Stability and the Routh-Hurwitz Stability Criterion

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1.0 Learning Outcomes

After studying this ACE-Lab learning guide, you should be able to demonstrate the Learning Outcomes listed in Table 1. On completion of the ACE-Lab learning guide, revisit Table 1 and confirm whether each Learning Outcome has been achieved.

Table 1: *Learning Outcomes from the Routh-Hurwitz Stability Criterion Learning Guide*

ID	Learning Outcome	Achieved
1.	Explain the importance of stability and the consequences of an unstable closed-loop control system, which can include system failure, destruction and even fatality.	☐
2.	Explain how the relative degree of the open-loop system indicates whether a closed-loop control system with a proportional control gain K_p can become unstable.	☐
3.	Apply the Routh-Hurwitz stability criterion to determine whether a closed-loop control system is stable.	☐
4.	Apply the Routh-Hurwitz stability criterion to determine the range of proportional control gain K_p values that gives a stable closed-loop control system and check the result using MATLAB.	☐

2.0 Introduction

The complex s-plane provides a visual method of plotting the poles and zeros of a closed-loop control system, and this graphical approach can be used to design a control system. The poles of a transfer function must be located in the left-half of the s-plane for a system to be stable, and that the location of the poles determines the system response. For example, a damping ratio of 0.6 gives a peak overshoot of approximately 10%.

This Learning Guide details the Routh-Hurwitz stability criterion. The criterion determines how many poles of the closed-loop transfer function lie in the unstable region of the s-plane, the right-half plane, without the poles having to be calculated. Knowing this allows the stability of the control system to be determined. The criterion is particularly useful when the relative degree of the open-loop system is greater than 2, and it can be used to find the effect of the proportional control gain K_p on the closed-loop control system, that is, the range of K_p values for which the system is stable. An important question therefore arises:

For which values of the controller gain does a closed-loop control system remain stable?

Why is the Routh-Hurwitz Stability Criterion Important?

The main reasons why the Routh-Hurwitz stability criterion is important are:

i. The method allows a closed-loop transfer function to be evaluated in terms of stability, that is, whether the closed-loop system response will be stable;
ii. The method shows whether a closed-loop control system containing a proportional control gain K_p will become unstable for a given value of K_p, which depends on the relative degree;
iii. Stability is of major importance at the design stage. In a model-based design approach, instability must be avoided, and this is checked by testing a range of conditions, that is, by exploring "what-if" scenarios.

3.0 Stability and the s-Plane

The poles of a transfer function are the roots of its denominator. As shown in Figure 1, poles with a negative real part lie in the left-half of the s-plane, the stable region, and give a response that settles to a steady state. Poles with a positive real part lie in the right-half plane, the unstable region. Poles on the imaginary axis give a sustained oscillation, and the system is then described as marginally stable.

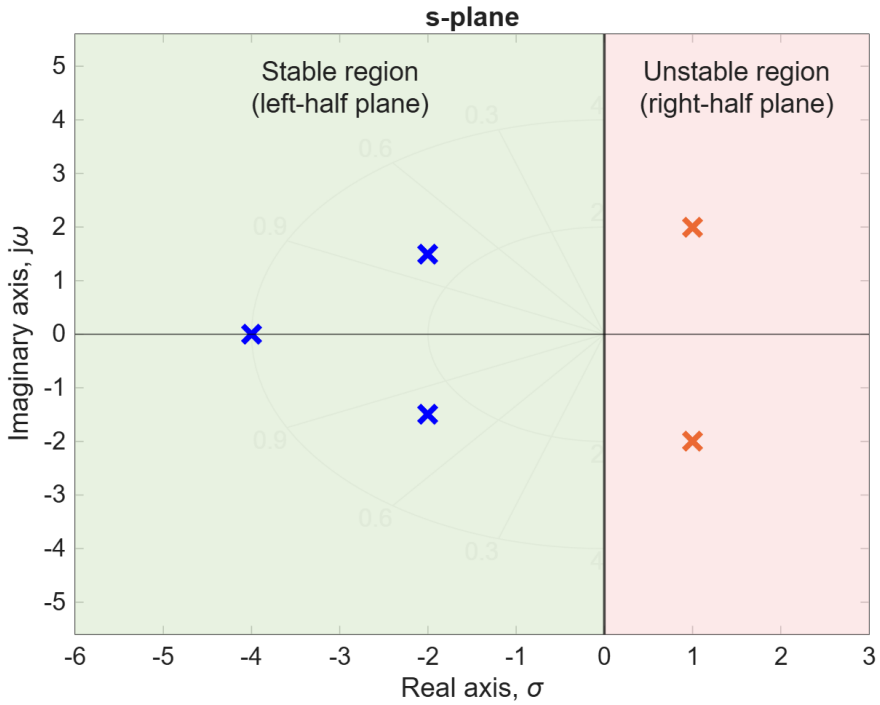


Figure 1: *The s-Plane, Showing the Stable and Unstable Regions and Lines of Constant Damping Ratio and Natural Frequency*

