Response Analysis of Second-Order Transfer Functions

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1.0 Learning Outcomes

After studying this ACE-Lab learning guide, you should be able to demonstrate the Learning Outcomes listed in Table 1. On completion of the ACE-Lab learning guide, revisit Table 1 and confirm whether each Learning Outcome has been achieved.

Table 1: *Learning Outcomes from the Response Analysis of Second-Order Transfer Functions Learning Guide*

ID	Learning Outcome	Achieved
1.	Convert a second-order transfer function into standard form and determine its system gain, natural frequency and damping ratio.	☐
2.	Explain how the damping ratio determines the type of response, and how the natural frequency determines the speed of response.	☐
3.	Explain the effect of the proportional control gain on the natural frequency, damping ratio, transient response and steady-state error (SSE) of a closed-loop control system.	☐
4.	Calculate the system performance criteria (SPC) of an underdamped system, such as rise time, overshoot and settling time, and use them to assess whether requirements are met.	☐

ID	Learning Outcome	Achieved
5.	Use MATLAB and Simulink to analyse and verify the response of second-order systems.	☐

2.0 Introduction

The Response Analysis of First-Order Transfer Functions Learning Guide showed that the standard form of a first-order transfer function, with its system gain and time constant, gives a quick way to describe and sketch a response. Many engineering systems, however, are second order. Examples include a mass-spring-damper, a DC motor driving an angle, and many closed-loop control systems, even when each component on its own is first order. Unlike first-order systems, second-order systems can overshoot and oscillate before settling.

This Learning Guide introduces the standard form of a second-order transfer function. It has three parameters: the system gain, K, which sets the final value; the natural frequency, ω_n, which sets the speed of the response; and the damping ratio, ζ, which determines whether, and how much, the response oscillates. MATLAB and Simulink are first used to show how these parameters, and the proportional control gain K_p, change the response. The system performance criteria (SPC), such as rise time, overshoot and settling time, are then introduced. These criteria describe the response numerically, so that it can be compared with the requirements of a control system.

Why use the Second-Order Standard Form?

The standard form of a second-order transfer function is useful because:

i. The system gain, K, gives the final value of the output for a step input, and so the steady-state error (SSE) of a closed-loop control system can be determined.
ii. The damping ratio, ζ, tells you the type of response (underdamped, critically damped or overdamped) without solving for the time solution.
iii. The natural frequency, ω_n, tells you how fast the response is.
iv. Together, these parameters give the system performance criteria, such as rise time, overshoot and settling time, which can be compared directly with design requirements.

3.0 Second-Order System Responses

The Final Value Theorem Learning Guide considered only the final value of the output. In this section, the transient part of the response is also considered, using the DC motor angle control system as a practical example.

DC Motor Angle Control

From the Final Value Theorem Learning Guide, the output of the closed-loop DC motor angle control system with a proportional control gain K_p and a reference of 180° is:

$$Y(s) = \left[\frac{113.7K_p}{s^2 + 3.398s + 113.7K_p}\right]\frac{180}{s} \tag{1}$$

This has been simulated using MATLAB and Simulink for $K_p = 0.5$, 1.0 and 1.5, and the results are shown in Figure 1 (the files are given in Appendix A). In every case the final value is 180°, as expected for a Type 1 system. However, the transient part of the response changes: as K_p increases, the output oscillates faster and overshoots further. The standard form in Section 4.0 explains why, and the SPC in Section 5.0 describe these responses numerically.

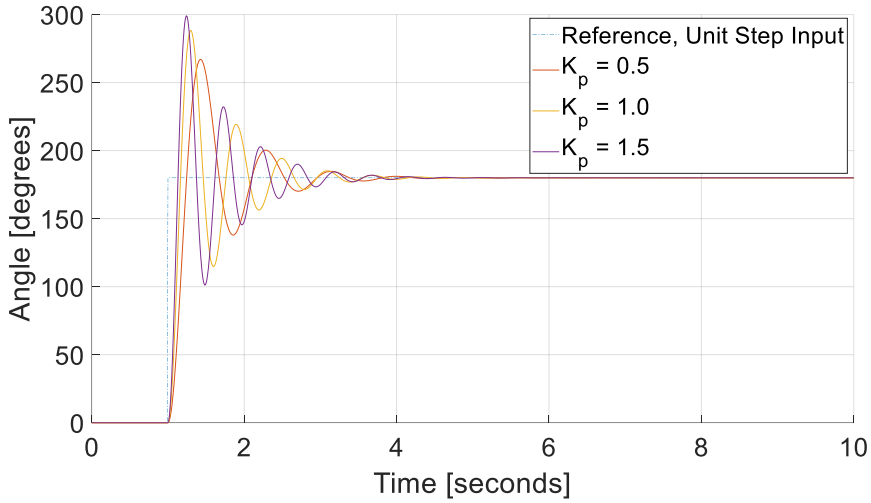


Figure 1: *Closed-Loop DC Motor Angle Control for Several Values of the Proportional Control Gain, K_p*

4.0 The Standard Form

The standard form of a second-order transfer function, which can represent an open-loop system or a closed-loop control system, is:

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

where K is the system gain, ω_n is the natural frequency [rad/s] and ζ is the damping ratio. For a unit step input, the final value of the output is $y_{ss} = K$.

