Homework 3 (Due Nov 15, 2026)

- Rookies: 37.4, 37.6
- Veterans: 37.4, 37.6, 48
- All stars: 37.4, 37.6, 45
- Hackers: 55.2
- Poets: 56

Problem 37.4 Assume the Axiom of Choice. Prove that, for every infinite set S, there exists an injective function $f : S \to S$ that is not surjective.

Note: It is not sufficient to choose an element $x \in S$ and simply assert that $|S| = |S \setminus \{x\}|$, thereby obtaining a bijection from S to the proper subset $S \setminus \{x\}$. If you wish to use this approach, you must first prove that $|S| = |S \setminus \{x\}|$.

Problem 37.6 Theorem 1.8.1 assumes that S has neither a maximum nor a minimum element. Does the conclusion of the theorem still hold if S is allowed to have a maximum element? Justify your answer.

Problem 55.2 With the help of an LLM, find a set-theoretic theorem, lemma, or proposition whose statement your fellow students can understand but would find difficult to believe at first glance. In other words, the result should be both comprehensible and counterintuitive.