Unstable Systems

The responses of first- and second-order systems considered previously were for stable closed-loop transfer functions, with poles in the left-half of the s-plane. Many open-loop systems are stable by design, and the closed-loop control system is then used to achieve a set of requirements, that is, reference tracking and system performance requirements. Some systems, however, are designed to be open-loop unstable, with the closed-loop control providing stabilisation; fighter aircraft are an example.

Because of design errors, a closed-loop control system can be unstable, or can become unstable under certain loading conditions, for example disturbances or changes in operating conditions. Two examples are shown in Figure 2: the failure of a wind turbine, and the Boeing 737 MAX aircraft accidents in Figure 3.



Figure 2: *Example of an Unstable System: Wind Turbine Failure (video: https://www.youtube.com/watch?v=oAWMpxX60KM)*



Figure 3: *Example of a Control Design Problem: the Boeing 737 MAX Accidents (video: https://www.youtube.com/watch?v=H2tuKiiznsY)*

A model-based design approach provides an understanding of the system performance and stability. The control method can then be introduced and configured to achieve a list of design requirements, and "what-if" scenarios can be tested to ensure that the system will not fail under the expected loading conditions. This is very important, as an unstable system can be costly and, in some cases, fatal.

An unstable closed-loop transfer function has one or more poles in the right-half of the s-plane. Only one such pole is needed for the system to be unstable. The system response then does not reach a steady state, and instead continues to grow in magnitude, which in engineering terms can lead to failure or self-destruction of the system.

4.0 The Routh-Hurwitz Stability Criterion

Consider the closed-loop control system in Figure 4, which contains a proportional control gain K_p, an actuator, a process and an ideal sensor.

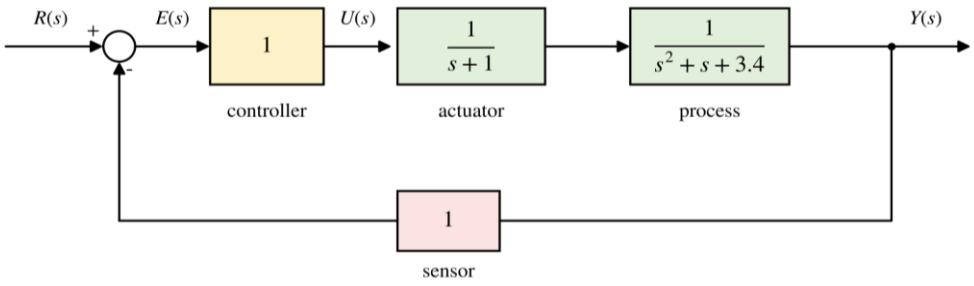


Figure 4: *Closed-Loop Control System with a Proportional Control Gain (here $K_p = 1$)*

With negative unity feedback, the general closed-loop transfer function is given by:

$$G_{CL}(s) = \frac{K_p N(s)}{D(s) + K_p N(s)} \tag{1}$$

where $G(s) = N(s)/D(s)$ is the open-loop system transfer function, here the product of the actuator and process transfer functions. For the system in Figure 4, with $K_p = 1$, this gives:

$$G_{CL}(s) = \frac{1}{s^3 + 2s^2 + 4.4s + 4.4} \tag{2}$$

In practice, components such as actuators and processes are modelled using combinations of first- and second-order transfer functions, as in Figure 4, so the overall transfer function is of a higher order, here third order. In industry, for example in automotive, aerospace, defence and manufacturing processes, an overall transfer function can be of the order of 100, with many sub-systems and sub-components.

Necessary Conditions

Before the Routh array is formed, the coefficients of the characteristic equation are checked against two necessary conditions:

i. All the coefficients of the characteristic equation (the denominator of the closed-loop transfer function) are present, that is, non-zero;
ii. All the coefficients have the same sign, either all positive or all negative.

If either condition fails, the system is not stable, and no further test is needed. If both are met, the system may still be unstable, and the full criterion must be applied. Equation (2) has all coefficients present and positive, so the Routh-Hurwitz stability criterion can be applied.

