al. (2009), Yeh et al. (2010), Psillaki et al. (2010), Premachandra et al. (2011) and Li et al. (2014). Some analysts have used DEA as a direct algorithm to assign a score to each Decision Making Unit (DMU) (Cielen et al., 2004; Min & Lee, 2008; Paradi et al., 2004; Shetty et al., 2012). Paradi et al. (2004) proposed a new frontier construction method in DEA and used a layered technique to predict bankruptcy. Their basic idea of 'Worst Practice Frontier' (WPF) is to put inefficient DMUs at the frontier, and they thus discovered that the combination of layered Best and Worst Practice Frontier DEA models yielded impressive classification accuracy. Given the fact that bank efficiency performance, as calculated by DEA, is evaluated over several time periods (Casu et al., 2004; Portela & Thanassoulis, 2010; Wheelock & Wilson, 2009). Li et al. (2017)

were therefore able to extend corporate failure prediction into dynamic assessment, using Malmquist DEA and hazard models to classify failed and non-failed companies over multiple periods. The results showed that the financial distress hazard model could be enhanced when combined with the dynamic DEA scores.

This paper proposes a nonparametric approach to dynamically predict bank failure using mutative datasets with mutative scores under the Malmquist DEA framework. The efficiency of bank peers over multiple periods allows for the detection of early signals of potential bank insolvency with a robust predictive power. This paper also makes some valuable comparisons with static traditional DEA models and some extended dynamic models. The paired t-test results of evaluation indicators show that the model we proposed, Malmquist DEA with WPF and global reference, has better predictive power than the static models and other extended models. Given that the empirical results use data from 4426 US banks from 2002 to 2016—a period which covered the last subprime crisis—we argue that this work offers valuable insights for stakeholders and regulators on how to react to the global depression which has now been triggered by the COVID-19 pandemic, where credit risk will increase and banks may fail.

The rest of the paper is organised as follows. We summarize the literature related to bank efficiency analysis and bank failure prediction in Sect. 2. Section 3 introduces the methodology and shows the research design. Section 4 includes the data description and variable selection. We present the comparison analysis and robust test in Sect. 5. Section 6 concludes our work, and we discuss future implications at the end.

## 2 Literature review

### 2.1 Bank failure prediction

The prediction of bank failure has a long history in the study of banking and finance, starting with Meyer and Pifer (1970), who input 32 financial measures and four types of variations into Linear Discriminant Analysis (LDA), which was introduced by Altman (1968), and found that only 10 out of 160 variables were significant in distinguishing between good and bad banks. They also established some criteria for bankruptcy prediction, crucially, to let the data speak for itself, instead of building on mere conjecture. Later, Martin (1977) proved that the linear discriminant function is a special case of the logistic function when multivariate normal distribution is met, thus becoming the first to apply Logistic Regression (LR) to bank failure prediction. Logistic Regression has thus become the most widely used method in firm and bank bankruptcy prediction (Imbierowicz & Rauch, 2014; Jin et al., 2011; Lanine & Vennet, 2006; West, 1985). Canbas et al. (2005) considered LDA, Logit and Probit models together in a multi-level analytical framework. They employed a Principal Component Analysis (PCA) on 12 financial ratios, and extracted three factors to detect problems in 40 commercial Turkish banks. Other than LDA, Logit and Probit, Survival Analysis is another statistical method common in bankruptcy prediction. It models the duration time to failure which can be affected by many deterministic variables, including Time-Varying Covariates, such as macroeconomic factors. Applications of bank failure prediction can be found in Lane et al. (1986), Henebry (1996), DeYoung (2003) and González et al. (2021), who all used Cox Proportional Hazard models to predict the probability of failure, given a bank could survive until a certain point.

Statistical models are typical parametric methods, where the relationships between explanatory variables and the target variable are specified, while nonparametric methods cannot provide specific statistical parameters. In Meyer and Pifer (1970), the four types of variation of a financial variable were calculated through the interactions of its values for the six years leading up to the year of the study. However, a more popular, modern way of introducing interaction into analysis is to consider the interactions of different variables. Trait recognition (TR) is one such nonparametric approach that measures all possible interactions between variables. It tracks the traits which exist in complex interactions between predictive variables (Lanine & Vennet, 2006). Kolari et al. (2002) and Lanine and Vennet (2006) compared TR with the Logit model, and both concluded that TR outperformed Logistic Regression in the prediction of bank failure for the holdout sample.

Neural Networks, another classical nonparametric method, frequently appears in the literature related to bank failure prediction. It was first introduced into the field by Tam (1991), who investigated its performance on 59 pairs of failed and non-failed banks. The performance of Neural Networks was compared to a group of other common algorithms, including LDA, LR, k-Nearest Neighbor and Decision Tree, and promising results were found. Neural Networks was then developed with many alterations and adjustments to improve its predictive accuracy (Alam et al., 2000; Boyacioglu et al., 2009; Tam & Kiang, 1992).

## 2.2 Bank efficiency analysis

DEA is a mathematical programming approach used to calculate the relative efficiency of DMUs, given that each of them utilise a number of inputs to produce various favorable outputs. DEA can identify a group of efficient units lying on the efficient frontier as reference points for other units to be compared with. In this way, the performance or productivity of DMUs can be evaluated through the efficiency shown by DEA models. Many studies employ DEA to analyze bank performance compared to their peers (Mahmoudabadi & Emrouznejad, 2019; Ouenniche & Carrales, 2018; Yang, 2014). There are also some studies combining other methods to measure banks' efficiency, such as DEA with network analysis (Antunes et al., 2021; Tan et al., 2021), or bootstrap methods (Dia et al., 2020) etc.. Stochastic Frontier Analysis (SFA), as a parametric production efficiency frontier analysis tool, is also widely used to evaluate the operating efficiency of banks (Ngo & Tripe, 2017). SFA can also be used to measure bank failure, for instance in Sanchez González et al. (2020), who applied the SFA and Bayesian approach to commercial banks in the USA to estimate the effect of inefficiency on bank failure. Unlike parametric productivity analytical tools such as SFA, since DEA is a nonparametric method, it is more advantageous than SFA in terms of the free distribution of variables and multiple outputs, though it is SFA which has attracted more interest in the investigation of bank performance (Behr, 2010; Casu et al., 2004; Lampe & Hilgers, 2015).

In a similar way that corporate efficiency is involved in bankruptcy prediction (Avkiran & Cai, 2014; Cielen et al., 2004; Mousavi & Ouenniche, 2018; Ouenniche & Tone, 2017; Premachandra et al., 2011; Psillaki et al., 2010), bank efficiency has also been found to be helpful in detecting potential bank failure. Barr et al. (1994) first proposed the use of DEA as a nonparametric tool to imply management quality and so to detect in advance those insolvent banks with significantly lower efficiency scores than solvent banks. Wheelock and Wilson (1995) interpreted the inefficiency calculated by DEA to be a cause of bank failure, and concluded that technically inefficient banks were more likely to fail than efficient ones. Brockett et al. (1997) suggested that DEA be used as an EWS to identify bad banks from good ones, and were therefore able to generate overall ratings based on the CAMEL

criteria (Capital, Assets, Management, Equity, Liquidity). Avkiran and Cai (2014) used a non-oriented DEA super-SBM model to flag the bank holding companies, and the results supported both the DEA's discriminant and predictive power. Scholars also compared the DEA method with the Logit model in failure prediction performance, and proved that DEA has higher estimation accuracy (Premachandra et al., 2009; Sanchez González et al., 2020).

When banks are observed over multiple time periods, their performance can be evaluated not only in cross sections with their peers, but they can also be compared to themselves over previous years in a time series. Malmquist DEA is one such model that can fulfil this task (Cooper et al., 2006). Many authors have documented empirical studies on banks regarding their performance dynamics and productivity growth over time (Casu et al., 2004; Kao & Liu, 2014; Sturm & Williams, 2004; Wheelock & Wilson, 2009). The literature related to bank failure prediction is shown in Table 1 that most of the literature is written from the perspective of static research, and that there has been no research which directly predicts bank failure by employing a multi-period DEA in a dynamic analysis of bank efficiency. This paper extends the application of the DEA to failure prediction, and proposes a nonparametric method to classify failed/non-failed banks directly and dynamically. Dynamic efficiency score is a potential early warning indicator to assess bank performance, and we introduce its methodology in the next section.

### 3 Methodology

#### 3.1 DEA and Malmquist DEA

Previous papers linking efficiency measures calculated by DEA to bank failure prediction have been limited to cross-sectional analysis. A dynamic analysis under different macroeconomic conditions is thus recommended in this paper for better bankruptcy prediction. We propose a nonparametric method, Malmquist DEA with Worst Practice Frontier, to assess the bankruptcy risk of banks over multiple periods.

In a production process, productivity or efficiency is usually defined as the weighted outputs divided by the weighted inputs:

$$\theta_j = \frac{\mathbf{u}^T \mathbf{y}_j}{\mathbf{v}^T \mathbf{x}_j} = \frac{\sum_{r=1}^q u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \quad j = 1, 2, \dots, n \quad (1)$$

where  $x_{ij}$  ( $i = 1, \dots, m$ ) and  $y_{rj}$  ( $r = 1, \dots, q$ ) are the inputs and outputs for the  $j$ th DMU to be evaluated;  $m$  and  $q$  are the numbers of inputs and outputs;  $v_i$  ( $i = 1, \dots, m$ ) and  $u_r$  ( $r = 1, \dots, q$ ) are their respective weights.

