Covariant and contravariant tensor pdf

What is the difference between contravariants and covariant tensors, and why do they change differently when the coordinates change? When I first studied this material I was able to apply formulas, but was very confused by what concepts intuitively meant, and could not distinguish the difference between esoteric pathways called covariant and contravariant. Background Information The purpose of this article is pedagogical, and students often find the notion of tensor itself confusing and intimidating, so we explain it first. I assume that the reader already has a basic familiarity with tensor, so examples of what is and is not a tensor to help conceptual clarity. Tensor is a geometric object. This is a more physical requirement/interpretation. For example, the temperature in each place in the room is a scale (field), a specific case of tensor. The temperature at the point does not depend on the coordinates used. It does not take the value of the 1st coordinate to say, in each place, in arbitrary coordinates. This gives a number, but not a scalar. This has nothing to do with physical reality. (See Norton, Einstein Stumble? No 4.2 for vaguely related historical discussions.) Vectors are tensors. They are geometric objects and are transformed as expected. As See (2013, No.1.4) put it, Tensor is what is transformed as a tensor. Kristoffel's symbols are not tensor. They have the law of transformation when the coordinates change, but first, the transformation is not linear: tensor components are transformed linearly when the coordinates change. Tensors are linear. Mathematical definition: linear functionality from vectors (in tangent space at point) to realities: , or on double space: , or several copies of them. Double vectors are tensors. A familiar example of a double vector is a gradient. Imagine the temperature distribution mentioned above (Collier No.5.3.4.1). Given its (spatial) gradient, put the vector and it tells you a total temperature change in the direction of this vector. In other words, the gradient sends vectors to real numbers. But that's what a double vector does! Pseudotensors are not tensors. Like all our non-empires, they (inevitably) rely on a choice of coordinate system. Some of them are tensors before the orientation determined by the coordinates. More generally, the physical quantities were derived from them, for example, Einstein used a pseudo-sensor for gravitational field energy to make successful predictions about gravitational waves, however this was seen as a little doubtful. By the way, the terminology is contravariant, covariant and mixed for tensors from Ricci. The term tensor from Einstein and Grossman (Rosenfeld 1988, No8). Discussion Briefly, the counter-vavariant tensor turns opposite the covariant tensor, that is their transform back when the base changes. The counter-vavariant tensor has components written with raised indices, such as the 4-speed, while the components of the covariant tentor are written in lower indices, such as a metric. There are also tengors with mixed indices, for example, Riemann tensor is often given as . We'll get more precise definitions soon. But for now, pay attention to this terminology of contravariant and covariant is old-fashioned (Schutz No. 3.3 p.60). It is better to refer to raised and lowered indices (either at the top and bottom, as in Misner, Thorne and Wheeler 1973, No.3.2) or, more generally, type tensor (1.3) in the example of Riemann tensor above. From now on, I'm only assuming one index. In this case, the tenor with one index raised is called a vector, and a lowered index - a double vector, a 1-shape or perhaps a co-vector. Let's say we want to transform from coordinates to. Recall that if the 'a' vector has components in the original coordinates, its components are in the new coordinates. On the other hand, let's assume that the double vector 'b' has components in the original coordinates, then its converted components. (The Einstein Convention is supposed to be all over.) From the first formula, vector components undergo a linear transformation at each point given by a matrix of partial derivatives. On the other hand, the components of the dual vector are transformed (I rolled the indices). Recall, they are called Jacobian matrix and are reversed to each other, so we say that the components are transformed back. (I have to add an example.) We can raise and lower indices and. This was a source of confusion because it seemed that one object could be transformed in two different ways, depending on whether it was written or. It seemed the covariant and counter-variant conversion were very different concepts. But the resolution is simple, which corresponds to different objects, double vector and vector respectively. Yes, that confusing, both vector and its duality are usually written as well in notation without indices. This is because there is a natural correspondence between them, given the metric (see above, the rise and decrease of the formula). Notation is convenient, but you just have to avoid delusion. In index-free notation, Schutz (No.3)selects notation for