Relative Degree

The relative degree of the open-loop transfer function indicates whether instability may occur when the proportional control gain K_p is increased. It is given by:

$$rd = n - m \tag{3}$$

where n is the number of poles and m is the number of zeros of the open-loop system. This applies when the controller is a proportional gain K_p only, since this introduces no additional poles or zeros.

A relative degree of 2, for example two poles and no zeros, gives a closed-loop control system that is stable for all values of K_p. This is shown in Figure 5: as K_p increases, only the imaginary parts of the poles change, and their real part stays the same. The system is therefore described as unconditionally stable. The MATLAB code used to generate Figure 5 is given in Appendix A.

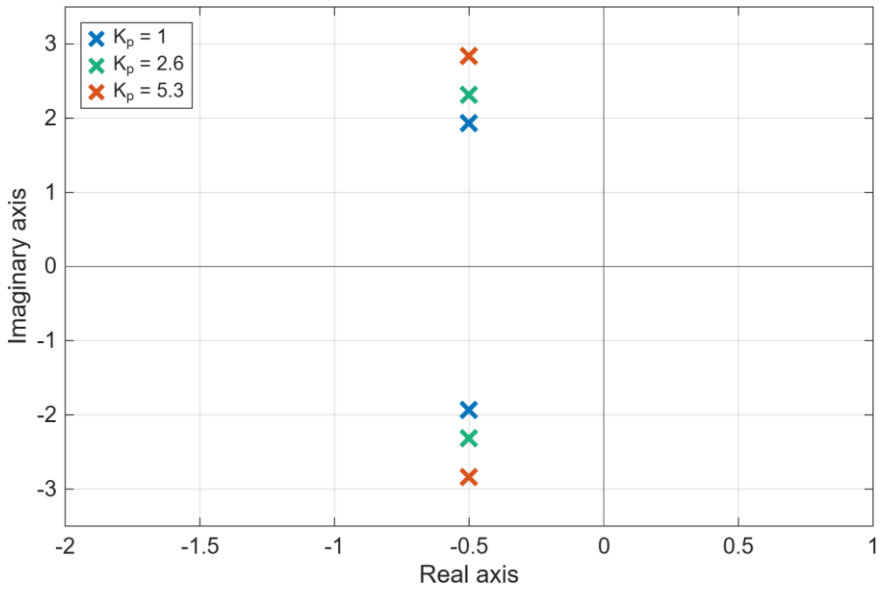


Figure 5: *Pole Locations as the Proportional Control Gain K_p is Increased, for a Transfer Function with a Relative Degree of 2*

A relative degree of 3, for example three poles and no zeros, gives a closed-loop control system that becomes unstable for a sufficiently large K_p: two of the poles move across into the right-half of the s-plane. It is therefore important to determine the value of K_p that gives a marginally stable system, with two poles on the imaginary axis, so that values of K_p greater than or equal to this are not selected. This is shown in Figure 6. One pole moves along the negative real axis, while the other two form a complex conjugate pair that crosses the imaginary axis. The three branches approach asymptotes that are 120° apart. The MATLAB code used to generate Figure 6 is given in Appendix A.

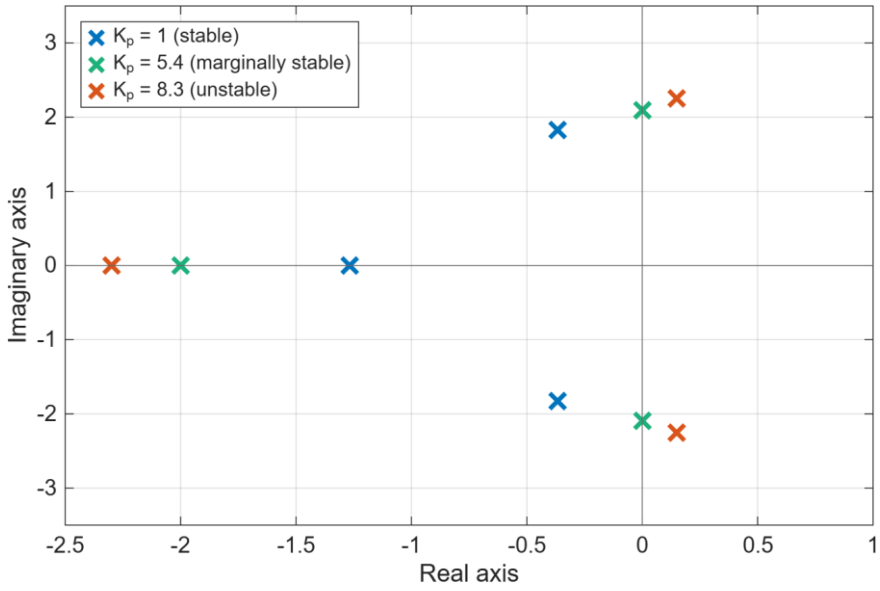


Figure 6: *Pole Locations as the Proportional Control Gain K_p is Increased, for a Transfer Function with a Relative Degree of 3*