A second-order transfer function is often written in the general form:

$$G(s) = \frac{K\beta}{s^2 + \alpha s + \beta} \tag{3}$$

Comparing Equations (2) and (3), the natural frequency and damping ratio are found from the coefficients α and β:

$$\omega_n = \sqrt{\beta} \tag{4}$$

$$\zeta = \frac{\alpha}{2\omega_n} \tag{5}$$

When the response oscillates, it does so at the damped natural frequency, ω_d, which can be seen in Figure 1:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{6}$$

Example 1: Standard form parameters

Determine the natural frequency, damping ratio and damped natural frequency of the second-order transfer function:

$$G(s) = \frac{4}{s^2 + 2.4s + 4}$$

Solution:

Comparing with Equation (3), $\alpha = 2.4$ and $\beta = 4$, and the numerator $K\beta = 4$ gives $K = 1$. Using Equations (4) to (6):

$$\omega_n = \sqrt{4} = 2 \text{ rad/s}$$

$$\zeta = \frac{2.4}{2(2)} = 0.6$$

$$\omega_d = 2\sqrt{1-0.6^2} = 1.6 \text{ rad/s}$$

Damping Ratio

The damping ratio determines the type of response, as summarised in Table 2. Figure 2 shows one example of each type: $4/(s^2 + 2.4s + 4)$ (underdamped), $4/(s^2 + 4)$ (undamped), $4/(s^2 + 4s + 4)$ (critically damped) and $8/(s^2 + 6s + 8)$ (overdamped). The Simulink files are given in Appendix B. If the damping ratio is negative, the poles have a positive real part and the response is unstable, as covered in a later Learning Guide.

Table 2: *Response Types and the Corresponding Damping Ratio and Poles*

Response type	Damping ratio	Poles	Response to a step input
Overdamped	$\zeta > 1$	Two different real poles, $(s+a)(s+b)$	No overshoot; slower than critically damped
Critically damped	$\zeta = 1$	Two equal real poles, $(s+a)^2$	Fastest response with no overshoot
Underdamped	$0 < \zeta < 1$	Two complex poles with a negative real part	Overshoot and a decaying oscillation
Undamped	$\zeta = 0$	Two imaginary poles, $s^2 + \omega_n^2$	Sustained oscillation (marginally stable)

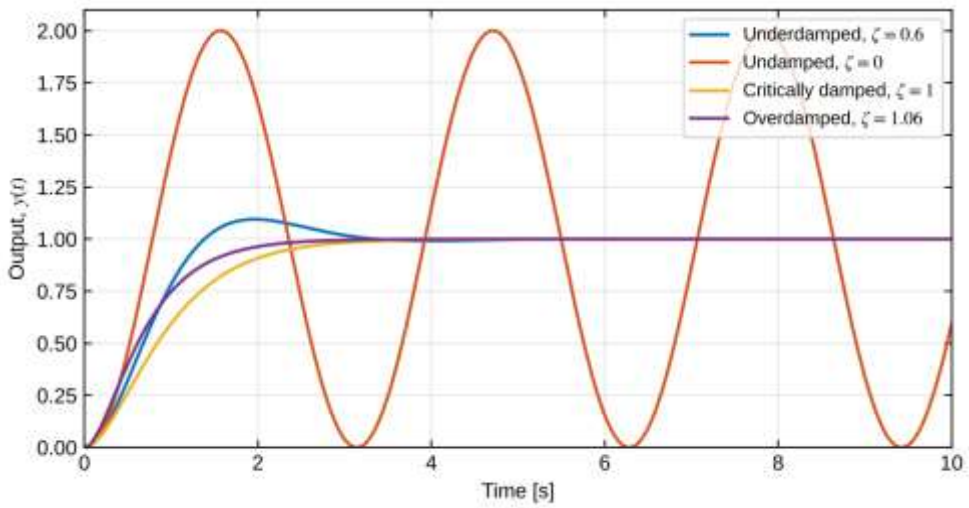


Figure 2: *The Four Types of Second-Order Response*

Natural Frequency

To investigate the effect of the natural frequency, the following transfer function, with $K = 1$ and $\zeta = 0.6$, is used:

$$G(s) = \frac{\omega_n^2}{s^2 + 2(0.6)\omega_n s + \omega_n^2}$$

The step responses for ω_n from 1 to 7 rad/s are shown in Figure 3 (the MATLAB code is given in Appendix C). As ω_n increases, the response becomes faster and reaches steady state in less time. Because ζ is the same in every case, every response has the same shape and the same overshoot of about 9.5%; only the time scale changes.