DEA is an optimising technique to identify the most efficient DMUs, which form the efficient frontier, and the relative efficiency of other DMUs is measured by the distance to the reference points on the frontier. Suppose there are  $n$  DMUs, a Slacks-Based Measure (SBM) model (Tone, 2001) under Variable Returns to Scale (VRS) conditions can give feasible solutions, and their input-oriented and output-oriented programming questions are written as:

**Table 1** Summary of literature in bank failure prediction

Literature	Sample	Method	Style
Martin (1977)	US banks	LR	Static
West (1985)	US banks	Factor Analysis, LR	Static
Lane et al. (1986)	US banks	HM	Static
Tam (1991)	US banks	NN, LR, DA, K-NN, DT	Static
Tam and Kiang (1992)	US banks	DA, LR, k-NN, NN, DT	Static
Barr et al. (1994)	US banks	DEA	Static
Wheelock and Wilson (1995)	US banks	DEA, HM	Static
Barr and Siems (1997)	US banks	DEA	Static
Bell (1997)	US banks	NN, LR	Static
Olmeda and Fernández (1997)	Spanish banks	NN, LR, DT, DA, etc	Static
Alam et al. (2000)	US banks	NN, Fuzzy Clustering	Static
Swicegood and Clark (2001)	US banks	DA, NN	Static
Kolari et al. (2002)	US banks	TR, LR	Static
Luo (2003)	US banks	DEA	Static
Kao and Liu (2004)	Taiwan banks	DEA	Static
Canbas et al. (2005)	Turkish banks	DA, LR, Probit	Static
Lanine and Vennet (2006)	Russian banks	LR, TR	Static
Halling and Hayden (2008)	Austrian banks	HM, LR	Dynamic
Celik and Karatepe (2007)	Turkish banks	NN	Static
Kick and Koetter (2007)	German banks	LR	Static
Davis and Karim (2008)	UK banks	LR, DT	Static
Ravi and Pramodh (2008)	Spanish banks	NN	Static
Zhao et al. (2009)	US banks	LR, DT, NN, k-NN	Static
Boyacioglu et al. (2009)	Turkish banks	NN, SVM, K-means, DA, LR	Static
Tsionas and Papadakis (2010)	Greek banks	DEA	Static
Reynaud (2010)	Turkish bank	DEA,SFA	Static
Fiordelisi and Mare (2013)	Italish banks	HM	Static
Zaghdoudi (2013)	Tunisian banks	LR	Static
Erdogan (2013)	Turkish banks	SVM	Static
Avkiran and Cai (2014)	UAE banks	DEA	Static
Wanke et al. (2015)	Brazilian banks	DEA	Dynamic
Almanidis and Sickles (2016)	US banks	SFA, HM	Static
Pagratris et al. (2017)	Greek banks	DEA	Static
Othman and Asutay (2018)	Islamic banks	DA, LR, Probit	Static
Halteh et al. (2018)	Islamic banks	DT, Gradient Boosting, RF	Static
Shrivastava et al. (2020)	Indian banks	SVM	Static
Manthoulis et al. (2020)	US banks	SVM, LR, Rusboost, etc	Dynamic
Shrivastava et al. (2020)	Indian banks	Lasso, Boosting	Static
Filippopoulou et al. (2020)	European banks	LR	Static
Pham and Ho (2021)	US banks	Adaboost, XGboost, Gradient boosting	Static
González et al. (2021)	US banks	Bayesian approach, HM, SFA	Static

*NN* Neural Network, *LR* Logistic regression or Logit, *DA* Discriminant Analysis, *KNN* K-nearest neighbour, *DT* Decision Tree, *TR* Trait Recognition, *HM* Hazard model, *SVM* Support Vector Machine, *SFA* Stochastic Frontier Analysis, *RF* Random Forest

VRS-Input-SBM:

$$\begin{aligned}
 \min \rho &= 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / x_{i0}^t \\
 \text{s.t. } X^t \lambda + s^- &= x_0^t \\
 Y^t \lambda &\geq y_0^t \\
 e\lambda &= 1 \\
 \lambda \geq 0, s^- &\geq 0
 \end{aligned} \tag{2}$$

VRS-output-SBM:

$$\begin{aligned}
 \min \rho &= \frac{1}{1 + \frac{1}{q} \sum_{i=1}^q s_r^+ / y_{r0}^t} \\
 \text{s.t. } X^t \lambda &\leq x_0^t \\
 Y^t \lambda - s^+ &= y_0^t \\
 e\lambda &= 1 \\
 \lambda \geq 0, s^+ &\geq 0
 \end{aligned} \tag{3}$$

where  $DMU_0$  with inputs and outputs vector  $(x_0, y_0)$  is the bank to be evaluated;  $X$  and  $Y$  are the matrices of inputs and outputs of the banking group;  $s^-$  and  $s^+$  are vectors of slacks;  $\lambda$  is a non-negative vector and  $\sum_{j=1}^n \lambda_j = 1$ ;  $e$  is a column vector of ones with dimension of  $n \times 1$ .

If the constraint condition  $e\lambda = 1$  is deleted, the efficiency score is calculated under the assumption of Constant Returns to Scale (CRS). In order to adapt to a situation where there are negative values in both input and output sides, Sharp et al. (2007) further developed the SBM model, which is called modified-SBM (MSBM) and only adapt to the VRS condition. The MSBM model is obtained as follows:

VRS-input-MSBM:

$$\begin{aligned}
 \min \rho &= 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / R_{i0} \\
 \text{s.t. } X\lambda + s^- &= x_0 \\
 Y\lambda &\geq y_0 \\
 e\lambda &= 1 \\
 \lambda \geq 0, s^- &\geq 0 \\
 R_{i0} &= x_{i0} - \min(x_i)
 \end{aligned} \tag{4}$$

VRS-output-MSBM:

$$\begin{aligned}
 \min \rho &= \frac{1}{1 + \frac{1}{q} \sum_{i=1}^q s_r^+ / R_{r0}} \\
 \text{s.t. } X\lambda &\leq x_0 \\
 Y\lambda - s^+ &= y_0 \\
 e\lambda &= 1 \\
 \lambda &\geq 0, s^+ \geq 0 \\
 R_{r0} &= \max(y_r) - y_{r0}
 \end{aligned} \tag{5}$$

In a single period, suppose  $\theta_0^t(x_0^t, y_0^t)$  is the optimal solution to Problem (4) or (5). If DMU<sub>0</sub> is observed over two continuous periods  $t$  and  $t + 1$ , the optimal solutions within each period are denoted by  $\theta_0^t(x_0^t, y_0^t)$  and  $\theta_0^{t+1}(x_0^{t+1}, y_0^{t+1})$ , and then clearly Problem (4) and (5) in period  $t + 1$  become:

VRS-input-MSBM:

$$\begin{aligned}
 \min \rho &= 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / R_{i0}^{t+1} \\
 \text{s.t. } X^{t+1}\lambda + s^- &= x_0^{t+1} \\
 Y^{t+1}\lambda &\geq y_0^{t+1} \\
 e\lambda &= 1 \\
 \lambda &\geq 0, s^- \geq 0 \\
 R_{i0}^{t+1} &= x_i^{t+1} - \min(x_i^{t+1})
 \end{aligned} \tag{6}$$

VRS-output-MSBM:

$$\begin{aligned}
 \min \rho &= \frac{1}{1 + \frac{1}{q} \sum_{i=1}^q s_r^+ / R_{r0}^{t+1}} \\
 \text{s.t. } X^{t+1}\lambda &\leq x_0^{t+1} \\
 Y^{t+1}\lambda - s^+ &= y_0^{t+1} \\
 e\lambda &= 1 \\
 \lambda &\geq 0, s^+ \geq 0 \\
 R_{r0}^{t+1} &= \max(y_r^{t+1}) - y_{r0}^{t+1}
 \end{aligned} \tag{7}$$

This is a simple shift of inputs and outputs from period  $t$  to period  $t + 1$ , where the reciprocal efficiency  $\theta_0^t(x_0^{t+1}, y_0^{t+1})$  or  $\theta_0^{t+1}(x_0^t, y_0^t)$  are the optimal solutions to Problem (4) and (6) for input orientation, with the reference set being formed by DMUs in period  $t$  or in period  $t + 1$  as follows:



**Table 2** Number of observations in the dataset

Number of banks				Number of observations in Panel	
Year	Failed	Active	Total	Panel	Total
2002	0	4426	4426		
2003	0	4426	4426	2002–2003	8852
2004	1	4425	4426	2002–2004	13,277
2005	1	4424	4425	2002–2005	17,701
2006	1	4423	4424	2002–2006	22,124
2007	3	4420	4423	2002–2007	26,544
2008	12	4408	4420	2002–2008	30,952
2009	60	4348	4408	2002–2009	35,300
2010	87	4261	4348	2002–2010	39,561
2011	43	4218	4261	2002–2011	43,779
2012	29	4189	4218	2002–2012	47,968
2013	13	4176	4189	2002–2013	52,144
2014	5	4171	4176	2002–2014	56,315
2015	6	4165	4171	2002–2015	60,480
2016	4	4161	4165	2002–2016	64,641

$$\begin{aligned}
\min \theta_0^t(x_0^{t+1}, y_0^{t+1}) &= 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / R_{i0}^t \\
s.t. \quad X^t \lambda + s^- &= x_0^{t+1} \\
Y^t \lambda &\geq y_0^{t+1} \\
e\lambda &= 1 \\
\lambda &\geq 0, s^- \geq 0
\end{aligned} \tag{8}$$

$$\begin{aligned}
R_{i0}^t &= x_{i0}^{t+1} - \min(x_i^t) \\
\min \theta_0^{t+1}(x_0^t, y_0^t) &= 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / R_{i0}^{t+1} \\
s.t. \quad X^{t+1} \lambda + s^- &= x_0^t \\
Y^{t+1} \lambda &\geq y_0^t \\
e\lambda &= 1 \\
\lambda &\geq 0, s^- \geq 0 \\
R_{i0}^{t+1} &= x_{i0}^t - \min(x_i^{t+1})
\end{aligned} \tag{9}$$