Characteristic Equation

The characteristic equation is obtained by setting the denominator of the closed-loop transfer function equal to zero. From Equation (1), for a closed-loop control system containing a proportional control gain K_p:

$$D(s) + K_p N(s) = 0 \tag{4}$$

The characteristic equation therefore contains both the numerator and denominator of the open-loop transfer function. From Equation (2), with $K_p = 1$:

$$s^3 + 2s^2 + 4.4s + 4.4 = 0 \tag{5}$$

This equation determines the stability of the closed-loop control system, and it is the starting point of the Routh-Hurwitz stability criterion.

The Routh Array

For an nth-order closed-loop transfer function, the characteristic equation is:

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \cdots + a_1 s + a_0 = 0 \tag{6}$$

The corresponding Routh array is given in Table 2. The first two rows are filled directly with alternate coefficients of the characteristic equation, and each later row is calculated from the two rows above it.

Table 2: *General Form of the Routh Array*

Row	Column 1	Column 2	Column 3	Column 4
s^n	a_n	a_{n-2}	a_{n-4}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
s^{n-2}	b_1	b_2	b_3	$\cdots$
s^{n-3}	c_1	c_2	c_3	$\cdots$
$\vdots$	$\vdots$	$\vdots$		
s^1	l_1			
s^0	m_1			

For a third-order system, the characteristic equation is:

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \tag{7}$$

and the Routh array is given in Table 3.

Table 3: *Routh Array for a Third-Order Characteristic Equation*

Row	Column 1	Column 2	Column 3
s^3	a_3	a_1	0
s^2	a_2	a_0	0
s^1	b_1	b_2	
s^0	c_1		

The coefficients a_3, a_2, a_1 and a_0 are taken directly from the characteristic equation. The coefficient b_1 is calculated from:

$$b_1 = \frac{1}{-a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \tag{8}$$

where a_{n-1} is termed the pivotal element and | | denotes the determinant. The coefficient b_2 uses the same pivotal element and first column, with the next pair of coefficients:

$$b_2 = \frac{1}{-a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} \tag{9}$$

For a third-order system, a_{n-4} and a_{n-5} are both zero, so the determinant is zero and $b_2 = 0$. In the next row, b_1 becomes the pivotal element and the first row of the array is no longer used:

$$c_1 = \frac{1}{-b_1}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix} \tag{10}$$

For a third-order system, with $b_2 = 0$, this always gives $c_1 = a_0$. Once the array is complete, the test is applied to the first column, the pivotal column:

The number of sign changes in the first column of the Routh array equals the number of closed-loop poles in the right-half of the s-plane.

One sign change therefore means one unstable pole, two sign changes two unstable poles, and so on. The system is stable only if there are no sign changes.

Example 1: Stability of a third-order closed-loop control system

The closed-loop control system in Figure 4 has the closed-loop transfer function denominator $s^3 + 2s^2 + 4.4s + 4.4$. Determine:

i. the characteristic equation;

ii. whether the system is stable, using the Routh-Hurwitz stability criterion.

Solution:

(i) The characteristic equation is:

$$s^3 + 2s^2 + 4.4s + 4.4 = 0$$

(ii) Both necessary conditions are met. The first two rows of the Routh array are filled from the coefficients, $a_3 = 1$, $a_2 = 2$, $a_1 = 4.4$ and $a_0 = 4.4$, and the remaining coefficients are calculated:

$$b_1 = \frac{1}{-2}\begin{vmatrix} 1 & 4.4 \\ 2 & 4.4 \end{vmatrix} = \frac{(1)(4.4) - (4.4)(2)}{-2} = \frac{-4.4}{-2} = 2.2$$

$$b_2 = \frac{1}{-2}\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0, \qquad c_1 = \frac{1}{-2.2}\begin{vmatrix} 2 & 4.4 \\ 2.2 & 0 \end{vmatrix} = \frac{(2)(0) - (4.4)(2.2)}{-2.2} = 4.4$$

The completed Routh array is:

s^3	1	4.4	0
s^2	2	4.4	0
s^1	2.2	0	
s^0	4.4		

There are no sign changes in the first column (1, 2, 2.2, 4.4), so there are no poles in the right-half plane, and the closed-loop control system is **stable**. As expected for a third-order system, $b_2 = 0$ and $c_1 = a_0$; they are calculated here for completeness.