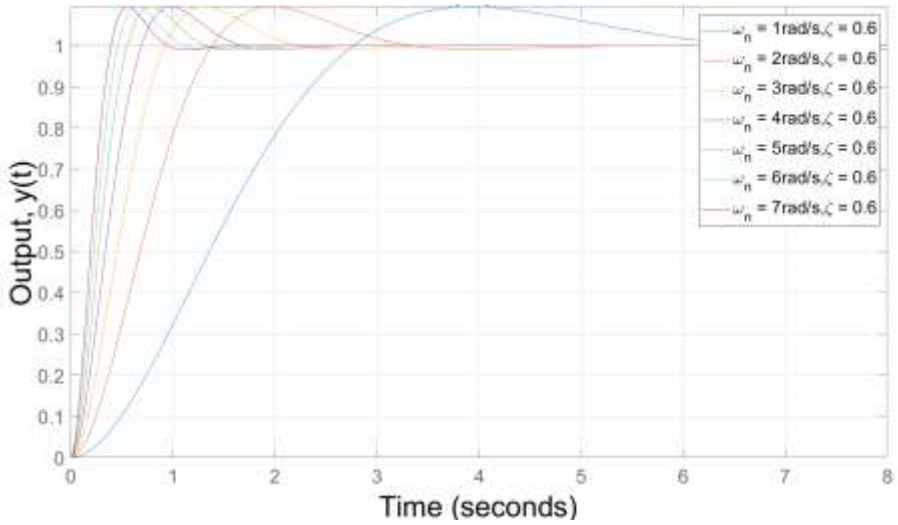


Figure 3: *Step Responses for Several Values of the Natural Frequency, ω_n*

Effect of the Proportional Control Gain

A closed-loop control system with a proportional control gain K_p, an ideal sensor and a unit step reference is now considered, with the open-loop system:

$$G(s) = \frac{N(s)}{D(s)} = \frac{4}{s^2 + 2.4s + 4}$$

The closed-loop transfer function is:

$$G_{CL}(s) = \frac{K_p\, N(s)}{D(s) + K_p\, N(s)} = \frac{4K_p}{s^2 + 2.4s + 4 + 4K_p}$$

For example, for $K_p = 1$, $G_{CL}(s) = 4/(s^2 + 2.4s + 8)$. The responses for K_p from 1 to 7 are shown in Figure 4 (the files are given in Appendix D), and the corresponding values of ζ and ω_n are listed in Table 3.

As K_p increases, the SSE decreases, because the closed-loop system gain $K = 4K_p/(4 + 4K_p)$ moves closer to 1. At the same time, $\omega_n = \sqrt{4 + 4K_p}$ increases and $\zeta = 2.4/(2\omega_n)$ decreases. The response therefore becomes faster but more oscillatory, with a larger overshoot. Reducing the SSE with proportional control alone comes at the cost of a poorer transient response.

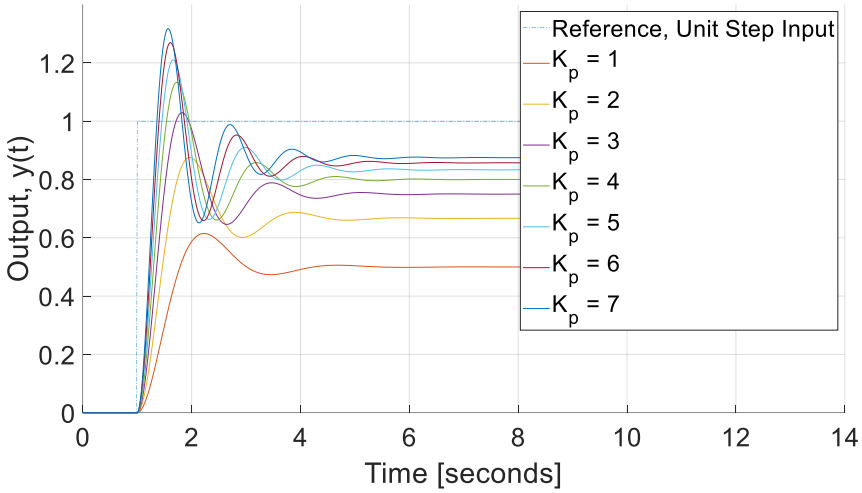


Figure 4: *Effect of the Proportional Control Gain, K_p, on the Closed-Loop Response*

Table 3: *Effect of the Proportional Control Gain on the Closed-Loop Damping Ratio and Natural Frequency*

K_p	**Damping ratio, ζ**	**Natural frequency, ω_n [rad/s]**
1	0.42	2.83
2	0.35	3.46
3	0.30	4.00
4	0.27	4.47
5	0.24	4.90
6	0.23	5.29
7	0.21	5.66

5.0 System Performance Criteria

The **system performance criteria (SPC)** describe the transient and steady-state response of a second-order system numerically. They are very useful when designing and evaluating a closed-loop control system, because they can be linked directly to the requirements. For example, a temperature control system might be required to reach its set point within a given time (the settling time) without large initial temperature swings (the overshoot).

The SPC are mainly used for underdamped responses, although some of them, such as the settling time, also apply to critically damped and overdamped responses. For an underdamped system with $K = 1$, the time solution for a unit step input is:

$$y(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta\omega_n}{\omega_d}\sin\omega_d t\right) \quad (7)$$

The exponential term, $e^{-\zeta\omega_n t}$, forms a first-order envelope that shows how quickly the oscillation decays, as illustrated in Figure 5.