**Table 3** Descriptive statistics of financial ratios

Variables	Active bank				Failed bank				Mean diff t-test	Wilcoxon rank Test (Z)
	N	Mean	Std	Min	Max	N	Mean	Std	Min	Max
TETA	58,254	10.96	3.75	– 1.90	100.00	2226	9.26	5.75	– 5.32	92.95
TCETA	58,254	10.68	3.75	– 1.90	100.00	2226	8.90	5.57	– 5.32	92.95
LADSTF	58,254	35.37	18.93	0.02	432.80	2226	25.20	24.23	1.04	493.10
CLATA	58,254	61.05	15.44	0.10	97.30	2226	71.41	13.74	2.38	97.05
GrowthTA	58,254	6.55	17.96	– 89.28	895.94	2226	15.12	33.83	– 89.35	523.97
GGCLA	58,254	7.02	22.68	– 100.00	944.67	2226	17.62	49.41	– 81.55	903.13
GNCLAs	58,254	7.04	22.80	– 100.00	938.87	2226	17.35	49.77	– 81.92	921.84
LLRGCLA	58,254	1.53	0.90	0.00	31.82	2226	1.99	1.59	0.16	21.29
UILTE	58,254	0.67	15.85	– 652.76	932.10	2226	34.66	104.99	– 616.84	910.88
ROAE	58,254	8.68	10.08	– 661.64	148.77	2226	– 10.92	56.92	– 840.16	314.52
OPATE	58,254	10.55	11.02	– 661.64	233.42	2226	– 8.58	57.48	– 839.63	314.52
ROAA	58,254	0.92	0.93	– 22.16	17.47	2226	– 0.24	2.69	– 26.79	15.95
IIAIEA	58,254	4.94	1.36	0.04	23.34	2226	6.20	1.46	1.16	17.08
IEAIBL	58,254	1.61	0.99	0.00	7.34	2226	2.57	0.99	0.00	6.74
GLACCD	58,254	74.56	21.07	0.00	785.41	2226	87.96	20.18	8.84	444.60
IECDACD	58,254	1.53	0.97	– 0.04	6.26	2226	2.50	1.01	0.00	6.74
NONIOR	58,254	15.72	16.09	– 978.01	437.33	2226	14.05	39.85	– 445.29	919.29
LATA	58,254	30.38	15.73	0.02	99.08	2226	20.93	12.55	0.93	91.69
CAAR	58,254	3.07	1.35	0.06	70.91	2226	3.61	1.95	0.17	26.70
LACMILAC	58,254	38.73	17.67	0.00	103.78	2226	53.36	18.87	0.00	113.69
LAASMTA	58,254	33.96	16.09	0.02	99.08	2226	22.19	12.89	0.94	91.69

The full names and descriptions of ratios are showed in Appendix

\*\*\*p &lt; 0.01, \*\*p &lt; 0.05, \*p &lt; 0.1

**Table 4** The eigenvalue results of PCA

Component	Eigenvalue	Proportion (%)	Cumulative (%)	Component	Eigenvalue	Proportion (%)	Cumulative (%)
<b>1</b>	<b>5.53</b>	<b>26.32</b>	<b>26.32</b>	<b>12</b>	<b>0.39</b>	<b>1.86</b>	<b>96.91</b>
<b>2</b>	<b>3.30</b>	<b>15.70</b>	<b>42.01</b>	<b>13</b>	<b>0.28</b>	<b>1.32</b>	<b>98.23</b>
<b>3</b>	<b>2.49</b>	<b>11.84</b>	<b>53.86</b>	<b>14</b>	<b>0.18</b>	<b>0.86</b>	<b>99.09</b>
<b>4</b>	<b>2.08</b>	<b>9.92</b>	<b>63.77</b>	<b>15</b>	<b>0.10</b>	<b>0.47</b>	<b>99.56</b>
<b>5</b>	<b>1.81</b>	<b>8.62</b>	<b>72.39</b>	<b>16</b>	<b>0.03</b>	<b>0.13</b>	<b>99.69</b>
<b>6</b>	<b>1.37</b>	<b>6.52</b>	<b>78.91</b>	<b>17</b>	<b>0.03</b>	<b>0.11</b>	<b>99.82</b>
<b>7</b>	<b>0.91</b>	<b>4.32</b>	<b>83.23</b>	<b>18</b>	<b>0.02</b>	<b>0.04</b>	<b>99.92</b>
<b>8</b>	<b>0.75</b>	<b>3.57</b>	<b>86.80</b>	<b>19</b>	<b>0.01</b>	<b>0.03</b>	<b>99.97</b>
<b>9</b>	<b>0.69</b>	<b>3.30</b>	<b>90.10</b>	<b>20</b>	<b>0.01</b>	<b>0.00</b>	<b>100.00</b>
<b>10</b>	<b>0.63</b>	<b>3.00</b>	<b>93.10</b>	<b>21</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>
<b>11</b>	<b>0.41</b>	<b>1.94</b>	<b>95.04</b>				
Kaiser–Meyer–Olkin measure of sampling adequacy (KMO test)							
							<b>0.705</b>

**Table 5** Regression results

	Logit regression		Rare event logit regression		Worst practice Frontier DEA	Best practice Frontier DEA
	Coef	St. Err	Coef	St. Err		
Component 1	0.523***	0.025	0.522***	0.061	Output	Input
Component 2	− 0.540***	0.015	− 0.538***	0.034	Input	Output
Component 3	0.100***	0.031	0.103***	0.063	Output	Input
Component 4	0.505***	0.035	0.504***	0.078	Output	Input
Component 5	− 0.561***	0.047	− 0.558***	0.146	Input	Output
Component 6	0.259***	0.024	0.259***	0.039	Output	Input
Constant	− 6.342***	0.096	− 6.332***	0.201	—	—

**Table 6** Robust test results

Independent variable: growth of NPLs

Variables	(1) POLS	(2) FE	(3) FE	(4) FE
Component 1	5.675*** (0.288)	11.260*** (0.547)	3.740*** (0.354)	11.482*** (0.758)
Component 2	− 4.355*** (0.367)	− 6.728*** (0.507)	− 3.369*** (0.362)	− 4.127*** (0.503)
Component 3	1.597*** (0.363)	0.802* (0.472)	0.577 (0.367)	1.348*** (0.513)
Component 4	10.727*** (0.451)	10.073*** (0.689)	5.308*** (0.763)	13.051*** (1.404)
Component 5	− 6.279*** (0.480)	− 4.480*** (0.865)	− 4.031*** (0.570)	− 6.431*** (0.970)
Component 6	0.494 (0.541)	0.202 (0.942)	0.341 (0.509)	1.727* (0.904)
Constant	34.458*** (0.639)	34.449*** (0.034)	27.294*** (2.731)	12.973*** (3.229)
Observations	60,480	60,480	60,480	60,480
R-squared	0.024	0.027		0.037
Number of id		4426	4426	4426
Entity FE		YES		YES
Time FE			YES	YES

Robust standard errors in parentheses

\*\*\*p &lt; 0.01, \*\*p &lt; 0.05, \*p &lt; 0.1

**Table 7** Mean score results of the dynamic model

Year	Active		Failed		Mean Diff t-test	Wilcoxon rank test (Z)
	Obs	Score	Obs	Score		
2002–2003	8322	0.830	530	0.850	– 0.020***	– 13.427***
2002–2004	12,483	0.841	794	0.857	– 0.017***	– 14.922***
2002–2005	16,644	0.828	1057	0.846	– 0.018***	– 22.514***
2002–2006	20,805	0.829	1319	0.847	– 0.018***	– 23.451***
2002–2007	24,966	0.832	1578	0.853	– 0.021***	– 27.394***
2002–2008	29,127	0.816	1825	0.836	– 0.020***	– 29.191***
2002–2009	33,288	0.810	2012	0.831	– 0.021***	– 30.309***
2002–2010	37,449	0.804	2112	0.838	– 0.034***	– 29.171***
2002–2011	41,610	0.808	2169	0.839	– 0.031***	– 23.763***
2002–2012	45,771	0.805	2197	0.835	– 0.030***	– 17.843***
2002–2013	49,932	0.802	2212	0.832	– 0.030***	– 12.806***
2002–2014	54,093	0.801	2222	0.826	– 0.025***	– 10.135***
2002–2015	58,254	0.800	2226	0.819	– 0.019***	– 5.567***

'Score' is the efficiency score under the global reference framework

\*\*\*p < 0.01, \*\*p < 0.05, \*p < 0.1

And for output orientation, Problems (5) and (7) are rewritten accordingly, as:

$$\begin{aligned}
 \min \theta_0^t(x_0^{t+1}, y_0^{t+1}) &= \frac{1}{1 + \frac{1}{q} \sum_{i=1}^q s_r^+ / R_{r0}^t} \\
 \text{s.t. } X^t \lambda &\leq x_0^{t+1} \\
 Y^t \lambda - s^+ &= y_0^{t+1} \\
 e\lambda &= 1 \\
 \lambda &\geq 0, s^+ \geq 0 \\
 R_{r0}^t &= \max(y_r^t) - y_{r0}^{t+1}
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 \min \theta_0^{t+1}(x_0^t, y_0^t) &= \frac{1}{1 + \frac{1}{q} \sum_{i=1}^q s_r^+ / R_{r0}^{t+1}} \\
 \text{s.t. } X^{t+1} \lambda &\leq x_0^t \\
 Y^{t+1} \lambda - s^+ &= y_0^t \\
 e\lambda &= 1 \\
 \lambda &\geq 0, s^+ \geq 0 \\
 R_{r0}^{t+1} &= \max(y_r^{t+1}) - y_{r0}^t
 \end{aligned} \tag{11}$$

The Malmquist Production Index or MI is a measure of comparing the efficiency of two periods. According to Caves et al. (1982) and Färe et al. (1992), the MI under CRS conditions

from period  $t$  to period  $t + 1$ , can be defined as

$$M_t^{t+1} = \sqrt{\frac{\theta_0^t(x_0^{t+1}, y_0^{t+1})}{\theta_0^t(x_0^t, y_0^t)} \cdot \frac{\theta_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{\theta_0^{t+1}(x_0^t, y_0^t)}} \quad (12)$$