Using MATLAB to Check Example 1

MATLAB can be used to check the solution to Example 1. MATLAB Script 1 calculates the pivotal column of the Routh array for a third-order system, and MATLAB Script 2 calculates the closed-loop poles and plots them on the s-plane.

MATLAB Script 1: calculation of the pivotal column of the Routh array for a third-order closed-loop transfer function

```
clear; clc; close all;
%% Routh-Hurwitz stability criterion (3rd order)
a3 = 1;
a2 = 2;
a1 = 4.4;
a0 = 4.4;
m1 = [a3 a1; a2 a0];   % 2 x 2 matrix
b1 = (1/-a2)*det(m1);  % pivotal element multiplied by the determinant
c1 = a0;
check = [a3 a2 b1 c1]'   % pivotal column
```

Output:

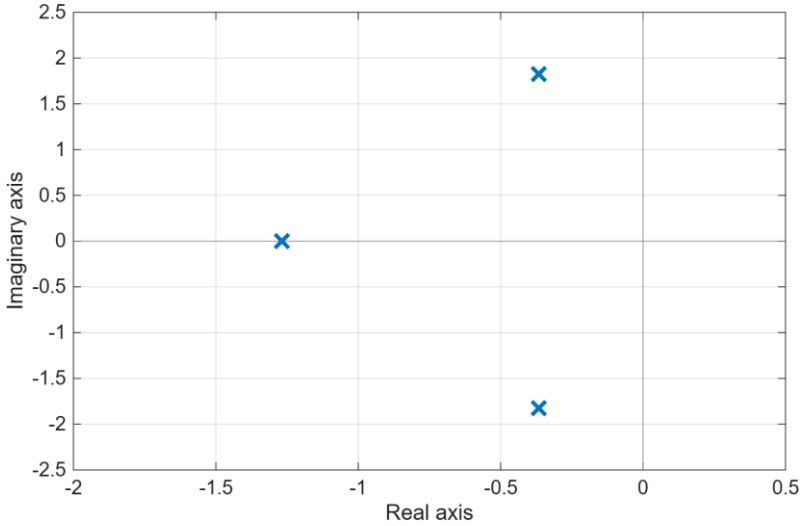
```
check =
    1.0000
    2.0000
    2.2000
    4.4000
```

MATLAB Script 2: closed-loop poles and their location on the s-plane, an alternative check of stability

```
clear; clc; close all;
%% Closed-loop control system transfer function
num = [1];              % numerator coefficients
den = [1 2 4.4 4.4];    % denominator coefficients
G_CL = tf(num,den)
%% s-plane pole-zero map
pzmap(G_CL)
grid on
%% Poles
pole(G_CL)
```

Output:

```
ans =
  -0.3663 + 1.8268i
  -0.3663 - 1.8268i
  -1.2675 + 0.0000i
```



All three poles have negative real parts, which confirms that the system is stable.

5.0 Stability Range of the Proportional Control Gain

A closed-loop control system containing a proportional control gain K_p, with a relative degree of 3 or more, can become unstable as K_p is increased. The Routh-Hurwitz stability criterion can find the range of K_p values that gives a stable system. This is done by keeping K_p as a symbol, with no numerical value, in the closed-loop transfer function. The characteristic equation and the Routh array then contain K_p, and simple algebra gives the range of K_p for which there are no sign changes in the first column.

Example 2: Range of proportional control gain for stability

For the closed-loop control system in Figure 4 with a general gain K_p, the closed-loop transfer function is:

$$G_{CL}(s) = \frac{K_p}{s^3 + 2s^2 + 4.4s + \left(3.4 + K_p\right)}$$

Determine:

i. the characteristic equation;

ii. the range of K_p values that gives a stable system, using the Routh-Hurwitz stability criterion.