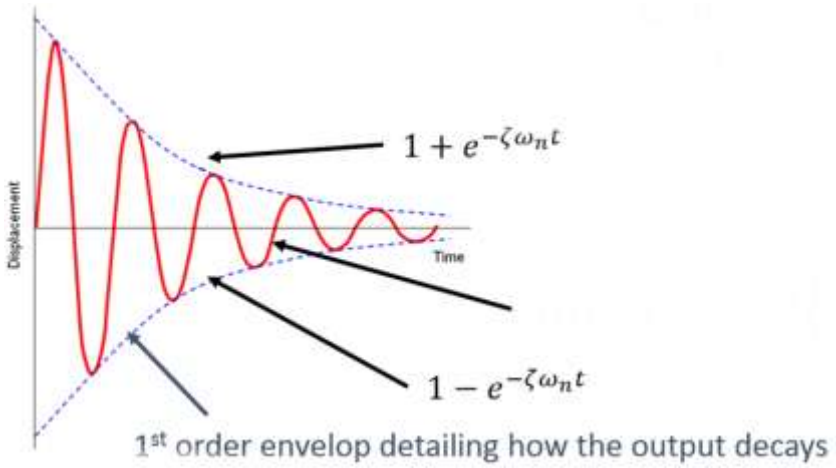


Figure 5: *First-Order Envelope Showing How the Oscillation Decays*

The SPC are described in Table 4 and labelled in Figure 6.

Table 4: *System Performance Criteria (SPC)*

Criterion	Description
Rise time, t_r	The time taken for the output to first reach its final value, y_{ss}
Peak time, t_p	The time taken to reach the first (largest) peak
Periodic time, T	The time taken for one complete oscillation
Peak amplitude, A_p	The value of the output at the first peak; the overshoot is often expressed as a percentage of y_{ss}
Settling time, t_s	The time after which the output stays within a given percentage (for example 5%) of y_{ss}
Number of oscillations, N	The number of oscillations that occur before the settling time

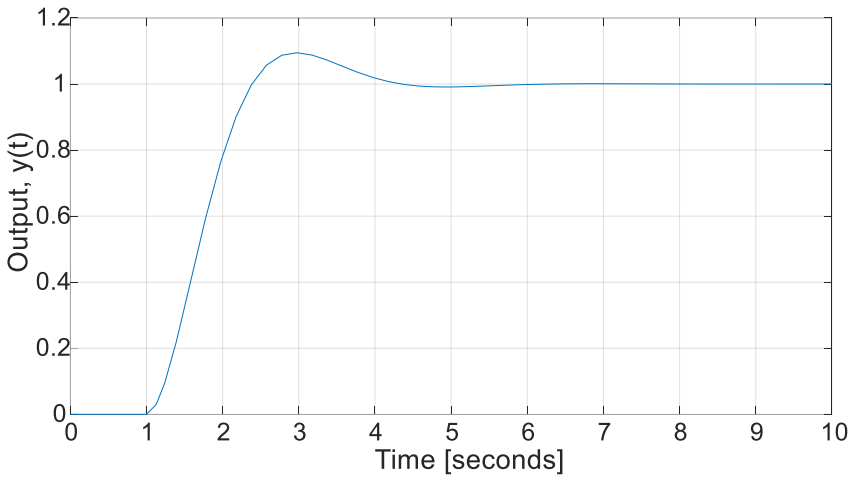


Figure 6: *System Performance Criteria for an Underdamped Response*

Calculating the SPC

The SPC are calculated from the natural frequency, damping ratio and damped natural frequency, which are found as in Example 1. It is assumed that a unit step input is applied, so that the final value is $y_{ss} = K$.

Rise time:

$$t_r = \frac{1}{\omega_d}\left(\pi - \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{8}$$

Peak time and periodic time: the first peak occurs after half an oscillation, so:

$$t_p = \frac{\pi}{\omega_d} \tag{9}$$

$$T = \frac{2\pi}{\omega_d} \tag{10}$$

Peak amplitude and percentage overshoot: substituting $t = t_p$ into Equation (7), and multiplying by K, gives:

$$A_p = K\left(1 + e^{-\pi\zeta/\sqrt{1-\zeta^2}}\right) \tag{11}$$

The percentage overshoot depends only on the damping ratio:

$$M_p = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}}\ \% \tag{12}$$

Settling time: the oscillation lies within the envelope $e^{-\zeta\omega_n t}$. Setting the envelope equal to the tolerance δ (for example, $\delta = 0.05$ for 5%) and taking the natural logarithm of both sides gives $-\zeta\omega_n t_s = \ln(\delta)$, and so:

$$t_s = \frac{-\ln(\delta)}{\zeta\omega_n} \tag{13}$$

Since $\ln(0.05) \approx -3.0$, the 5% settling time is $t_s \approx 3/(\zeta\omega_n)$. Similarly, the 2% settling time is $t_s \approx 3.9/(\zeta\omega_n)$, and the 10% settling time is $t_s \approx 2.3/(\zeta\omega_n)$. These envelope-based values are estimates, which are most accurate for lightly damped systems.