Färe et al. (1992) decomposed this formula into efficiency change and technical change. A technical change greater than 1 means that the production frontier has moved forward and production efficiency therefore increases. When this formula is based on CRS, the efficiency change can be further decomposed. Ray and Desli (1997) proposed an alternative method in VRS and CRS. They decomposed  $M_t^{t+1}$  into technical change (TECH), pure efficiency change (PEFFCH) and scale efficiency change (SCH) with VRS. If  $\theta_0^t(x_0^t, y_0^t|V)$  indicates the efficiency score under VRS and  $\theta_0^t(x_0^t, y_0^t|C)$  indicates the efficiency score under CRS, the decomposition can be expressed in the following form:

$$M(x^{t+1}, y^{t+1}, x^t, y^t) = TECH \times PEFFCH \times SCH \quad (13)$$

$$TECH = \sqrt{\frac{\theta_0^t(x_0^t, y_0^t|V)}{\theta_0^{t+1}(x_0^t, y_0^t|V)} \cdot \frac{\theta_0^t(x_0^{t+1}, y_0^{t+1}|V)}{\theta_0^{t+1}(x_0^{t+1}, y_0^{t+1}|V)}} \quad (14)$$

$$PEFFCH = \frac{\theta_0^{t+1}(x_0^{t+1}, y_0^{t+1}|V)}{\theta_0^t(x_0^t, y_0^t|V)} \quad (15)$$

$$SCH = \sqrt{\frac{S_0^t(x_0^{t+1}, y_0^{t+1})}{S_0^t(x_0^t, y_0^t)} \cdot \frac{S_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{S_0^{t+1}(x_0^t, y_0^t)}} \quad (16)$$

$$S_0^t(x_0^t, y_0^t) = \frac{\theta_0^t(x_0^t, y_0^t|C)}{\theta_0^t(x_0^t, y_0^t|V)} \quad (17)$$

If Eq. (14) multiplies Eq. (15), we can obtain the MI under VRS. In other words, the CRS MI can be decomposed into a VRS MI and a scale component (Färe et al., 1998) in Eqs. (18) and (19):

$$M(x^{t+1}, y^{t+1}, x^t, y^t) = M(x^{t+1}, y^{t+1}, x^t, y^t|V) \times SCH \quad (18)$$

$$M(x^{t+1}, y^{t+1}, x^t, y^t|V) = PEFFCH \times TECH \quad (19)$$

When DMUs are observed over multiple periods  $1, 2, \dots, T$ , we can calculate any efficiency  $\theta_a^b$  at period  $a$  ( $a = 1, 2, \dots, T$ ) with reference to another period  $b$  ( $b = 1, 2, \dots, T$ ) where  $a \geq b$ . For the model we proposed in the context of bankruptcy prediction, we have a priori information from year 1 to year  $c$  and, based on the data of  $c$  years, we are therefore able to make predictions on what is happening in year  $c + l$ , depending on how many years in advance we want to give early warnings ( $l$  being the number of years of early warnings). In this sense, we expect the efficiency score to be measured as precisely as possible. In the dynamic model we propose, rather than comparing units for one period, we are able to calculate the dynamic DEA scores with reference to the efficient frontier formed by the most efficient units which exist over the entire period from year 1 to  $c$ , which is referred to as the ‘global reference’ (Pastor & Lovell, 2005).

### 3.2 Research design

In the case of bank failure prediction, we have banks' financial ratio information for  $T$  years. In order to evaluate the predictive performance of our proposed model, we run the DEA model based on data in years  $[1, t]$  ( $t = 1, 2, \dots, c$ ) first in a global sense, and we then predict failure in terms of specific years  $[t + 1, t + l]$  ( $t = 1, 2, \dots, T - l$ ) with the modelling window moving along with the increase of  $t$ . As time goes by, dynamic efficiency scores incorporate more information, since additional data is added to the global reference set. Predictive accuracy is evaluated on different panels with varying time gaps of early warnings. In order to test whether our dynamic panel DEA model has a more robust and reliable predictive power than the static model, we set up the traditional DEA model as the benchmark model, which is built on data for the current year. In Sect. 5, we also attempt various extensions and robustness tests to explore optimal Malmquist DEA on our 'moving datasets'. The research framework is illustrated in Fig. 1.

Since 1985, DEA has been applied to evaluate the efficiency of banks, and this method has become popular throughout the banking industry. This paper employs simple DEA as a benchmark, since we expect to be able to identify those banks which are likely to fail in advance, while they are still running concurrent to other, healthy banks.

A bank's annual reports and balance sheets reflect their real operations and profitability, which are the key indicators of its stability. Conventionally, the values in a bank's balance sheet can be further converted into financial ratios, as either input or output variables in DEA. Starting from Beaver (1966), financial ratios have dominated bankruptcy prediction models for decades (Lane et al., 1986; Lanine & Vennet, 2006; Martin, 1977) and they have also been used as inputs and outputs in DEA models in previous literature (Cielen et al., 2004;

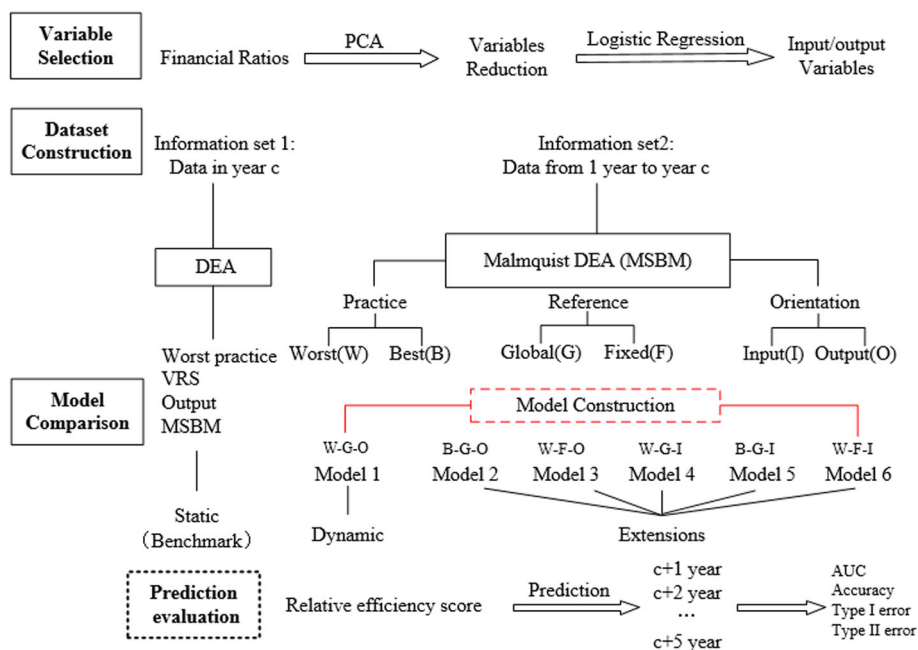
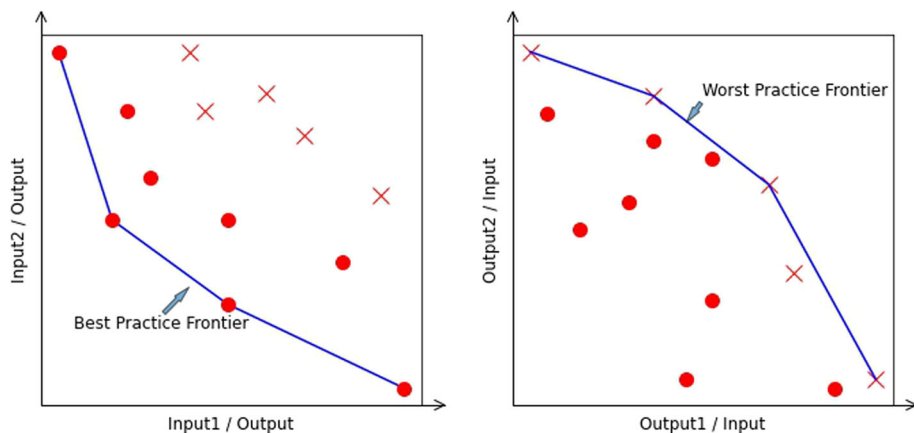


Fig. 1 Research design



**Fig. 2** VRS best & worst practice frontier. The dots represent active banks and the crosses represent failed ones

Min & Lee, 2008; Premachandra et al., 2011). However, the inclusion of financial ratios may raise the issue of negative values, which cause efficiency to be incorrectly measured in DEA models. To tackle this, we follow Sharp et al. (2007) and employ the Modified-SBM model (Problem (6) and (7)), which allows negative values to exist on both the input and output sides of DEA.

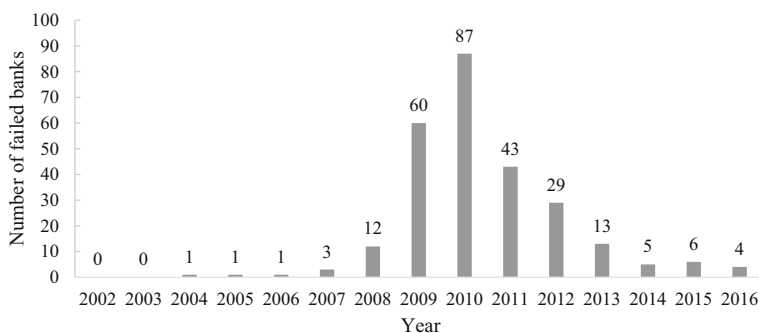
In terms of the various applications of DEA in bankruptcy prediction, other than calculating the efficiency in terms of best practice, Paradi et al. (2004) suggested an alternative application to calculate a bank's inefficiency, by referring to a 'Worst Practice Frontier'. Intuitively, a Best Practice Frontier DEA identifies the most efficient units on the frontier, whereas the Worst Practice Frontier DEA identifies the most inefficient units on the frontier (as illustrated in Fig. 2). This can be done by simply switching the sides of inputs and outputs and solving problems (7), (10) and (11). This modelling setting is Malmquist DEA with VRS-Output-MSBM and Worst Practice Frontier.

## 4 Data and variable selection

### 4.1 Sample description

Orbis Bank Focus is one of BvD's products which includes information pertaining to over 135,000 banks across the world, and so it is used as our data source. In this paper, US banks from 2002 to 2016 are selected as the sample in our analysis. The homogeneity of the DMUs is an important assumption in DEA theory, because they should be comparable as peers. As Dyson et al. (2001) suggested, DMUs (in this study, banks) should firstly be engaged in similar activities and produce similar products; they should secondly employ similar resources in production; and thirdly act in a similar market environment. For banks, the former two conditions can be easily met, however international banks are not comparable in the DEA framework. We decide that only savings banks and commercial banks in the US should be chosen as the samples, because the conditions in retail banking and their market environment are similar, and a considerable number of failed banks are needed to effectively test the models.