the dual vector and for the vector, but this is not a standard convention, and in relativity → may imply a three-dimensional spatial vector in particular. One option is to just write in words... double vector a.... Given that tensor is a vector or a double vector, the law of transformation is defined. Vector components are re-transformed into dual vector components. (Besides, just to add more confusion, the components transform to transform at the base of the vectors. And the components of the double vectors are transformed back into the double base vectors.) This reverse transformation gives rise to the word double in double vector space. The conversion property with base vectors generates co in the covariant vector and its shorter shape of the cowector. Because the components of conventional vectors are transformed in front of the main vectors... they are often referred to as contravariant vectors. Most of these names are old-fashioned; vectors and double vectors or one form are modern names. The reason they have given up cooperation and dissent is because they mix two very different things: the transformation of the foundation is an expression of new vectors from the point of view of the old; converting components is an expression of the same object in terms of a new framework. (Schutz No.3.3) Some mathematical definitions are overdue, although they do belong elsewhere, and are only abbreviated resumes, not introductions. Recall that in curved geometry the vector exists only in the tangent space at a given point and does not extend from one point of diversity to another, as is often thought of in a flat space. Thus, the vector is an element of tangent space. The double vector displays vectors to real numbers, in other words, it is linear functionality on a tangent space. In addition, we can think of the vector as a double vector display (see Schutz No.3.6 on Circular Justification?). Recall that for this system of coordinates are indicated vectors of the coordinate base, as well as vectors of the double coordinate base with property. The final clarification, the modern consensus, seems to be that the term covariant has a broader meaning of any quantity that turns into a tensor, i.e. it is transformed appropriately with the coordination of change. Therefore, we can say that both vectors and double vectors, like all other tenors, are treacherous! Like everything, the meaning must be cleared out of context. Summary Instead of a counter-vavariant, say, raised indices, tensor type, or vector (in the case of a one-dimensional tentor). Instead of treacherous, say, lowered indices, tensor type, or double vector/covector/1-shape. The rise or decline of indices, for example, corresponds to different tengors. They are usually written with the same variable (here 'a') because the metric determines the natural correspondence between them. There is only one type of conversion, given a certain type of tentor. (Yes, maybe one could count on the conversion and its reverse as two types ... but I express it in a way that combats my previous delusion, see above). Use covariant to express that the quantity is tensor, meaning it is converted appropriately when changed It doesn't depend on whether its indices are raised, downgraded or mixed. Indexes. different, especially in old sources, but the meaning should be cleared out of context. Update: From further survey textbooks, obviously the terminology Schutz criticizes is very widespread. For example, the leading mathematical reference to differential geometry (O'Neill 1983, p.37) uses this language, although later books are worth checking out. Adding to the rise and decline of indices: when the tenors are not symmetrical, you need to be careful with the position of the index. For example, taking into account tensor, the decline of the first or second indices gives different (1.2)-tensors and (Schutz No.3.7). The references I drew were largely on the relativity of the Schutz tutorial, which is introductory but known for its accuracy and clear explanation. The author emphasizes the geometric nature of tensors, not the properties of transformation of their components (No.3.9, see also recommended sources). Many other sources will contain similar materials. Note Schutz No. 3 concerns only a special theory of relativity, but only to replace , Lorenz impulse arbitrary transformation of coordinates, etc. Collier (2012), The most incomprehensible thing: Notes to the very mild introduction to the mathematics of relativity has a lengthy pedagogical section in No.5.3 This topic refers to geometry in general, not just relativity in particular. So try also books on differential geometry, or field theory (such as electromagnetism), for example, and let me know about any useful ones. Those. covariant and contravariant tensor pdf. covariant and contravariant tensor ppt. difference between covariant and contravariant tensors. examples of covariant and contravariant tensors. define covariant and contravariant tensor. physical meaning of covariant and contravariant tensors. product of covariant and contravariant metric tensor. covariant and contravariant metric tensor