Number of oscillations before settling:

$$N = \frac{t_s}{T} \tag{14}$$

Example 2: SPC of a closed-loop control system

The closed-loop control system from Section 4.0 with $K_p = 1$ has the transfer function:

$$G_{CL}(s) = \frac{4}{s^2 + 2.4s + 8}$$

For a unit step reference, determine the rise time, peak time and periodic time, peak amplitude, 5% settling time and number of oscillations before settling. Sketch the response, labelling the SPC.

Solution:

Comparing with Equation (3), $\alpha = 2.4$ and $\beta = 8$, so $K = 4/8 = 0.5$. Then:

$$\omega_n = \sqrt{8} = 2.83 \text{ rad/s}, \qquad \zeta = \frac{2.4}{2(2.83)} = 0.42, \qquad \omega_d = 2.83\sqrt{1 - 0.42^2} = 2.56 \text{ rad/s}$$

Rise time:

$$t_r = \frac{1}{2.56}\left(\pi - \tan^{-1}\frac{\sqrt{1 - 0.42^2}}{0.42}\right) = \frac{\pi - 1.13}{2.56} = 0.78 \text{ s}$$

Peak time and periodic time:

$$t_p = \frac{\pi}{2.56} = 1.23 \text{ s}, \qquad T = \frac{2\pi}{2.56} = 2.45 \text{ s}$$

Peak amplitude:

$$A_p = 0.5\left(1 + e^{-\pi(0.42)/\sqrt{1-0.42^2}}\right) = 0.5(1 + 0.23) = 0.61$$

The overshoot is 23% of the final value of 0.5.

5% settling time:

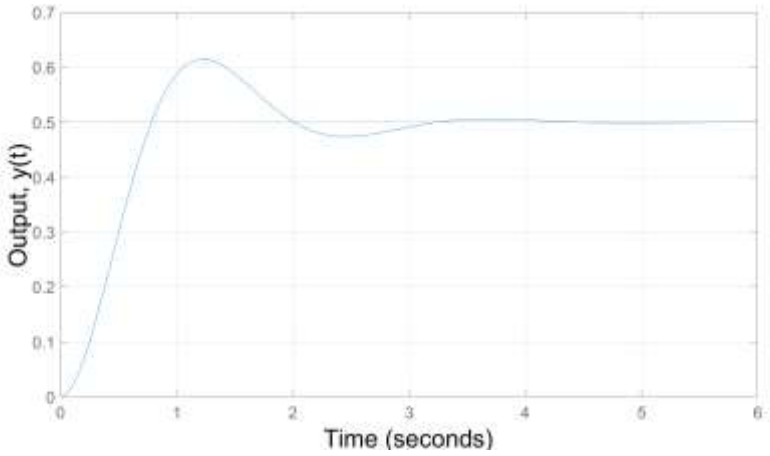
$$t_s = \frac{3}{\zeta\omega_n} = \frac{3}{(0.42)(2.83)} = 2.50 \text{ s}$$

Number of oscillations before settling:

$$N = \frac{2.50}{2.45} = 1.02$$

The response is shown below. Note that the final value is 0.5, so the SSE is

50%.



Example 3: SPC of the DC motor angle control system

For the closed-loop DC motor angle control system of Equation (1) with $K_p = 0.5$, determine the rise time, peak time and periodic time, peak amplitude, 5% settling time and number of oscillations before settling. Sketch the response, labelling the SPC.

Solution:

With $K_p = 0.5$, the closed-loop transfer function is:

$$G_{CL}(s) = \frac{(0.5)(113.7)}{s^2 + 3.398s + (0.5)(113.7)} = \frac{56.85}{s^2 + 3.398s + 56.85}$$

So $K = 1$ and:

$$\omega_n = \sqrt{56.85} = 7.54 \text{ rad/s}, \qquad \zeta = \frac{3.398}{2(7.54)} = 0.225, \qquad \omega_d = 7.54\sqrt{1 - 0.225^2} = 7.35 \text{ rad/s}$$

Rise time:

$$t_r = \frac{1}{7.35}\left(\pi - \tan^{-1}\frac{\sqrt{1 - 0.225^2}}{0.225}\right) = \frac{\pi - 1.34}{7.35} = 0.245 \text{ s}$$

Peak time and periodic time:

$$t_p = \frac{\pi}{7.35} = 0.428 \text{ s}, \qquad T = \frac{2\pi}{7.35} = 0.855 \text{ s}$$

Peak amplitude (for a unit step):

$$A_p = 1 + e^{-\pi(0.225)/\sqrt{1-0.225^2}} = 1 + 0.48 = 1.48$$

The overshoot is 48%, so for the 180° reference the angle peaks at about $1.48 \times 180 = 267°$.

