Area of shaded region calculus

 When we learn for the first time to find areas through integration, we take representative rectangles vertically. The rectangles have a base of a small change in #x) and elevations that correspond to the larger #y (the one on the top curve) minus the lower #y value (the one on the bottom curve). We then integrate from the smallest #x value to the largest #x value. For this new problem, we could use two such intergrals (see Jim S's answer), but it is very valuable to learn to turn our minds #90. We take representative rectangles horiontally. The rectangles have the height #dy a (a small change in #y) and bases that correspond to the larger #x (the one on the right side of the curve) minus the value less #x (the one on the left curve). We then integrate from the smallest #y value to the largest #y value. Note the duality of :(vertical , iff , horizontal), (dx, iff, dy), (upper, iff, rightmost), (lower, iff, leftmost), (x, iff, y): . The expression from the smallest #x value to the largest #x value indicates that we are integrating from left to right. (Towards increasing the #x values.) The expression from the smallest #y to the largest #y value indicates that we are integrating from bottom to top. (Towards increasing the #y values.) Here is an

image of the region with a small rectangle: The range is #int_1 2 (y-1/ y2) dy = 1 Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a narrow screen width (i.e. They are probably on a mobile phone). Due to the nature of mathematics on this page, it is the best view in landscape mode. If your device is not in landscape mode, many of the equations run off the side of the device (should be able to scroll to see them), and some of the menu items are truncated due to the narrow screen width. In this section, we will look at finding the area between two curves. There are actually two cases that we will look into. In the first case, we would like to see the range between the interval (y = f'left(x 'right)') and '(y = g'left(x 'right)') in the interval '('left[
'a,"""""""""""""""""""" Take a look at the following sketch to get an idea of what we're going to look at first. In the Area and Volume Formulas section of the Extras chapter, we derived the following formula for the range in this case. int_ The second case is almost identical to the first case. Here the range between the range (x = f'left(y 'right)') and '(x = g'left(y 'right)') in the interval
'("""""""""""""""""""" y - right) - g'left(y'right)',dy"label'eq:eq2'end'equation'] Now the formulas eqref and eqref are perfectly neglected, but sometimes it is easy to forget that they always require the first function to be the larger of the two functions. Instead of these formulas, we use the following word formulas to make sure that we remember that the range is always the larger function minus the smaller function. Im ersten Fall verwenden wir, ,,['begin''equation'' ""int_"""""""""""""""""""Array, c-Mbox-[-begin--Gleichung int_
(""""""""""""""""""" Die Verwendung dieser Formeln zwingt uns immer dazu, darüber nachzudenken, was mit jedem Problem vor sich geht, und sicherzustellen, dass wir die richtige Reihenfolge der Funktionen haben, wenn wir die Formel verwenden. Let us give an example. Example 1 Determine the range of the range enclosed by the ranges '(y = 'x'2') and '(y = 'sqrt x'). Show solution first, what we mean by area surrounded by. This means that the region we are interested in must have one of the two curves on every border of the region. So here is a diagram of the two functions shaded with the enclosed region. Note that we do not occupy any part of the region to the right of the intersection of these two charts. In this area there is no boundary on the right and therefore is not part of the closed area. Remember that one of the specified functions must be at each boundary of the included range. From this diagram it also becomes clear that the top function depends on the range of '(x)' that we use. For this reason, you should always sketch a chart of the region. Without a sketch, it is often easy to confuse which of the two functions is the larger one. In this case, most would probably say that the upper function of (y = x2 is the upper function, and they would be for the vast majority of .(x)' it's right. In this case, however, it is the bottom of the two functions. The limits of integration for this are the intersections of the two curves. In this case, it's pretty easy to see that they're cutting each other around the two, so these are the limits of integration. So, the integral that we have to calculate, to find the area is, A & - int_, a, b, left("""""""""""""""""""-,-End-Array- -,,,dx" & = = = x - x , dx, dx & = ... {1} {3} {1} {3} {3} {2} {2} {3}, a chart is required for almost all of these problems.

