Distributed Non-Convex Least Squares Localization Problem

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Abstract

We propose an algorithmic framework to solve a non-convex localization problem using a non-convex least squares formulation and squared range measurements. Our main contribution is a distributed method based on ADMM (Alternating Direction Method of Multipliers) by solving local non-convex quadratic subproblems. We prove convergence towards a stationary solution.

Keywords: Wireless Sensor Networks, Distributed Optimization, Localization

1. Introduction

Localization problems arise in a variety of scenarios and applications, and have been amply studied in the literature Beck et al. (2008); Tarrío et al. (2008); Lin and So (2011). However, these approaches are centralized, i.e., a fusion node knows the location of every sensor and the ranged measurements. In this paper, we focus on learning the position of a target node in a distributed manner using noisy measurements of distances between the target and the network nodes. This formulation arises, for example, in the estimation of a user's location that is reached by different base stations within a cellular network. Given a set of nodes $\mathcal{Q} = \{1, \ldots, Q\}$ with known position $s_i \in \mathbb{R}^p$, the distance measurements d_i between the network node $i \in \mathcal{Q}$ and the target node, we estimate the target node's position:

$$\min_{x} \quad \sum_{i \in \mathcal{Q}} \left(\|x - s_i\|^2 - d_i^2 \right)^2, \tag{1}$$

where $x \in \mathbb{R}^p$ and p is typically 2 or 3, if the localization is in the plane or in a three-dimensional space, respectively. However, (1) is non-convex and presents certain difficulties.

Decentralized approaches such as Chen and Sayed (2012); Di Lorenzo and Scutari (2016) minimize a global function such as (1) by minimizing a local version, but use first order methods to simplify the procedure. However, by exploiting the nonconvex quadratic formulation, we can easily improve the convergence results. Our proposed algorithm uses an ADMM decomposition (Alternating Direction Method of Multipliers). However, ADMM has no convergence guarantees when the problem is non-convex, as is the case in (1), so we analyze its convergence towards the global solution of the problem.

1.1. Algorithmic Framework

We reformulate the original problem (1) as follows:

$$\begin{aligned} \min_{\mathbf{t}, \mathbf{x}, z} \quad & \sum_{i \in \mathcal{Q}} (t_i - d_i^2)^2 \\ \text{s.t.} \quad & \|x_i - s_i\|^2 - t_i = 0, \qquad x_i = z \quad \forall i \in \mathcal{Q} \end{aligned} \tag{2}$$

where $\mathbf{x} = (x_i)_{i \in \mathcal{Q}}$, $\mathbf{t} = (t_i)_{1 \in \mathcal{Q}}$, $t_i \in \mathbb{R}$ and x_i, $z \in \mathbb{R}^p$. Problem (2) is non-convex, but includes slack variable z to facilitate a separable formulation. In particular, the augmented Lagrangian can be decomposed as follows:

$$L_\rho(\mathbf{t}, \mathbf{x}, z, \boldsymbol{\mu}) = \sum_{i \in \mathcal{Q}} L_\rho^i(t_i, x_i, z, \mu_i) \tag{3}$$

where the local Lagrangians have the following form:

$$L_\rho^i(t_i, x_i, z, \mu_i) = (t_i - d_i^2)^2 + \mu_i^T (x_i - z) + \rho \|x_i - z\|^2, \tag{4}$$

and $\boldsymbol{\mu} = (\mu_i)_{i \in \mathcal{Q}}$, $\mu_i \in \mathbb{R}^p$ are dual variables associated with the cost of disagreement between the x_i and z; $\rho > 0$ weights the proximal term that induces strong convexity in x_i; and, variables x_i, t_i have to satisfy $\|x_i - s_i\|^2 = t_i$.

From the Lagrangian (3), we can derive the ADMM steps:

1. Solve $\forall i \in \mathcal{Q}$ and assign the solution to t_i^{k+1} and x_i^{k+1}:

$$\operatorname*{arg\,min}_{t_i, x_i} \quad L_\rho^i(t_i, x_i, z^k, \mu_i^k), \quad \text{s.t.} \quad \|x_i - s_i\|_2^2 - t_i = 0. \tag{5}$$

2. Update $z^{k+1} = \frac{1}{Q} \sum_{i \in \mathcal{Q}} (x_i^{k+1} + \frac{1}{2\rho} \mu_i^k)$.

3. Update $\mu_i^{k+1} = \mu_i^k + 2\rho(x_i^{k+1} - z^{k+1})$.

In step 1 every network node $i \in \mathcal{Q}$ solves a **non-convex** problem in parallel. Then, estimates are interchanged between nodes, or average consensus is executed, see e.g. Xiao et al. (2007), in order to update z. Finally, dual variables μ_i are updated locally.

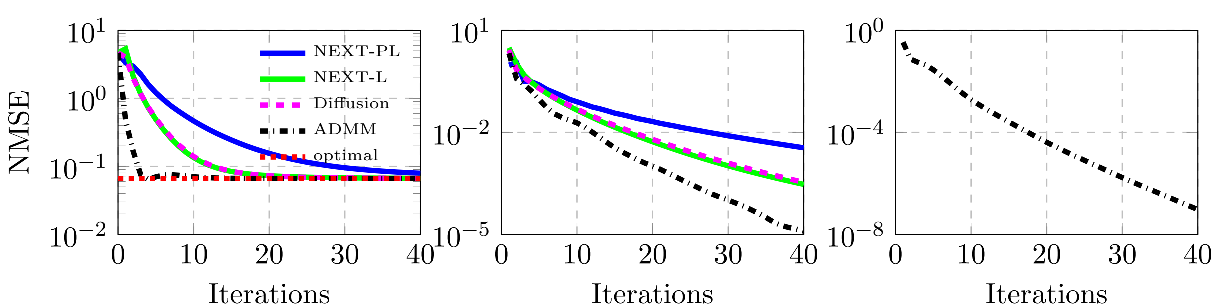

Figure 1: Convergence to solution, variabilitiy per iteration and divergence between nodes.

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