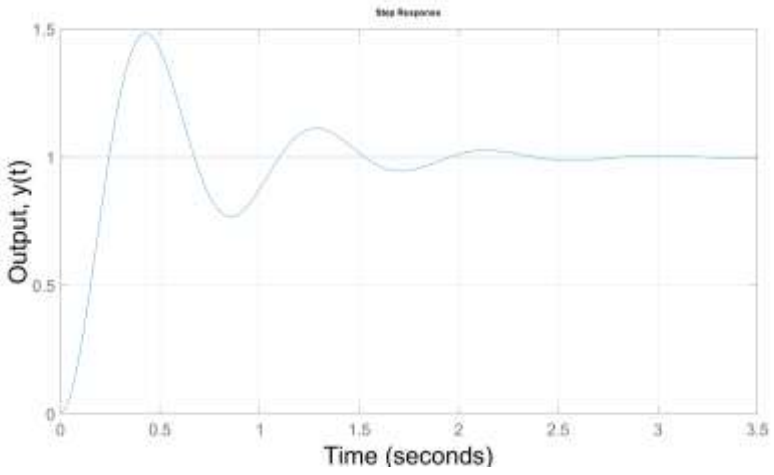
5% settling time:

$$t_s = \frac{3}{(0.225)(7.54)} = 1.77 \text{ s}$$

Number of oscillations before settling:

$$N = \frac{1.77}{0.855} = 2.06$$

The response for a unit step is shown below.



6.0 Verifying the Solution Using MATLAB

The hand calculations for Example 2 can be checked using the MATLAB script below, which calculates the SPC and plots the response.

MATLAB Script: calculating the SPC and plotting the response for Example 2

```
clear; clc; close all;

%% Closed-loop transfer function: K*beta/(s^2 + alpha*s + beta)
alpha = 2.4; beta = 8;          % transfer function coefficients
K = 0.5;                        % system gain
G = tf(K*beta, [1 alpha beta]);

%% Standard form properties
omega = sqrt(beta)              % natural frequency
zeta = alpha/(2*omega)          % damping ratio
omega_d = omega*sqrt(1-zeta^2)  % damped natural frequency

%% System performance criteria (SPC)
t_r = (pi - atan(sqrt(1-zeta^2)/zeta))/omega_d
T = 2*pi/omega_d
t_p = pi/omega_d
A_p = K*(1 + exp(-pi*zeta/sqrt(1-zeta^2)))
t_s = 3.0/(zeta*omega)          % 5% settling time
N = t_s/T

%% System response
step(G)
set(gca, 'FontSize', 20)
xlabel('Time', 'FontSize', 30);
ylabel('Output, y(t)', 'FontSize', 30);
```

```
grid on;
```

After clicking **Run**, the following values appear in the Command Window, and they agree with Example 2:

```
omega = 2.8284    zeta = 0.4243    omega_d = 2.5612
t_r = 0.7844      T = 2.4532       t_p = 1.2266
A_p = 0.6147      t_s = 2.5000     N = 1.0191
```

MATLAB's `stepinfo(G)` command can also be used to measure the rise time, overshoot and settling time directly from the simulated response. Note that `stepinfo` defines rise time as 10% to 90% of the final value by default, so its value differs from Equation (8).

7.0 Exercises

Exercise 1: consider the following second-order transfer functions, each subject to a unit step input:

a) $G(s) = \frac{4}{s^2+3.2s+4}$

b) $G(s) = \frac{9}{s^2+4.02s+9}$

c) $G(s) = \frac{4}{s^2+4.7s+5}$

For each transfer function:

i. determine the rise time, peak time and periodic time, peak amplitude, 10% settling time and number of oscillations before settling, where these apply

ii. sketch and label the response

Use the MATLAB script in Section 6.0 to check your answers. A step-by-step video solution can be found on YouTube. Answers are given in Appendix F.

Exercise 2: using MATLAB, determine the SPC of the closed-loop DC motor angle control system of Equation (1) for $K_p = 0.5$, 1.0 and 1.5 (the same values as in Figure 1). Tabulate the SPC for the three values and comment on the results. A fully worked solution is given in Appendix E.

8.0 Summary

This Learning Guide considered the transient and steady-state response of second-order transfer functions to step inputs. Through practical examples and the use of MATLAB and Simulink, it was shown how the system coefficients and the proportional control gain, K_p, affect the response.

The standard form introduced three parameters. The system gain, K, sets the final value; the natural frequency, ω_n, sets the speed of the response; and the damping ratio, ζ, determines whether the response is underdamped, critically damped or overdamped. For a closed-loop control system with proportional

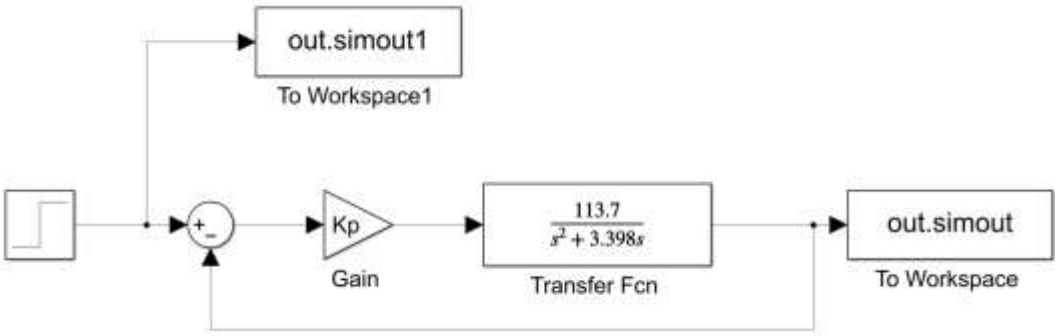
control, increasing K_p reduces the SSE but also reduces the damping ratio, giving a faster but more oscillatory response.