Solution:

(i) The characteristic equation is:

$$s^3 + 2s^2 + 4.4s + \left(3.4 + K_p\right) = 0$$

(ii) The first two rows are filled from the coefficients, and b_1 is calculated:

$$b_1 = \frac{1}{-2}\Big((1)\left(3.4 + K_p\right) - (2)(4.4)\Big) = \frac{K_p - 5.4}{-2} = \frac{5.4 - K_p}{2}$$

With $b_2 = 0$ and $c_1 = a_0 = 3.4 + K_p$, the completed Routh array is:

$$\begin{array}{llll} s^3 & 1 & 4.4 & 0 \\ s^2 & 2 & 3.4 + K_p & 0 \\ s^1 & \dfrac{5.4 - K_p}{2} & 0 & \\ s^0 & 3.4 + K_p & & \end{array}$$

For no sign changes in the first column, every element must be positive:

$$\frac{5.4 - K_p}{2} > 0 \;\Rightarrow\; K_p < 5.4, \qquad 3.4 + K_p > 0 \;\Rightarrow\; K_p > -3.4$$

The second condition is satisfied for all positive gains. The system is therefore stable for $0 < K_p < 5.4$. At $K_p = 5.4$ the first column contains a zero, and the system is marginally stable, with a pair of poles on the imaginary axis.

Using MATLAB to Check Example 2

MATLAB Script 3 plots the closed-loop poles for three values of K_p: within the stable range, at the limit and above it.

MATLAB Script 3: location of the closed-loop poles for different values of the proportional control gain K_p

```
clear; clc; close all;
%% Closed-loop transfer function 1: stable
Kp1 = 1;
G_CL1 = tf(Kp1,[1 2 4.4 3.4+Kp1]);
%% Closed-loop transfer function 2: marginally stable
Kp2 = 5.4;
G_CL2 = tf(Kp2,[1 2 4.4 3.4+Kp2]);
%% Closed-loop transfer function 3: unstable
Kp3 = 8.3;
G_CL3 = tf(Kp3,[1 2 4.4 3.4+Kp3]);
%% s-plane pole-zero map
```

```
pzmap(G_CL1,'b',G_CL2,'g',G_CL3,'r')
grid on
```

Output:

The output is the pole map in Figure 6. With $K_p = 5.4$, two poles lie on the imaginary axis at $s = \pm j2.098$; with $K_p = 8.3$, they lie in the right-half plane at $s = 0.150 \pm j2.251$.

6.0 Exercises

Attempt the following exercises. For each, write down the characteristic equation of the closed-loop control system and apply the Routh-Hurwitz stability criterion. MATLAB may be used to check the answers.

a) The closed-loop transfer function of a control system (with no disturbance) is $G_{CL}(s) = \frac{2}{s^3+s^2+3s+8}$. Determine whether the system is stable.

b) The closed-loop transfer function of a control system is $G_{CL}(s) = \frac{3}{s^3+2s^2+4s+7}$. Determine whether the system is stable.

c) For the closed-loop control system in Figure 7, with a proportional control gain $K_p = 1.1$ and an ideal sensor, determine whether the system is stable.

d) A closed-loop control system with negative unity feedback and a proportional controller K_p has an actuator $G_a(s) = \frac{1}{s+1}$ and a process $G_p(s) = \frac{1}{s^2+2s+1}$. Determine the range of K_p values that gives a stable system.

e) Repeat part (d) with $G_a(s) = \frac{1}{s+1}$ and $G_p(s) = \frac{6}{s^2+2s+6}$.

f) Repeat part (d) with $G_a(s) = \frac{1}{0.01s+1}$ and $G_p(s) = \frac{10}{2s^2+8s+10}$.

g) For the closed-loop control system in Figure 8, a unit step reference is applied and the system experiences a disturbance of $-0.1/s$. With an initial proportional control gain of $K_p = 1.2$, determine whether the system is stable.