**Fig. 3** Frequency of bank failures in the period 2002–2016

During the recent subprime and financial crises, many US banks suffered great losses and eventually filed for bankruptcy, were held in receivership by other organisations or simply remained in liquidation. Thus, bankruptcy, liquidation and receivership are the three failure events being considered in this research. 368 US failed banks have been observed over the periods 2002 to 2019, while 5068 US banks have survived over the same period. The active banks remain in the models set up each year for all time periods, whereas observations of the failed banks are removed from the sample upon filing for bankruptcy. It is in this way that the model for each year was set up based on balanced panel data. In our dataset, the sample consists of 4426 banks, of which 265 banks encountered one of the aforementioned failure events in the period from 2002 to 2016. The distribution of bank failures during the period 2002 to 2016 is presented in Fig. 3. Table 2 describes the number of banks and observations for each year.

It is clear from Fig. 3 that the occurrence of bank failure remained at a relatively low level before 2008, whereas shortly after the start of the crisis in 2008, many US banks collapsed. In particular, we can see that the impact of the financial crisis on the banking sector had a lagged effect, with the number of failed banks reaching its peak in 2010. In this paper, data related to banks from 2002 to 2016 have been selected as the sample in our analysis since there are no failed banks in the period 2017–2019. The financial ratios from 2002 to 2015 have been used in the model to calculate the dynamic relative efficiency scores, and the information as to which banks carried a bankruptcy label in the years 2004 to 2016 has been used to verify the overall predictive performance. Due to missing data on key variables, our final sample consists of 4161 active banks and 265 failed banks.

## 4.2 Variable selection

Efficiency calculated using DEA is sensitive to the selection of inputs and outputs, and there have been various types of selections used in the past literature. Berger and Humphrey (1997) proposed both the production and the intermediation approaches in selecting variables for bank efficiency studies. In the first approach, banks use labor and capital to provide loans and deposit services, whereas in the second approach, banks serve as intermediaries between borrowers and lenders. However, while neither approach is perfect, the selection of inputs and outputs is nevertheless very flexible, where generally deposits are used as inputs (Amin & Ibn Boamah, 2020; Razipour-GhalehJough et al., 2021) and loans are used as outputs

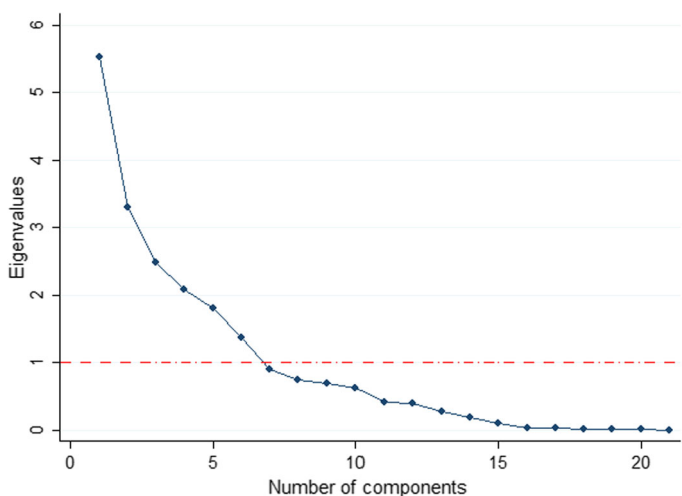
(Lartey et al., 2021), but there are also examples where deposits are used as outputs (Maudos et al., 2002) or both inputs and outputs (Li et al., 2021).

In bank efficiency analyses, it has also been popular to use financial ratios, examples being Isik and Uygur (2021) and Mohtashami and Ghiasvand (2020). Premachandra et al. (2009) gave a tip as to how best to choose financial ratios as inputs or outputs of DEA, so that the larger ratio value which could lead to bankruptcy be used as an input, or vice versa (if the model is in the Best Practice format). More specifically, Halkos and Salamouris (2004) used no inputs at all, instead only using ratios as outputs in their banking DEA model. We have chosen financial ratios as either the input or output variables in our work, and we solve the negative ratio value by applying MSBM (Sharp et al. 2007), giving us a result which is both translation invariant and units invariant.

According to the aforementioned CAMEL principle, 40 major financial ratios are extracted from the database, which can be grouped into five categories: capital, asset quality, management, earning and liquidity. However, the ratios related to management have more than 90% missing values and thus cannot be included in our research. The variables with over 20% missing values for the other four categories have also been deleted. 21 ratios are ultimately retained, and the full definitions are shown in the Appendix. We winsorize the continuous variables at the 1% and 99% levels, because the DEA efficiency frontier can be affected dramatically by outliers. The descriptive statistics of selected variables is shown in Table 3. The mean difference test and the Wilcoxon rank test show that the financial ratios we choose have significant differences between the active group and failed group at the 1% confidence level.

Since some of the ratios are in the same category, this may present a multi-collinearity problem. In addition, DEA optimisation becomes very complex when given a large number of input/output variables. Based on the above two considerations, we reduce the dimensions of the variables by Principal Components Analysis (PCA). PCA is a common statistical method which compresses information pertaining to all variables into several factors, thus maintaining information within an acceptable attribution. Its main idea is to establish a new multi-dimensional coordinate plane, then to project the most variable information onto the axis, on the condition that the number of axes is fewer than the number of variables. Examples of a PCA integrated DEA can be found in Štefko et al. (2021), Liang et al. (2009), Premachandra et al. (2009), Adler and Yazhemsy (2010), etc.. It has been shown that PCA can improve the discriminant power of the overall analysis (Adler & Yazhemsy, 2010). In general, the components with an eigenvalue of more than 1 are considered suitable to be included in the transformed factor. The first six components are chosen to fit into our model, and the cumulative proportion is up to 78.91%. The sample is appropriate for PCA if the KMO statistic is higher than 0.7. Our KMO statistic and the eigenvalue results are shown in the following Table 4 and Fig. 4.

In order to identify DEA inputs and outputs from the six components, logistic regression is applied to the sample. Much literatures such as Manthoulis et al. (2020) and Canbas et al. (2005) argue that bank failure can be detected by early warnings up to 3 years in advance. Thus, the dependent variable in regression is whether a bank has failed or not within a period of three years. We use both logistic regression and rare event logistic regression to determine the inputs/outputs, the latter of these models having been widely used in failure event studies (Eling & Jia, 2018; King & Zeng, 2001). We construct a Worst Practice Frontier DEA following the ideas of Cielen et al. (2004) and Premachandra et al. (2009), wherein variables negatively associated with the possibility of bank failure are determined as inputs, and those positively associated with failure are determined as outputs. In Table 5, the six components are all significant ( $p$ -value  $< 0.01$ ) in the prediction of bank failure. In the ‘Worst



**Fig. 4** The scree plot of PCA results

Practice’, the DMUs with the worst performance are put on the frontier and thus have large relative inefficiency scores. As such, Components 2 and 5 are therefore input variables in the Worst Practice Frontier DEA models, and outputs in the Best Practice Frontier DEA models. Components 1, 3, 4 and 6 are outputs for the Worst Practice or inputs for the Best Practice. The results of both practices will be compared in the next section.

To verify the judgement of the inputs and outputs selection and its relevance to bank failure, we implement panel regressions for robustness tests. The non-performing loan ratio (NPL) has a close positive correlation with failure within the banking industry. Indeed, a dramatic increase in non-performing loans is the main reason for a bank’s failure (Jin et al., 2011; Liu & Ngo, 2014). We therefore make the first group of robust tests by using the growth rate of NPLs as the proxy of the active/failed dummy variables to construct the panel regressions of Pooled OLS (POLS) and Fixed Effect (FE) models. The robust test results are shown in Table 6, where the signs of coefficients are not varied.

## 5 Results

### 5.1 Dynamic DEA score

The software package ‘MaxDEA X’ is used to calculate efficiency scores in our analysis, which contains many extensions of DEA and computes the data rapidly. The relative inefficiency for each DMU (i.e. each bank) can be calculated in the context of the MSBM and its global reference. The models are built dynamically each time when the information for a new year is added to the framework. In Year 2016, there are only 4 failed banks. Since then, no failed banks are observed so that we cannot further validate the predictive performance of our model. We therefore only present the results until 2015. Table 7 shows the dynamic mean scores of active and failed banks from 2003 to 2015, where all the relative efficiency scores pass the significant t-test and Wilcoxon rank test at the 1% confidence level. The sign

of the mean difference is also consistent with the economic intuition, i.e. the failed banks have higher relative inefficiencies under the Worst Practice Frontier DEA framework.

Table 8 shows the statistical *t*-test and Wilcoxon rank test of both the pure efficiency change (PEFFCH) and technical change (TC), obtained after the decomposition of the VRS Malmquist index with global reference as Eq. (19). We find that the MI is a good indicator to help discriminate between two types of banks. Compared to the MI, PEFFCH and TECH present no difference between the healthy and failed banks for most years studied in this sample. Thus, we can conclude that PEFFCH and TECH have poor discriminative ability and the decomposition of MI is not helpful in failure prediction.

In Malmquist DEA, by taking the global reference, the relative efficiencies of each bank can be compared not only in a cross-sectional dimension, but also in a time series. For example, Bank A (Trust Company Bank) and Bank B (The Woodbury Banking Company) are two banks that failed in 2016. Their inefficiency score in 2015 along with the average scores for all healthy banks are shown in Fig. 5. As we can see, when a bank is faced with the risk of failure, their inefficiency scores gradually increase over time. This score can thus effectively reveal early warning signs of imminent risk. In the Worst Practice Frontier, the higher the score, the more inefficient the bank. In 2015, Bank A has a score of 1 in year 2015, indicating that it was the most inefficient among all, and it did indeed fail. Further, a significant difference can be found between the healthy and failing banks. For example, Bank B had been less efficient than healthy banks since the subprime crisis, and it eventually failed in 2016. In the Best Practice model, if the score of a bank is declining, it may suffer some financial distress before its eventual liquidation.

The above pattern implies that those collapsed banks are likely to show an increase in their inefficiency and larger efficiency scores prior to their actual failure, which highlights the importance that both the bank's management and supervisors recognize the early signs of potential risks by monitoring any decrease in operating efficiency. For nearly half a century, bank efficiency has attracted constant academic interest, and now, in the aftermath of several financial crises we are learning that low productivity or efficiency in banks should be regarded as a potential core reason of eventual failure. It is commonly believed that low efficiency implies low quality management of the organisation, which could lead to an entity's bad performance in the ferociously competitive financial markets.