The system performance criteria (rise time, peak time, peak amplitude, settling time and number of oscillations) were then derived from ω_n and ζ. They allow the key features of an oscillatory response to be calculated and compared with the requirements of a control system, such as a maximum overshoot or settling time. The key progression introduced in this guide can be summarised as:

transfer function → standard form → K, ω_n and ζ → system performance criteria

Appendix A: DC Motor, Effect of the Proportional Control Gain

Simulink diagram:



MATLAB Script: simulating the DC motor for several values of K_p

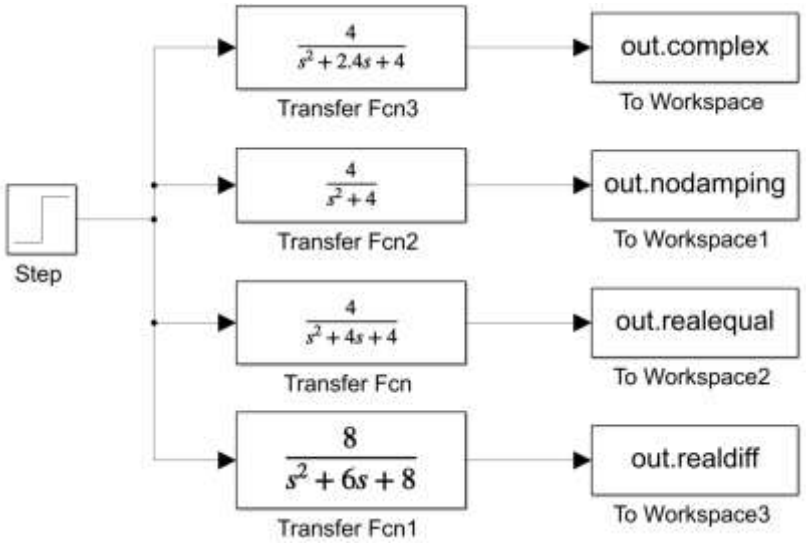
```
% MATLAB version 2021a used to create this script
clear; clc; close all;

%% Graphical properties
figure; hold on; grid on;
xlabel('Time [s]');
ylabel('Angle [degrees]');
set(gca, 'FontSize', 30);
idoitonce = 1;

%% Simulate a range of proportional control gains
for Kp = 0.5:0.5:1.5
    t = 5;        % simulation time
    sim('dc_motor_prop_effect')
    if idoitonce  % plot the reference once
        plot(ans.tout, ans.simout1(:,1), '-.');
        idoitonce = 0;
    end
    plot(ans.tout, ans.simout(:,1));
end
legend('Reference', 'K_p = 0.5', 'K_p = 1.0', 'K_p = 1.5')
```

Appendix B: Effect of the Damping Ratio

Simulink diagram:



MATLAB Script: simulating the four response types

```
% MATLAB version 2021a used to create this script
clear; clc; close all;

%% Simulink
sim('damping_investigation')

%% Graphical output
plot(ans.tout, ans.complex, ans.tout, ans.nodamping, ...
     ans.tout, ans.realequal, ans.tout, ans.realdiff)
xlabel('Time [s]');
ylabel('Output, y(t)');
set(gca, 'FontSize', 30);
grid on;
legend('Underdamped, {\zeta} < 1', 'Undamped, {\zeta} = 0', ...
       'Critically damped, {\zeta} = 1', ...
       'Overdamped, {\zeta} > 1')
```

Appendix C: Effect of the Natural Frequency

MATLAB Script: step responses for several natural frequencies

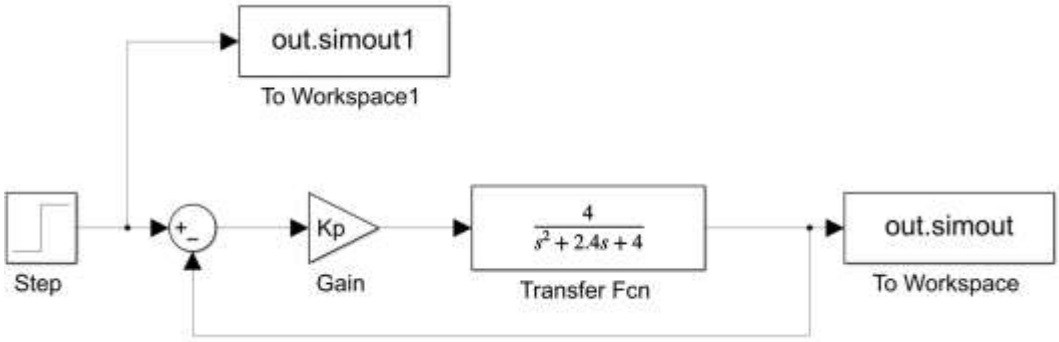
```
clear; clc; close all;

for w = 1:1:7  % natural frequency [rad/s]
    z = 0.6;   % damping ratio
    K = 1;     % system gain
    sys = tf(K*w^2, [1 2*z*w w^2]);
    step(sys); hold all
end
set(gca, 'FontSize', 20)
xlabel('Time', 'FontSize', 30);
```

```
ylabel('Output, y(t)', 'FontSize', 30);
grid on;
legend('{\omega_n} = 1', '{\omega_n} = 2', '{\omega_n} = 3', ...
       '{\omega_n} = 4', '{\omega_n} = 5', '{\omega_n} = 6', ...
       '{\omega_n} = 7')
```