Often, the boundary area that gives the limits of integration is difficult to determine without a diagram. Also, it can often be difficult to determine which of the functions is the upper function and which is the lower function without a graph. This is especially true in cases such as the last example where the answer to this question actually depended on the range of '(x)' that we used. Unlike the area below a curve that we looked at in the previous chapter, the range between two curves is always positive. If we get a negative number or zero, we can be sure that we made a mistake somewhere and need to go back and find it. Also note that sometimes, instead of saying region of us included, we will say region bound by. They mean the same thing. Let us work on a few more examples. Example 2 Determine the range of the range bounded by the axis (y = xbfe - x2 (x + 1 and (x = 2) and the axis (y). Solution Display In this case, let us know the last two information, the axis (x = 2)) and the axis ,,,,,), the right and left boundaries of the region. Also remember that the axis of . (y) is specified by the line (x = 0. Here is the diagram with the closed area in which is shaded. In contrast to the first example, the two curves do not meet here. Instead, we rely on two vertical lines to bind the left and right sides of the region, as we mentioned above here is the integral that will give the surface. A & - int_, a, b,
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,""""""""""""""""""""{1}{2} int_------------------ --- --- -------------- --"""""""""" '2' + x + example3 Determine the range of the range bounded by the boundaries of y = 2x + 10 and (y = 4x + 16). Show solution In this case, the intersections (which we finally need) will not be easily identified from the chart, so let's move on and get them now. Note that for most of these issues, you won't be able to accurately identify the intersections from the chart, so you need to be able to determine them by hand. In this case, we can retrieve the intersections by equaling the two equations. - 4x - 6 & = 0, 2 , left(x - 3 , right) & = 0, end, align. Align. (x = 3). If we need them, we can retrieve the values of ,(y), which correspond to each of these values, by putting the values back into one of the equations. We leave it to you to verify that the coordinates of the two intersections in the chart are (left (-1.12) and left (3.28) and right). Also note that if you do not know well in the graph that the intersections can help to at least start the graph. Here is a diagram of the region. With the diagram we can now identify the upper and lower functions and so we can now find the closed area. [-begin----int_Mbox-Funktion, Array ,,,,,,,,,,,,,, dx & = int_, 1, 3 4x 4x + 16 - links (2 x 2, 2, 2, rechts) int_ & - 1,3, 3, 2 x 2 , + 4x + 6, dx, & = , •links(' ''fra {3} {64}c{2}{3}''x'3'' + 2'x'2' + 6x"""""""""""""""""""" Einer der häufigsten Fehler, die Schüler bei diesen Problemen machen, ist die Vernachlässigung der Klammer im zweiten Semester. Example 4 Determine the range of the range bounded by the region ,y = 2x2 + 10 , (y = 4x + 16 and (x = - 2 and (x = 5). Show Solution This Way, the functions used in this problem are the same as the functions of the first problem. The difference is that we have expanded the limited range from the intersections. Since these are the same functions that we used in the previous example, we won't bother to find the intersections again. Here is a diagram of this region. Okay, we have a little problem here. Our formula requires that one function is always the top function and the other function is always the lower function and we clearly do not have that here. However, this is not the problem that it seems to be at first sight. There are three areas in which one function is always the top function and the other is always the lower function. So all we have to do is find the area of each of the three regions, what we can do, and then add them all up. Here's the area. A & = int_, - 2, - 1, 2, 2, 2 int_, 2, 2, 2, + 16 - ,,links ('2'x'2' + """"int_
int_""""""""""""""""""""- 2, - 1, 2 x 2 x 2 , 4x - 6, dx , dx , - int_, - 1, 3, 3, 3, 2 x 2 , + 4x + 6,dx , + , int_, 3, 5, 5, 2, 2, 2, 2x, 2x2, 2xx2, 2xx2, 2x, 2x, 6x, 6, 2x, 2x, 2x, 2x, 2x, 2x, 2x, 2x, 2x, 2x, 2xx, x 2, 6x, 6, 2x, 2x, 2x, 2x , 2x2, , ... ,dx' & = 'links' links (frac{2}{3} - x - 2 - - - . . . . links (-frac{2}{3} , links ('frac{2}{3}'x'3' - 2'x'2"""""""""{3} {64} {3} {64} {3} {14}""""""""""""""""""""