## 5.2 Comparison with the static model

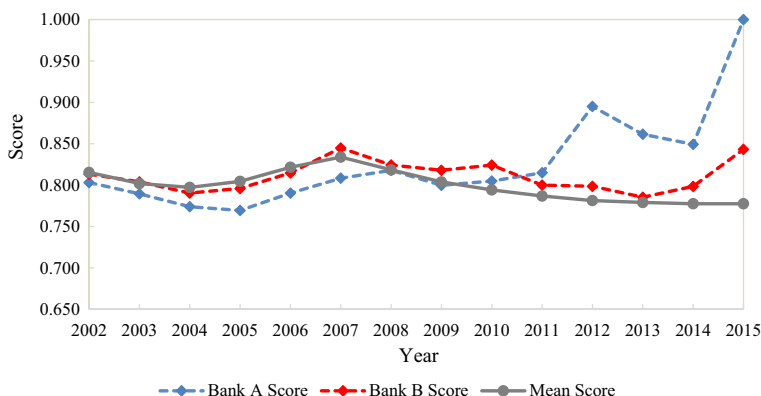
In this section, we use the traditional static model as the benchmark for comparison with our proposed model. Since the dynamic DEA models require at least two periods, we start the analysis from year 2003 and the dataset is then updated every year. In the dynamic model, the global reference set is formed with all observations in the dataset while in the static model, the reference set consists of only samples of the current year. In Table 9, the active and failed banks' scores of the static traditional model, built on the 13 cross-sectional datasets, have all passed the *t*-test and Wilcoxon rank test (1% confidence level). The difference between the static model and the dynamic models lies in whether the information of previous periods is taken into account when calculating the scores for each year. The dynamic DEA collect incremental information as the years go by, while the dataset of the static model only contains the data information of the current year in question. Figure 6 describes Tables 7 and 9, where the relative inefficiency scores in the dynamic model present a downward trend from 2003 to 2015 with some minor fluctuations, while the scores of the static model are up and down,

**Table 8** MI results of the dynamic model

Year	Active (mean)				Failed (mean)				Mean Diff. t-test				Wilcoxon rank test (Z)			
	Obs	MI	PEFFCH	TECH	Obs	MI	PEFFCH	TECH	MI	PEFFCH	TECH	MI	PEFFCH	TECH	MI	TECH
2003	8322	0.979	1.003	0.977	530	0.972	0.994	0.978	0.007***	0.009***	− 0.002**	4.461***	4.987***	− 3.060***		
2004	12,483	0.985	0.999	0.987	794	0.983	0.994	0.990	0.002*	0.005***	− 0.003***	0.178	3.951***	− 4.69***		
2005	16,644	0.995	0.991	1.005	1057	0.997	0.992	1.005	− 0.002***	− 0.002*	− 0.001	− 3.891***	− 2.404**	− 1.509		
2006	20,805	1.002	1.006	0.997	1319	1.008	1.010	0.999	− 0.006***	− 0.004***	− 0.002*	− 6.507***	− 2.735***	− 3.176***		
2007	24,966	1.004	0.985	1.022	1578	1.010	0.994	1.019	− 0.006***	− 0.008***	0.003**	− 9.867***	− 5.045***	− 0.443		
2008	29,127	1.000	0.989	1.015	1825	1.006	0.994	1.016	− 0.006***	− 0.005***	− 0.001	− 10.300***	− 1.908*	− 2.874***		
2009	33,288	0.998	0.997	1.004	2012	1.003	1.000	1.005	− 0.005***	− 0.003*	− 0.002	− 6.781***	0.190	− 3.969***		
2010	37,449	0.997	0.987	1.016	2112	1.007	0.999	1.013	− 0.010***	− 0.012***	0.003	− 10.178***	− 3.303***	− 0.014		
2011	41,610	0.996	1.008	0.996	2169	1.006	1.011	0.999	− 0.009***	− 0.004	− 0.003	− 8.357***	− 1.339	− 0.279		
2012	45,771	0.996	1.007	0.996	2197	1.003	1.013	0.995	− 0.007***	− 0.006	0.001	− 4.987***	− 2.053**	0.633		
2013	49,932	0.996	1.002	1.000	2212	1.003	1.006	1.002	− 0.007***	− 0.003	− 0.002	− 3.555***	− 1.272	0.128		
2014	54,093	0.996	1.007	0.995	2222	1.001	1.011	0.994	− 0.005***	− 0.005	0.001	− 1.982**	− 1.011	0.421		
2015	58,254	0.997	1.005	0.997	2226	1.005	1.013	0.998	− 0.009***	− 0.008	− 0.001	− 1.570	− 0.861	0.291		
Max		1.004	1.008	1.022		1.010	1.013	1.019								
Mean		0.979	0.985	0.977		0.972	0.992	0.978								
Min		0.995	0.999	1.001		1.000	1.002	1.001								

‘Score’ is the efficiency score under the global reference framework, ‘MI’ is the VRS Malmquist Index, ‘TECH’ and ‘PEFFCH’ are technical change and pure efficiency change calculated by the Eqs. [14] and [15] separately

\*\*\*p < 0.01, \*\*p < 0.05, \*p < 0.1



**Fig. 5** Comparison on the vertical and horizontal dimensions

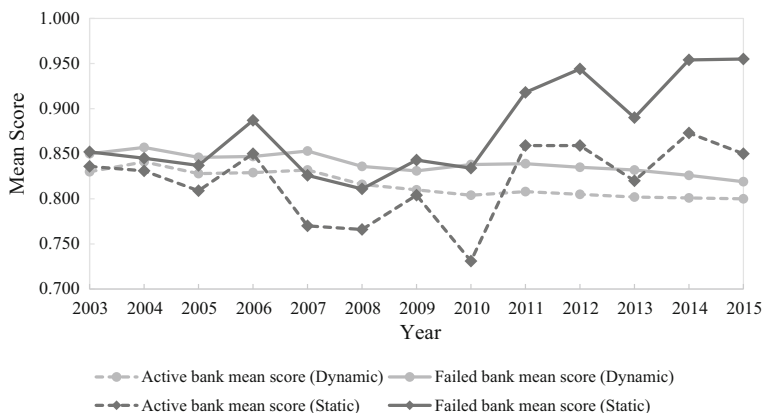
**Table 9** Mean score results of the static model

Year	Active		Failed		Mean diff. t-test	Wilcoxon rank test (Z)
	Obs	Score	Obs	Score		
2003	4161	0.836	265	0.852	− 0.016***	− 6.866***
2004	4161	0.831	264	0.845	− 0.015***	− 8.795***
2005	4161	0.809	263	0.837	− 0.028***	− 14.260***
2006	4161	0.850	262	0.887	− 0.037***	− 15.561***
2007	4161	0.770	259	0.826	− 0.056***	− 16.558***
2008	4161	0.766	247	0.811	− 0.045***	− 21.255***
2009	4161	0.804	187	0.843	− 0.040***	− 15.884***
2010	4161	0.731	100	0.834	− 0.103***	− 13.677***
2011	4161	0.859	57	0.918	− 0.060***	− 12.124***
2012	4161	0.859	28	0.944	− 0.085***	− 8.300***
2013	4161	0.820	15	0.890	− 0.070***	− 5.670***
2014	4161	0.873	10	0.954	− 0.081***	− 5.048***
2015	4161	0.850	4	0.955	− 0.105***	− 3.370***

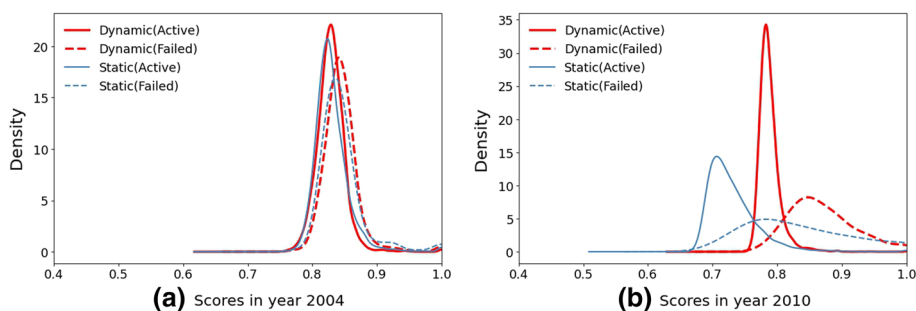
Italicized cells are the maximum and minimum mean differences of the two models separately

which is obviously inconsistent. In this way, it is reasonable to state that the static model has only a limited capacity to identify future failed banks.

We notice that the minimum and maximum mean differences between failed and active banks (italic values in Tables 7 and 9) are filed in year 2004 and 2010 respectively. We show their distributions of scores in Fig. 7. It is demonstrated that four distributions of scores are compacted together in 2004, while the scores in 2010 show distinguished discrepancy. After the financial crisis, the score gap between active and failed banks is enlarged. The classification of good and bad banks would be easier if given such a cut-off.



**Fig. 6** Mean score results of the dynamic model and the Static Model



**Fig. 7** The score distributions for years 2004 and 2010

We use AUC, accuracy, Type I error and Type II error rates to evaluate the predictive power of dynamic and static models. The cut-off of the scores is obtained at the quantile corresponding to the ratio of failed observations to total observations. The AUC (Area under the Receiver Operating Characteristic curve) is a common indicator to measure the discriminant power for binary outcomes. Accuracy is used as a ratio to measure the correct predictions over all observations. Type I error means that an active bank has been misclassified as having failed, where a Type II error means that a failed bank has been misclassified as active. If one model is superior to the other, it will have a higher AUC, accuracy, and lower type I error and type II error rates. We evaluate the predictive ability of efficiency scores in the 1 to 5 years before the bank's eventual failure separately. Table 10 shows the detailed AUC value of 1 to 5 early warning years for both the dynamic and static models.