Appendix D: Effect of the Proportional Control Gain

Simulink diagram:



MATLAB Script: simulating the closed-loop system for several values of K_p

```
% MATLAB version 2021a used to create this script
clear; clc; close all;

%% Graphical properties
figure; hold on; grid on;
xlabel('Time [s]');
ylabel('Output, y(t)');
set(gca, 'FontSize', 30);
idoitonce = 1;

%% Simulate a range of proportional control gains
for Kp = 1:1:7
    t = 13;       % simulation time
    sim('prop_effect')
    if idoitonce  % plot the reference once
        plot(ans.tout, ans.simout1(:,1), '-.');
        idoitonce = 0;
    end
    plot(ans.tout, ans.simout(:,1));
end
legend('Reference', 'K_p = 1', 'K_p = 2', 'K_p = 3', ...
       'K_p = 4', ...
       'K_p = 5', 'K_p = 6', 'K_p = 7')
```

Appendix E: Solution to Exercise 2

The MATLAB script in Section 6.0 can be adapted for each value of K_p. For example, for $K_p = 1.5$:

MATLAB Script: SPC of the DC motor angle control system for $K_p = 1.5$

```
clear; clc; close all;

Kp = 1.5;                          % proportional control gain
alpha = 3.398; beta = Kp*113.7;    % closed-loop coefficients
K = 1;                             % system gain (unit step)
G_CL = tf(K*beta, [1 alpha beta]);

omega = sqrt(beta)                 % natural frequency
zeta = alpha/(2*omega)             % damping ratio
omega_d = omega*sqrt(1-zeta^2)     % damped natural frequency

t_r = (pi - atan(sqrt(1-zeta^2)/zeta))/omega_d
T = 2*pi/omega_d
t_p = pi/omega_d
A_p = K*(1 + exp(-pi*zeta/sqrt(1-zeta^2)))
t_s = 3.0/(zeta*omega)             % 5% settling time
N = t_s/T

step(G_CL); grid on;
```

The script assumes a unit step reference; for the 180° reference, the peak angles are 180 times the peak amplitudes below. The results are summarised in Table 5.

Table 5: *SPC of the DC Motor Angle Control System for Three Values of K_p*

SPC	$K_p = 0.5$	$K_p = 1.0$	$K_p = 1.5$
Damping ratio, ζ	0.225	0.159	0.130
Rise time, t_r [s]	0.245	0.164	0.131
Peak time, t_p [s]	0.428	0.298	0.243
Periodic time, T [s]	0.855	0.597	0.485
Peak amplitude, A_p	1.48	1.60	1.66
Settling time (5%), t_s [s]	1.77	1.77	1.77
Number of oscillations, N	2.06	2.96	3.64

As K_p increases, the rise time, peak time and periodic time all reduce, so the response becomes faster, but the peak amplitude increases. The 5% settling time does not change, because $\zeta\omega_n = \alpha/2 = 1.699$ is the same for every value of K_p. However, the number of oscillations before settling increases, so the response oscillates more.

Appendix F: Answers to Exercise 1

Table 6: *Answers to Exercise 1 (10% Settling Time)*

Quantity			
System gain, K	1	1	0.8
Natural frequency, ω_n [rad/s]	2.00	3.00	2.24
Damping ratio, ζ	0.80	0.67	1.05
Damped natural frequency, ω_d [rad/s]	1.20	2.23	–
Rise time, t_r [s]	2.08	1.03	–
Peak time, t_p [s]	2.62	1.41	–
Periodic time, T [s]	5.24	2.82	–
Peak amplitude, A_p	1.015	1.059	–
Settling time (10%), t_s [s]	1.44	1.15	1.86
Number of oscillations, N	0.27	0.41	–

Transfer function (c) has $\zeta = 1.05 > 1$, so it is overdamped: its poles are real ($s = -1.63$ and $s = -3.07$), there is no overshoot and no oscillation, and Equations (8) to (14) do not apply. Its 10% settling time (1.86 s) has been found from the simulated response, for example using `stepinfo(G, 'SettlingTimeThreshold', 0.1)`. For (a) and (b), the damping is high enough that the overshoot is very small (1.5% and 5.9%), so the envelope-based settling time is only an estimate.

Recommended Citation

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