{14}"""""""""""""""""""" Example 5 Determine the range of the range enclosed by the axis ,(y = sin x), the axis ,(y = sin x'), the axis '(y' = 'sin x'{2}', and the axis '(y')'. Show Solution First let's get a chart of the region. So we have a different situation where we have to do two integrals to preserve the area. The intersection is where the interval of . ['sin x = 'cos x''. We leave it to you to check if this (x = fracppi is {4}. Der Bereich ist dann, ,,['begin'align*'A & = 'int_',',',','{4}""""""""""""""""""""{4} int_"""""""""""""""""""" ,pi ({2}) Sin x - écos x,dx & = ... left (sin x + cos x right) , right|_0 frac and {4} + left. left (- cos x - sin x (right {4}) pi ({2}; & = sqrt 2 - 1 + left(sqrt 2 - 1, right) & = 2 sqrt 2 - 2 = 0.828427 -end-align *'] We have to be careful with this next example. Example 6 Determine the range of the region enclosed by the region ,(x = 'frac{1}{2}'y'2'' - 3' and '(y = x - 1'). Show Solution Don't get upset with the first equation. We will occasionally have to deal with such equations so that we have to get used to them. As always, it will be helpful if we have the intersections for the two curves. In this case, we get the intersections by using the second equation for . (x) and then set it immediately. Here's this work, '['begin'align*'y + 1 & = 'frac{1}{2}'y'2' - 3'2y + 2 & = 'y'2' - 6' 0 & = 'y'2' - 2y - 8 ' 0 & = 'left('y - 4' 'right)' left('y + 2'
"""""""""""""""""""" y = 4), or if we need the full coordinates, they are: ,,,left(- 1, - 2 , right) and '('links' ('5,""')'). Here is a sketch of the two curves. Now we will have a serious problem if we are not careful. Up to this point, we used an upper function and a lower function. To do this here note that there are actually two parts of the region that have different lower functions. In the area of the parable n ('left[' - 3, - 1'' 'right]') the parable is actually both the upper and lower functions. To use the formula we used up to this point, the parable for the solution of . (y). This results in the + the upper part of the parable and the - the lower part. Here is a sketch of the entire area with each region that we would need if we were to use the first formula. The integrals for the area would then be [-begin-align&A & = int_, - 3, - 1 'sqrt '2x + 6' - 'left' - 'left' - 'sqrt '2x + 6',,dx', int_ ',dx', -1',5''sqrt '2x + 6 int_' - 'left' - 3, - 1, 2x2, 2x + 6, dx, dx, - int_, - 1.5, sqrt, 2x + 6, dx, - 1.5, 5, 2x2, sqrt, 2x2, 2x, dx, - . . . . 2x + 6 - x + 1, dx , & = int_, - 3, - 1, 2, 2x + 6, dx, 2x + 6, dx, 2x + 6, dx, dx, dx, - 1.5, 5, sqrt , 2x + 6, dx , + , int_, - 1, 5, 5, 5, X + 1, dx, Dx, & = ,{2}{3} , , , , , , ,{3}{2} , , , , , , , , , , , , ...
Sfrac{1}{3}, frac{3}{2} , , , , {16} , {1} {2} Remember that there is another formula for determining the range. It is the file [A int_ (mbox (left) and in our case we have a function that is always on the left and the other always on the right. So in this case, this is definitely the one away. Note that we need to rewrite the equation of the line because it must be in the form (x = fleft(y 'right),' but that's simple enough. Here is the chart for using this formula. Der Bereich ist, ,,,,,int_,,,,,,links
(""""""""""""""""""""-,-End-Array- -,,-int_,-,- 2,4, 4, links(éy + 1, rechts) - links frac{1}{2} - 2 - Rechts) und int_ - 2,4, 4, - sfrac{1}{2} , 2 , & 4, dy, & = , Dies ist dasselbe, was wir mit der ersten Formel erhalten haben, und dies war definitiv einfacher als die erste Methode. {1} {2} {1} {6} So in this last example, we saw a case where we could use one of the two formulas to find the range. The second, however, was definitely easier. Students often come into a calculation class with the idea that the only easy way to work with functions is to use them in the form. However, as we have seen in this previous example, there are definitely times when it will be easier to work with functions in the form(x = f'left(y 'right)'). In fact, there will be cases where this will be the only way to solve a problem, so make sure you can deal with features in this form. Let's take a look at another example to make sure we can handle functions in this form. Example 7 the area of the range, which is through the boundaries of x = - ,y + 10) and ,(x = links ('y -
""""""""""""""""""""we need intersections. [-begin-align-Direction2 - y2 + 10 & left (y - y (right) - y & y & y & y (4y + 4 0 & = 2 The intersections are the intersections (y = - 1) and (y = 3) and (y = 3) (y = 3). Here is a sketch of the region. This is definitely a region where the second territorial formula will be easier. If we were to use the first formula, three different regions that we would have to look at. The area in this case is,
""int_""""""""""""""""""""int_ -,-,-, -,-'
""""""""""""""""""""- 1,3, 3, - ,,y' 2' + 10 - 'links('y - '2'', """"""""""int_""""""""""""""""""""- 1,3, 3, 2, 2 , + 4y + 6, dy , & = , links ({3} {64}{2}{3} {3} {64}