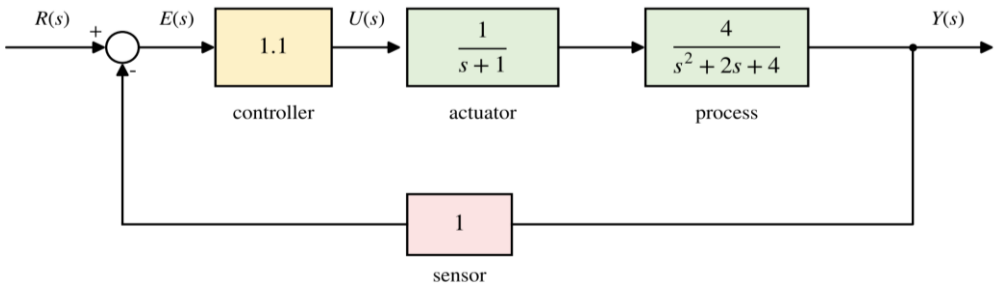


Figure 7: *Closed-Loop Control System for Exercise (c)*

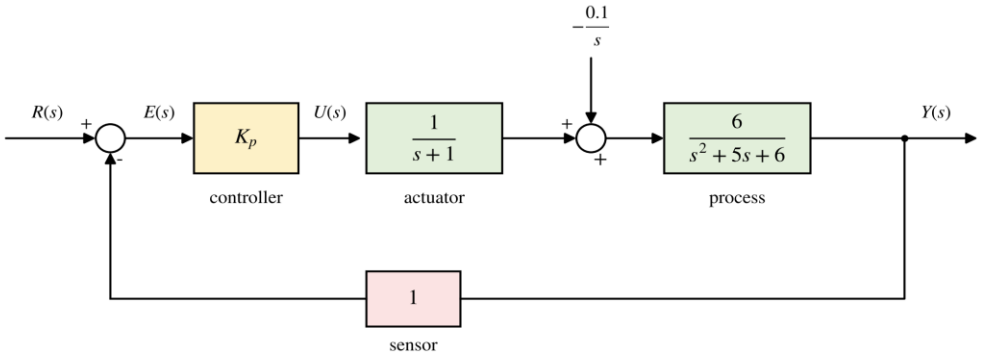


Figure 8: *Closed-Loop Control System for Exercise (g)*

Fully worked solutions to the above exercises can be found in **Appendix B**.

7.0 Summary

This Learning Guide detailed the importance of closed-loop control system stability. A system is stable when all of its closed-loop poles lie in the left-half of the s-plane; a single pole in the right-half plane makes it unstable, and the response then grows.

The Routh-Hurwitz stability criterion determines how many poles lie in the right-half plane without calculating them. The characteristic equation, the denominator of the closed-loop transfer function set equal to zero, is checked against the necessary conditions, and its coefficients are used to fill the Routh array. The number of sign changes in the first column equals the number of right-half plane poles.

The test is especially important when the relative degree is 3 or more, because a large enough proportional control gain K_p then moves poles into the right-half plane. Keeping K_p as a symbol in the Routh array gives the range of gains for which the system remains stable. MATLAB can be used to check the hand calculations.

Appendix A: MATLAB Code for Figures 4 and 5

MATLAB Script 4: pole locations for a relative degree of 2 (Figure 5)

```
clear; clc; close all;
Kp = [1 2.6 5.3];
col = {'b','g','r'};
for i = 1:3
    G_CL = tf(Kp(i),[1 1 3+Kp(i)]);
    pzmap(G_CL,col{i}); hold on
end
grid on
legend('K_p = 1','K_p = 2.6','K_p = 5.3')
```

MATLAB Script 5: pole locations for a relative degree of 3 (Figure 6)

```
clear; clc; close all;
Kp = [1 5.4 8.3];
col = {'b','g','r'};
for i = 1:3
    G_CL = tf(Kp(i),[1 2 4.4 3.4+Kp(i)]);
    pzmap(G_CL,col{i}); hold on
end
grid on
legend('K_p = 1','K_p = 5.4','K_p = 8.3')
```

Appendix B: Fully Worked Solutions to Exercises

Question a:

The characteristic equation is $s^3 + s^2 + 3s + 8 = 0$. Calculating b_1:

$$b_1 = \frac{(1)(8) - (1)(3)}{-1} = -5$$

$$\begin{array}{cccc} s^3 & 1 & 3 & 0 \\ s^2 & 1 & 8 & 0 \\ s^1 & -5 & 0 & \\ s^0 & 8 & & \end{array}$$

The first column (1, 1, −5, 8) has two sign changes, so two poles lie in the right-half plane and the closed-loop control system is **unstable**.

Question b:

The characteristic equation is $s^3 + 2s^2 + 4s + 7 = 0$. Calculating b_1:

$$b_1 = \frac{(1)(7) - (2)(4)}{-2} = 0.5$$

$$\begin{array}{cccc} s^3 & 1 & 4 & 0 \\ s^2 & 2 & 7 & 0 \\ s^1 & 0.5 & 0 & \\ s^0 & 7 & & \end{array}$$

There are no sign changes in the first column, so the system is **stable**.