It can be seen in Table 11 that the paired t-tests of all measures are significant at the 1% level, which means that the dynamic models show better results than their static counterpart. The dynamic model runs on a continually updating dataset, which refines itself by increasing its inclusion of effective active observations and the removal of all the observations of those banks which have failed. Therefore, the efficiency scores of each year will change with the updating of the reference set and the resultant change of the frontier. The ejecting of failed observations allows us to construct a new frontier, to better identify those banks that

**Table 10** AUC of 1–5 early warning years in dynamic and static models

	Dynamic model (Panel data)					Static model (Sectional data)						
	Early warning (years)	1	2	3	4	5	Early warning (years)	1	2	3	4	5
2002–2003		0.985	0.729	0.803	0.752	0.687	2003	0.979	0.709	0.778	0.739	0.624
2002–2004		0.999	0.915	0.786	0.641	0.677	2004	0.997	0.995	0.816	0.692	0.648
2002–2005		0.998	0.785	0.820	0.778	0.770	2005	0.996	0.778	0.834	0.771	0.757
2002–2006		0.740	0.819	0.817	0.809	0.817	2006	0.761	0.813	0.810	0.793	0.799
2002–2007		0.901	0.882	0.856	0.858	0.850	2007	0.883	0.868	0.825	0.825	0.815
2002–2008		0.950	0.934	0.929	0.917	0.917	2008	0.963	0.932	0.924	0.910	0.909
2002–2009		0.965	0.962	0.937	0.934	0.934	2009	0.878	0.876	0.855	0.851	0.852
2002–2010		0.990	0.971	0.973	0.974	0.973	2010	0.958	0.923	0.917	0.917	0.913
2002–2011		0.992	0.991	0.990	0.987	0.984	2011	0.981	0.977	0.975	0.970	0.967
2002–2012		0.992	0.992	0.986	0.982	–	2012	0.981	0.978	0.970	0.954	–
2002–2013		0.993	0.991	0.974	–	–	2013	0.903	0.930	0.923	–	–
2002–2014		0.987	0.982	–	–	–	2014	0.958	0.961	–	–	–
2002–2015		0.993	–	–	–	–	2015	0.987	–	–	–	–



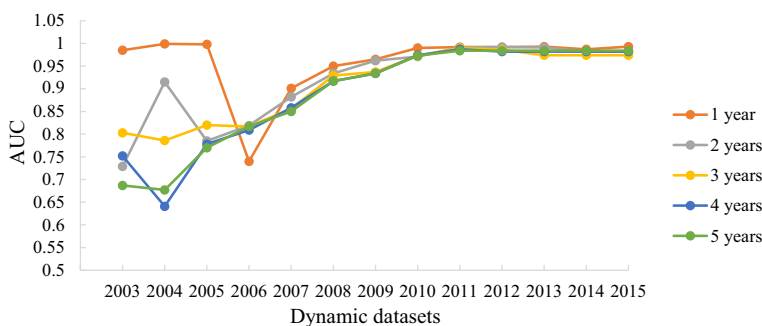
**Table 11** Paired *t* test of dynamic and static model

Metrics	Early warning (years)	1	2	3	4	5
AUC	Dynamic	0.960	0.919	0.911	0.891	0.888
	Static	0.940	0.902	0.890	0.869	0.855
	<i>t</i>	2.143	1.582	2.538	2.481	5.030
	<i>p</i> -value	0.027	0.070	0.013	0.015	0.000
Accuracy	Dynamic	99.392	98.920	98.355	97.712	97.078
	Static	98.906	98.392	97.810	97.179	96.551
	<i>t</i>	6.018	4.908	4.643	3.744	3.596
	<i>p</i> -value	0.000	0.000	0.000	0.002	0.002
Type I error	Dynamic	71.066	68.665	67.552	67.773	67.310
	Static	86.629	83.479	81.279	80.346	79.104
	<i>t</i>	− 4.638	− 4.661	− 4.718	− 4.345	− 4.185
	<i>p</i> -value	0.000	0.000	0.000	0.001	0.001
Type II error	Dynamic	0.251	0.513	0.828	1.174	1.520
	Static	0.330	0.644	0.989	1.358	1.710
	<i>t</i>	− 1.530	− 1.862	− 2.148	− 2.364	− 2.236
	<i>p</i> -value	0.076	0.044	0.027	0.019	0.023

may undergo future crises. This dynamically updated method makes the evaluation of the efficiency scores more stable and thus more completely reflect the current efficiency rates of the banking system. In addition, the global reference DEA allows us to evaluate the efficiency of banks over time. A continuous decrease in the inefficiency score indicates that the bank's operating conditions are rapidly deteriorating. In practice, the regulatory bodies want to discover the signals of potential risks as early as possible, in order to issue advance warnings. In previous studies, scholars verified the early warning time within a period of 3 years (Manthoulis et al., 2020). Here we validate the performance of early warnings for up to five years. Naturally, as the window extends further into the future, the predictive power of the efficiency scores decreases, but here too, the performance of the dynamic model is generally still better than the static model. As we use different datasets in a combination of various scenarios, there is no definitive statement to be made about the standard threshold of the evaluation indicators. According to the suggestion of Hosmer Jr et al. (2013), the model is to be considered acceptable when the AUC is over 0.7. In Fig. 8, we find that AUC values in some dynamic models are less than 0.7 when measured from a period of 4 years in advance, thus we confirm that the most effective early warning timespan is 3 years, according to our models.

### 5.3 Crisis and non-crisis time window analysis

In this section, we set up two dynamic models, covering the periods of economic crisis and non-crisis respectively. We also separate the period before crisis (2003 to 2007) and the period during the crisis (2008 to 2012), both of which span a window of 5 years. In addition to the changes in the time interval of the dynamic model, the global reference is still applied to calculate the efficiency scores, with 'failed banks' staying in the model until they officially



**Fig. 8** AUC performance of different early warning years of dynamic models

fail. Other settings remain the same as in the previous section. The results are shown in Table 12. The DEA efficiency score displays a better predictive power during periods of crisis than non-crisis. The sharp changes in the observed values during the crisis prompted the updating efficiency scores to become more discriminative, and the AUC to increase significantly. The increase in the number of bad samples during the crisis also resulted in fewer Type I errors and more Type II errors in the model compared to the periods of non-crisis. During crisis periods, the prediction performance of the dynamic models is significantly better than the static models at the mean level. In Table 12, during the period of non-crisis, although the AUC of the dynamic model at the two-year warning margin is lower than that of the static model, the difference is small, and the combined results of the AUC, Type I error and Type II error still support the hypothesis that the dynamic model is better overall than the static model.

## 5.4 Extensions of the dynamic DEA model

In the DEA framework, different settings will lead to different efficiency scores and relative rankings. In this section, we will further study how orientations, reference sets and the Worst/Best Practice Frontiers perform under our proposed dynamic framework, as illustrated in Fig. 1 (including the Best Practice with Output-MSBM and global reference settings). Model 1 and Model 2, then Model 4 and Model 5 make comparisons between the Worst Practice and Best Practice models. Input-orientation and output-orientation are two options available in the DEA framework (formulas are given in Sect. 3.1). The two orientation models evaluate the efficiency of the DMU from the perspective of input or output directions in terms of their respective impact on the efficiency scores and their rankings. Models 1–3 and 4–6 make comparisons between these two orientations. Reference sets are in fact where the Production Possibility Sets (PPS) come from. We use two types of reference sets: fixed reference and global reference, where the former takes a year of data as PPS, while the latter takes DMUs of all years as its PPS, as shown in Table 13. Models 3 and 6 are designed with a fixed reference using the sample for Year 2002. We therefore have a total of six models for comparison, all of which are presented in Table 14.

We continue to use AUC as an indicator of predictive ability for early warnings for the period of up to three years. The AUC depends on the rankings of scores. When the number of active banks is much larger than that of failed banks, the value of AUC would be large or

**Table 12** Mean of evaluation indicators in periods of crisis and non-crisis

Early warning (years)	Panel 1: Non-crisis periods				Panel 2: Crisis periods				
	Accuracy		Type I error		Accuracy		Type I error		
	AUC	Type II error	AUC	Type II error	AUC	Type II error	AUC	Type II error	
1	Dynamic	0.925	99.598	92.518	0.054	0.983	98.899	52.811	0.558
	Static	0.923	99.231	96.299	0.055	0.925	98.371	73.323	0.766
2	Dynamic	0.826	99.127	89.595	0.344	0.972	98.210	47.566	0.919
	Static	0.833	98.811	93.451	0.354	0.937	97.482	69.326	1.242
3	Dynamic	0.816	98.155	84.803	0.897	0.965	97.769	48.094	1.154
	Static	0.813	97.816	89.362	0.953	0.928	96.975	68.563	1.527

**Table 13** Types of reference sets

Global (G)	$\text{Reference}^G = \{(x_j^1, y_j^1)\} \cup \{(x_j^2, y_j^2)\} \cup \dots \cup \{(x_j^T, y_j^T)\}$
Fixed (F)	$\text{Reference}^F = \{(x^{\text{Fixed}}, y^{\text{Fixed}})\}$

**Table 14** Model specification

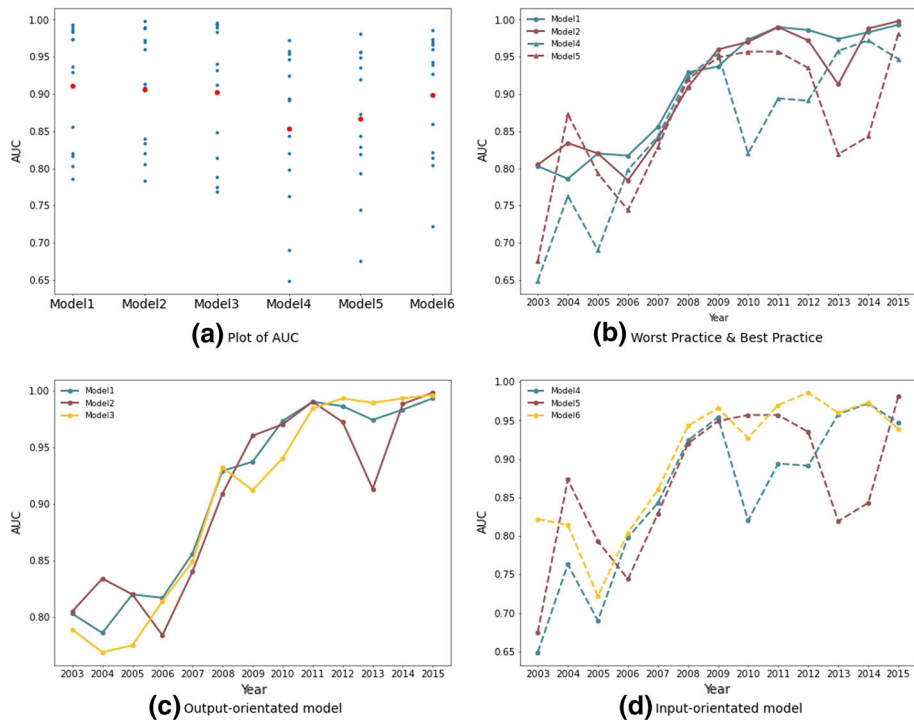
Model number	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Orientation (O/I)	O	O	O	I	I	I
Worst/Best Practice (W/B)	W	B	W	W	B	W
Reference (G/F)	G	G	F	G	G	F

close to 1, as found in Manthoulis et al. (2020). In order to eliminate the effect caused by the sample size of failed banks, we add paired *t*-tests to compare the results.