Question c:

Using the block diagram algebra form $G_{CL}(s) = \frac{N_a C N_p}{D_a D_p + H N_a C N_p}$, with $C(s) = 1.1$:

$$G_{CL}(s) = \frac{(1)(1.1)(4)}{(s+1)(s^2+2s+4) + (1)(1)(1.1)(4)} = \frac{4.4}{s^3 + 3s^2 + 6s + 8.4}$$

The characteristic equation is $s^3 + 3s^2 + 6s + 8.4 = 0$, and:

$$b_1 = \frac{(1)(8.4) - (3)(6)}{-3} = \frac{-9.6}{-3} = 3.2$$

$$\begin{array}{cccc} s^3 & 1 & 6 & 0 \\ s^2 & 3 & 8.4 & 0 \\ s^1 & 3.2 & 0 & \\ s^0 & 8.4 & & \end{array}$$

There are no sign changes, so the closed-loop control system is **stable**.

Question d:

$$G_{CL}(s) = \frac{K_p}{(s+1)(s^2+2s+1)+K_p} = \frac{K_p}{s^3+3s^2+3s+(1+K_p)}$$

The characteristic equation is $s^3+3s^2+3s+(1+K_p)=0$, and:

$$b_1 = \frac{(1)(1+K_p)-(3)(3)}{-3} = \frac{8-K_p}{3}$$

s^3	1	3	0
s^2	3	$1+K_p$	0
s^1	$\frac{8-K_p}{3}$	0	
s^0	$1+K_p$		

For no sign changes, $8-K_p>0$, so $K_p<8$; the last element $1+K_p$ is positive for all positive gains. The system is stable for $0<K_p<8$.

Question e:

$$G_{CL}(s) = \frac{6K_p}{(s+1)(s^2+2s+6)+6K_p} = \frac{6K_p}{s^3+3s^2+8s+(6+6K_p)}$$

The characteristic equation is $s^3+3s^2+8s+(6+6K_p)=0$, and:

$$b_1 = \frac{(1)(6+6K_p)-(3)(8)}{-3} = 6-2K_p$$

s^3	1	8	0
s^2	3	$6+6K_p$	0
s^1	$6-2K_p$	0	
s^0	$6+6K_p$		

For no sign changes, $6-2K_p>0$, so $K_p<3$. The system is stable for $0<K_p<3$.

Question f:

Each transfer function is first written with a unity leading coefficient in its denominator. Dividing the process by 2 and multiplying the actuator numerator and denominator by 100 gives:

$$G_p(s) = \frac{5}{s^2+4s+5}, \qquad G_a(s) = \frac{100}{s+100}$$

$$G_{CL}(s) = \frac{500K_p}{(s+100)(s^2+4s+5)+500K_p} = \frac{500K_p}{s^3+104s^2+405s+(500+500K_p)}$$

The characteristic equation is $s^3+104s^2+405s+(500+500K_p)=0$, and:

$$b_1 = \frac{(1)(500 + 500K_p) - (104)(405)}{-104} = 400.19 - 4.808K_p$$

s^3	1	405	0
s^2	104	$500 + 500K_p$	0
s^1	$400.19 - 4.808K_p$	0	
s^0	$500 + 500K_p$		

For no sign changes, $400.19 - 4.808K_p > 0$, so $K_p < 83.24$. The system is stable for $0 < K_p < 83.24$. The fast actuator (time constant 0.01 s) allows a much larger gain than in parts (d) and (e).

Question g:

The disturbance does not affect stability, which depends only on the characteristic equation. With $K_p = 1.2$:

$$G_{CL}(s) = \frac{(1)(1.2)(6)}{(s+1)(s^2+5s+6)+(1)(1)(1.2)(6)} = \frac{7.2}{s^3+6s^2+11s+13.2}$$

The characteristic equation is $s^3 + 6s^2 + 11s + 13.2 = 0$, and:

$$b_1 = \frac{(1)(13.2) - (6)(11)}{-6} = \frac{-52.8}{-6} = 8.8$$

s^3	1	11	0
s^2	6	13.2	0
s^1	8.8	0	
s^0	13.2		

There are no sign changes, so the closed-loop control system is **stable** with $K_p = 1.2$. (Keeping K_p general gives $b_1 = (60 - 6K_p)/6$, so the system is stable for $0 < K_p < 10$.)

Recommended Citation

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Pickering, J. E. (2026). Stability and the Routh-Hurwitz Stability Criterion. ACE-Lab. Teaching Notes: ACE-Lab Learning Guide, Version 1.0. https://doi.org/10.5281/zenodo.23511336