As shown in Table 15, the last three rows present the maximum, average and minimum values of the AUC. The results show that from the average performance of annual predictions, the dynamic model we originally proposed (Malmquist DEA with global reference and WPF) has the best overall predictive ability (mean AUC equals to 0.911), while the mean AUC of Model 4, with the input-orientated DEA with global reference and WPF, is the lowest. Comparing the results of Models 1–3 with those of Models 4–6, we see that no matter which reference is used, the output-oriented models outperform the input-oriented models, since

**Table 15** AUC in dynamic models

Year	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
2003	0.803	0.805	0.789	0.649	0.675	0.822
2004	0.786	0.834	0.769	0.763	0.873	0.814
2005	0.820	0.820	0.775	0.690	0.793	0.722
2006	0.817	0.784	0.814	0.798	0.744	0.804
2007	0.856	0.840	0.849	0.843	0.829	0.860
2008	0.929	0.909	0.932	0.924	0.920	0.943
2009	0.937	0.960	0.912	0.954	0.949	0.966
2010	0.973	0.970	0.940	0.820	0.957	0.927
2011	0.990	0.990	0.984	0.894	0.957	0.970
2012	0.986	0.972	0.993	0.891	0.935	0.986
2013	0.974	0.913	0.989	0.958	0.819	0.960
2014	0.983	0.988	0.993	0.972	0.843	0.973
2015	0.993	0.998	0.996	0.947	0.981	0.939
Max	0.993	0.998	0.996	0.972	0.981	0.986
Mean	0.911	0.906	0.903	0.854	0.867	0.899
Min	0.786	0.784	0.769	0.649	0.675	0.722



**Fig. 9** AUC analysis

output-oriented models have larger mean, maximum and minimum values. The difference between these two orientations is the objective, which creates the different scores' results and rankings. Given a background objective of failure prediction, the scores of output orientation models display a much better performance. Models 1–3 all reached the maximum AUC value in 2015, while the input-oriented models do not show such a pattern. Figure 9 shows the AUC comparisons for different aspects: Fig. 9a shows the AUC for the years from 2003 to 2015, where the red points mark the mean values. Figure 9b shows the comparison between the Worst and Best Practice Frontier models, where the dash-lines show the output-orientation, while the dotted lines show the input-orientation. Figure 9c, d show the AUC results of input and output orientation separately.

As explained previously, we use a paired  $t$ -test to verify whether there is a significant difference in the predictive power between two models (Ling et al., 2003). In Table 16, it is noticed that some pairs of AUCs show no significant difference, while Model 1 outperforms many of others. Between the Worst and Best practices, there is little difference recorded. However, in the Worst Practice Frontier, the model automatically places the most inefficient cases on the frontier, which makes them easy to identify and therefore be used to generate an early warning to the impending risks. After discussing the predictive capabilities of different models under the framework of the dynamic update data set, we find that the output-oriented Malmquist DEA with global reference and WPF is the one of the best predictive capabilities.

**Table 16** Paired *t* test of AUC

<i>p</i> -value	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Model 1	0					
Model 2	0.477	0				
Model 3	0.081	0.715	0			
Model 5	<b>0.003</b>	<b>0.010</b>	<b>0.004</b>	0		
Model 6	<b>0.024</b>	<b>0.014</b>	0.096	0.554	0	
Model 7	<b>0.021</b>	0.490	0.672	<b>0.006</b>	0.117	0

Values in bold are significant at 5% level

## 6 Conclusions and implications

Previous literature linking efficiency calculated by DEA to bank failure has been limited to cross sectional analyses. A dynamic analysis given regularly updated datasets is therefore recommended for bankruptcy prediction. This paper proposes a nonparametric method based on Malmquist DEA, and uses some extensions to dynamically assess the bankruptcy risk of banks over multiple periods. DEA models require no prior information about the relationships between variables and the outcome, though we do have to specify inputs and outputs for DEA. By combining different settings, including slacks, output orientation, global reference and WPF, we calculate the relative inefficiency scores for banks. This method shows a better predictive performance than the static model and other extensions.

We can draw several conclusions based on these results. First, because a dynamic model contains historical information, it generates robust and reliable scores to use as bank failure signals. This score is comparable not only among bank peers (cross-sectional) but also for individual banks over multiple periods (time series). This is the best-case scenario, the one most conducive to the creation of an internal early warning system for banks and supervision agencies alike, which can further be used in stress testing. The regular updates to the inefficiency score create a dynamic evaluation of a bank's performance on a long-term basis, so this dynamic model displays better predictive power than its static counterpart. Second, the models with BPF and WPF have similar resolutions, and the two can form a good mutual authentication, but the WPF in specific scenarios puts inefficient banks on the frontiers, which is preferable since we are focusing on bankruptcy. Third, we demonstrate that an output-orientated model has a better AUC performance than an input-orientated one in terms of discriminating between good and bad banks. By making use of a global reference, the extended models of the global reference take the worst or best DMUs to ever exist in history to the frontier. When the time-period is sufficiently long, this relative efficiency may become absolute efficiency. A general benchmark of this kind would be helpful for managers and regulators to implement strategies and policies. The Malmquist DEA is a useful tool not only to estimate productivity growth but also to predict bank failure. It is therefore of great value in maintaining the overall stability of the financial system.

Practically speaking, our research also provides some implications for bank risk management and regulation. Our results indicate that banks with lower efficiency rates are more likely to suffer distress or eventual collapse. The presence of low efficiency should raise the awareness of the management team, and managers can thus react quickly to change their operations and strategies. In the last few decades, there have been several financial crises to

hit the financial system. Regulators can use the models we propose in this research and create their own early warning signals of any potential crises within the banking industry. The Basel Accord requires measures such as stress testing and the assessment of value-at-risk (VaR) in banks. Though the DEA framework is nonparametric, the efficiencies can be incorporated into a parametric model for further analysis. The Malmquist DEA model proposed in this paper can provide a reliable method for predicting eventual bank failure, as well as a useful guide for bank management and supervision in terms of early warning against risk.

Based on the current work there are several avenues for future research. First, we can add super efficiency into future modelling, as Tan et al (2021) and Li et al (2017) have done. Super efficiency can distinguish DUMs on the frontier and make them comparable, which will increase the discriminatory power of models in the current context. Second, since the banking industry is a network system where businesses are intersected, a dynamic network DEA model, such as that proposed by Wanke et al. (2015), could also be considered for dynamic analysis. Further studies are therefore needed in the domain of bank failure prediction.

**Author contributions** ZL: Conceptualization, Methodology, Writing—original draft, Resources, Supervision. CF: Data curation, Formal analysis, Writing—original draft, Visualization. YT: Software, Investigation, Funding acquisition.

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**Data availability** This article uses publicly disclosed data obtained from BvD-Orbis Bank Focus.

**Code availability** The programming code can be provided upon request.

## Declarations

**Conflict of interest** All authors declare that they have no conflict of interest.

## Appendix

See Table 17.

**Table 17** Description of selected variables.

Category	Variable	Description	Total	Missing	Missing%
Capital	TETA	Total equity/Total assets	73,799	7081	9.59
	TCETA	Tangible common equity/Tangible assets	73,799	7081	9.59
Asset	CLATA	Customer loans & advances/Total assets	73,799	7900	10.70
	GrowthTAs	Growth in total assets	73,799	8133	11.02
	GGCLA	Growth in gross customer loans & advances	73,799	9012	12.21
	GNCLA	Growth in net customer loans & advances	73,799	9011	12.21
	ILGCLA	Impaired loans/Gross customer loans & advances	73,799	13,412	18.17
	LLRGCLA	Loan loss reserves/Gross customer loans & advances	73,799	8062	10.92
Earning	UILTE	Unreserved impaired loans/Total equity	73,799	8082	10.95
	ROAE	Return on average equity	73,799	8143	11.03
	OPATE	operating profit/average total equity	73,799	8143	11.03
	ROAA	Return on average assets	73,799	8143	11.03
	IIAIEA	Interest income/average interest earning assets	73,799	8203	11.12
	IEAIBL	Interest expense/average interest bearing liabilities	73,799	8697	11.78
	IILAGCLA	Interest income on loans/Average gross customer loans & advances	73,799	8942	12.12
	IECDACD	Interest expense on customer deposits/Average customer deposits	73,799	8799	11.92
Liquidity	CAAR	Cost to average asset ratio	73,799	8118	11.00
	LATA	Liquid assets/Total assets	73,799	7096	9.62
	LAASMTA	Liquid assets including available for sale & held to maturity/Total assets	73,799	7096	9.62
	LACTA	Loans & advances to customers/Total assets	73,799	7900	10.70
	LACMILAC	Loans & advances to customers maturing in < 1 year/Loans & advances to customers	73,799	8014	10.86
	GLACCD	Gross loans & advances to customers/Customer deposits	73,799	7958	10.78
	LADSTF	Liquid assets/Deposits & short-term funding	73,799	7789	10.55



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